INVESTIGATION OF FEL GAIN IN THE OPTICAL KLYSTRON CONFIGURATION WITH THE GPT SIMULATION CODE

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Abstract

A new element for the General Particle Tracer code, GPT, has been created to include the radiation provided by electrons in interaction with the electro-magnetic field. With this new element, which provides the evolution of the propagating longitudinal TEM_{00} modes as a function of time, the synchrotron radiation is simulated. Furthermore, the complete interaction between an electron bunch and a laser pulse leading to the laser amplification process can be simulated, solving over time the evolution of the distribution profiles of both the bunch and the pulse.

1 INTRODUCTION

An accurate evaluation of the gain of a Free Electron Laser (FEL) is important in order to understand the dynamics of this system and to predict the laser characteristics. This evaluation can be done with a full numerical approach and also by means of an analytical expression. These two methods are complementary. The analytical method is valid up to a certain limit determined by the approximations used, but this method gives a deep insight into the physics it describes. An analytical expression of the gain exists for an FEL with an undulator [1] and has been tested accurately. An expression of the FEL gain in the case of an optical klystron has been given [2] and is valid in the small signal gain regime. In principle for the numerical approach one may start from first principles, so the domain of applications often extends beyond what is covered by analytical theory. But this method is often CPU-time consuming. With the General Particle Tracer code (GPT) the numerical approach has been chosen. GPT is a tracking code that solves the equations of motion of charged particles. We have implemented a new element in the code which couples the equations of motion with a differential equation for the electromagnetic field [3]. Thus the energy exchange from the electro-magnetic point of view is taken into account, and the interaction between charged particles and electro-magnetic field is complete.
We use GPT to study the gain of a FEL in two cases, the undulator case, in order to validate the numerical approach, and the optical klystron case. In the first part we present the FEL simulations with the GPT code, and we analyze the FEL spontaneous emission obtained from the simulations. In the second part we analyze the FEL gain for the case of the optical klystron, taking into account the optical klystron parameters as retrieved from the spontaneous emission analysis.

2 GPT FEL SIMULATIONS

The GPT code is a 3D simulation platform for the study of charged particle dynamics in electromagnetic fields [4]. The code solves the equations of motion for each particle, taking into account the electromagnetic field felt by the particle. The results are in a file containing the time evolution of the phase space coordinates of each particle and the electro-magnetic field felt by the particle. The electromagnetic field radiated or absorbed by each particle is calculated by a differential equation which is coupled to the equations of motion. We derived this differential equation assuming the propagating electro-magnetic field is described by the TEM_{00q} expression given by Kogelnick [5]. The electro-magnetic field is the sum of the fields of each longitudinal propagating mode q, where q is associated with the q^{th} eigen mode of a given optical cavity. The result, of which an example is shown in figure 1, is the frequency spectrum of the propagating electro-magnetic field. Using that code, it is then possible to make a one pass FEL simulation, taking a specific design of a FEL with its specific optical cavity and undulator characteristics. The first step is to validate the simulation of the spontaneous emission. Taking an undulator and varying its parameters, the calculated radiated field should compare with the known analytical expression [2]:

$$I_{sp,und} = \frac{e^2}{4\pi^2 c}\left(\frac{KN\lambda_u}{\gamma}\right)^2 (JJ(\xi))^2 \left(\frac{\nu}{c}\right)^2 \left(\frac{\sin(\delta)}{\delta}\right)^2, \tag{1}$$

where e is the electron charge, c the speed of light in vacuum, K the undulator strength, λ_u the undulator period length, γ the Lorentz factor of the electron; $JJ(\xi)$ is the difference between the Bessel functions of order 0 and 1 with the argument $\xi = \frac{K^2}{4+2K^2}$, $\nu_r = \frac{c\gamma^2}{\lambda_u\left(1+\frac{K^2}{2}\right)}$ is the resonant frequency, and $\delta = \frac{\pi N}{\nu_r}(\nu_r - \nu)$.

