DEPENDENCE OF THE PHOTON BEAM CHARACTERISTICS ON ELECTRON BEAM PARAMETERS IN THIRD GENERATION SYNCHROTRON LIGHT SOURCES

M. Ivanyan[†], Y. Martirosyan, V. Tsakanov, CANDLE, Yerevan, Armenia

Abstract

The results of regular study of the photon beam main characteristics - spectral brightness and coherence in third generation synchrotron light sources are presented in this report. The low and high beta lattice designs have been studied to obtain the maximum spectral brightness in x-ray region keeping the dynamical aperture of the ring sufficiently large. The results of the study have been applied for design study of CANDLE light source to be constructed in the republic of Armenia

1 INTRODUCTION

The brightness is an important figure of merit of the synchrotron radiation. The high brightness is required for the experiments that involve samples or optics with very small phase space acceptance or the techniques that exploit beam coherence. For Gaussian beam distribution the brightness is given by

$$B = N_{ph} \big/ 4\pi^2 \sigma_{px}\sigma_{px'}\sigma_{py}\sigma_{py'} \qquad (1)$$

where N_{ph} is the photon flux, $\sigma_{pu}, \sigma_{pu'}$ are the effective rms size and divergence of the photon source in the horizontal $(u = x)$ and vertical $(u = y)$ plains. The source characteristics $\sigma_{pu}, \sigma_{pu'}$ are defined by the convolution of the electron $\sigma_{bu}, \sigma_{bu'}$ and photon $\sigma_\gamma, \sigma_{\gamma'}$ beam sizes and divergences as [1]

$$\sigma_{pu} = \sqrt{\sigma_{bu}^2 + \sigma_\gamma^2}\,,\ \ \sigma_{pu'} = \sqrt{\sigma_{bu'}^2 + \sigma_{\gamma'}^2}\,,$$
$$\sigma_{bx} = \sqrt{\varepsilon_x\beta_x + \sigma_\varepsilon^2\eta_x^2}\,,\ \sigma_{by} = \sqrt{\varepsilon_y\beta_y}\,, \qquad (2)$$
$$\sigma_{bx'} = \sqrt{\varepsilon_x/\beta_x}\,,\ \sigma_{by'} = \sqrt{\varepsilon_y/\beta_y}\,.$$

with ε_u emittance, β_u, η_u beta function and dispersion at the source point, σ_ε relative energy spread. The photon beam size and divergence are the diffraction limited characteristics of the point source that for undulator are given by [1]

$$\sigma_\gamma = \sqrt{\lambda L/2}\big/2\pi\,,\ \sigma_{\gamma'} = \sqrt{\lambda/2L}\,, \qquad (3)$$

where L is the undulator length and λ is the radiation wavelength. The optimal beta value that provides the maximum brightness of the photon source (minimum of $S_u = \sigma_{pu}\cdot\sigma_{pu'}$) is then $\beta_{x,y} = L/2\pi$ for non dispersive source location. As an example, the optimum beta value for 4m long undulator source is 0.64 m. However, an impact of the optimal beta value on spectral brightness becomes essential when the electron beam emittance is at the level of diffraction limited photon beam emittance $\varepsilon_\gamma = \sigma_\gamma\sigma_{\gamma'} = \lambda/2\pi$. As an example, a diffraction limited electron beam capable of producing 10 keV X-Rays would have an emittance of 0.01 nm·rad. The new generation of rings under design or construction has design values of 5-15 nm·rad horizontal emittance [2]. For this range of electron beam emittance, the optimisation of the lattice optical properties to reach maximum spectral brightness not necessarily implies the minimum beta lattice as the dilution of the source size along the insertion device affects the brightness of the electron beam. As a result, the optimal beta at the source point may significantly differ from the diffraction limited case and may be based on the balance to obtain high spectral brightness of the photon beams and the stable machine operation (large dynamical aperture). As an example, for 3 GeV CANDLE synchrotron light source, the horizontal beta value at the insertion has been optimised at the level of 8 m that allows to approach the maximum brightness of the photon beam keeping large dynamical aperture of the ring [3]. The same problem is discussed in the Ref. [4], where for the particular cases of DIAMOND straight section parameters were shown that the brightness may be larger for large horizontal beta values in high photon energy regions.

In present report, the detailed study of the spectral brightness on the lattice performance, the beta values in horizontal and vertical planes, the beam evaluation along the undulator device is given.

2 BRIGHTNESS OF DILUTED BEAM

For a real electron beam, the diffraction limited photon emittance becomes diluted due to beam and source appearance effects. The diluted photon beam emittance along the optical axis of the undulator can be presented as [1]

$$\sigma_{pu}^2\sigma_{pu'}^2 = \varepsilon_u^2\left(1 + L^2/12\beta_u^2\right) + \varepsilon_u\sigma_{\gamma'}^2\beta_u + \qquad (4)$$
$$+\varepsilon_u\sigma_{\gamma'}^2 p_u\big/\beta_u + d_u^2\sigma_{\gamma'}^2\,,$$
$$p_u = L^2/12 + d_u^2\big/\sigma_{\gamma'}^2\,,$$
$$d_x^2 = \sigma_\varepsilon^2\eta_x^2 + \sigma_\gamma^2 + a^2\,,\quad d_y^2 = \sigma_\gamma^2\,,$$

[†]ivanian@ASLS.candle.am

where $a = \lambda_p K / (2\pi\gamma)$ is the electron beam oscillations amplitude that causes the widening of the source size, λ_p is the undulator period, K is the strength parameter and γ is the Lorentz factor.

Diffraction limiting case (Zero electron beam emittance). The factor $S_u = \sigma_{pu}\sigma_{pu'}$ is equal to $d_u^2\sigma_{\gamma'}^2$. The maximal possible brightness can be written then as $B_{\max} = N_{ph} / 4\pi^2 d_x \sigma_\gamma \sigma_{\gamma'}^2$. Fig.1 shows the diffraction limited brightness dependence on photon energy in dispersion free and dispersion presence cases. The dispersion ($\eta = 0.18$) influence leads to the significant decrease of diffraction limited brightness.