In figure 1 the spectrum radiated by one particle crossing one undulator, calculated with GPT, is compared with expression (1), showing a perfect agreement as expected. Varying the energy of the particles, the number of undulator periods, the period length and the undulator strength value gives also a perfect agreement with expression (1), as one can see in figures 2 and 3.

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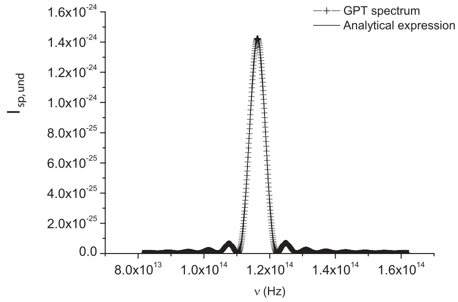

Figure 1: Spontaneous emission in an undulator calculated with GPT (+) and evaluated with expression (1), for the parameters $K = 1$, $N = 20$, $\gamma = 196$, $\lambda_u = 0.1$ m.

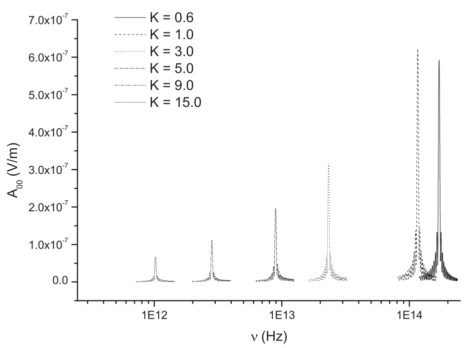

Figure 2: Amplitude Spectrum of one electron crossing an undulator, $N = 40$, $E_0 = 100$ MeV; K varies between 0.6 and 15.

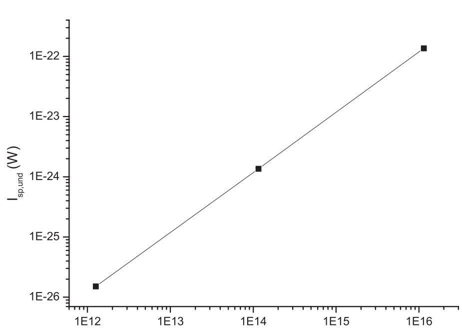

Figure 3: Power of the spectrum of one electron crossing an undulator, $N = 20$, $K = 1$; E_0 varies between 10 MeV to 1 GeV.

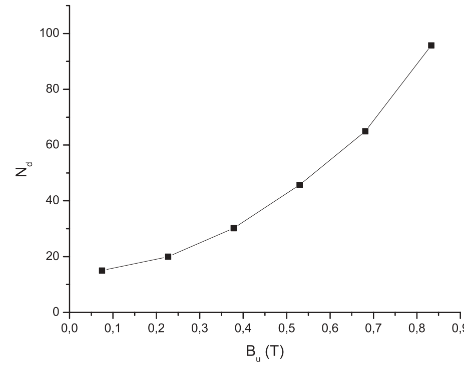

Figure 4: Interference order N_d of the power spectrum in an optical klystron vs. the magnetic field of the dispersive section. The modulator and the radiator have both 20 periods of 5.5 cm. The dispersive section has been designed here like a small undulator of 4 periods of 11 cm.

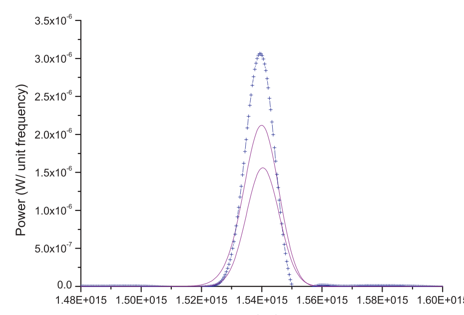

Figure 5: Energy echange between one electron, 900 MeV, and an initial light pulse. The lower curve shows the initial power spectrum of the pulse, the middle and the upper curve give the difference between the power spectrum of the spontaneous emission and the power spectrum at the end of the undulator of 20 periods and the end of the optical klystron. The gain is 44% in the undulator, and 116% in the optical klystron.

3 THE OPTICAL KLYSTRON GAIN

An optical klystron consists of two undulators, equivalent in our case, separated by a dispersive section. The first undulator is used as an energy modulator, the dispersive section transforms the energy modulation in space modulation providing micro-bunching and then enhancing the radiation flux in the second undulator. As a consequence the FEL gain, which is proportional to the derivative of the radiation spectrum, is stronger than for an undulator of the same length as the optical klystron.

The one pass FEL small signal gain can be evaluated from the expression [2]:

$$G_{OK} = \frac{4 r_e \pi^{\frac{5}{4}} N_e}{\sigma_x \sigma_y \sigma_z \gamma^3} \lambda_u^2 N^2 K^2 \left(JJ(\xi)\right)^2 \left(N + N_d\right) f \\ F_f \frac{\sin\left(\delta\right)}{\delta} \sin\left(2\pi \left(N + N_d\right) \frac{\nu}{\nu_r}\right) \tag{2}$$

where r_e is the classical electron radius, N_e the number of electrons in the bunch; σ_x, σ_y and σ_z give the bunch dimensions; N_d is the interference order between the radiation from the first and the second undulator, f the modulation rate of the spectrum depending on the energy spread of the bunch, σ_ϵ, $f = f_0 e^{\left(-8\pi^2 (N+N_d)^2 \sigma_\epsilon^2\right)}$. f_0 is a constant close to 1. F_f is the filling factor.

4 GPT RESULTS

An important parameter for the optical klystron gain is the interference order N_d between the two undulators. In order to know this number one should have a corresponding table in which N_d is scaled with the magnetic field amplitude B_d of the dispersive section. And for doing this one has to calculate the path of the particle through the dispersive section depending on the particle energy and the magnetic field shape. Another way to do this evaluation is using GPT simulations, by fitting the spontaneous emission in an optical klystron with the following expression [2]

$$I_{sp,OK} = I_{sp,und} \left(1 + f \cos\left(\alpha\right)\right) \tag{3}$$

where $\alpha = 2\pi \left(N + N_d\right) \frac{\nu}{\nu_r}$. Figure 4 presents the interference number N_d vs. B_d for a given optical klystron and for a 500 MeV electron. Using the analysis of the optical klystron spectrum calculated with GPT the optimization of the FEL gain of a specific design can be done, remembering that the maximum gain for a given electron bunch is for $N + N_d = \frac{1}{4\pi\sigma_\epsilon}$.

Then the gain in an optical klystron can be calculated with GPT, and in figure 5 we present the result obtained for one particle interacting with an initial gaussian light pulse. Although it is not possible to apply expression (2) to a one particle bunch, it is important to check how the code treats the energy exchange between charged particles and the electro-magnetic field, before running a full simulation. The resulting gain of 116% should be taken as the ideal gain for the ideally short bunch with zero energy spread. Then the situation of a realistic electron bunch is expected to give a lower value of the gain, however larger than or equal to the value given by expression (2).

5 CONCLUSION

The new element of the GPT code, modelling the interaction of the electro-magnetic field with charged particles, has been applied for performing free electron laser simulations. The results of the code give the spontaneous emission for both the undulator and the optical klystron cases. The element can be used to study the design of an optical klystron and its associated free electron laser gain. The energy exchange is treated properly and it should be possible to use the element for evaluating the single pass free electron laser gain for a given electron bunch traversing an undulator or optical klystron.

6 REFERENCES

[1] Madey J.M.J, Journ. Appl. Phys., 42, 5, 1906-1913, (1971).

[2] Elleaume P. *et al.*, Journ. de Phys. (Paris), 44 C1, 353, (1893).

[3] Thomas C. A. *et al.*, proc. PAC2001, Chicago, (2001).

[4] M.J. de Loos *et al.*, Proc. 5th Eur. Part. Acc. Conf., 1241, (1996).

[5] Kogelnik H. *et al.*, Appl. Opt., 8, 1687-1693 (1969).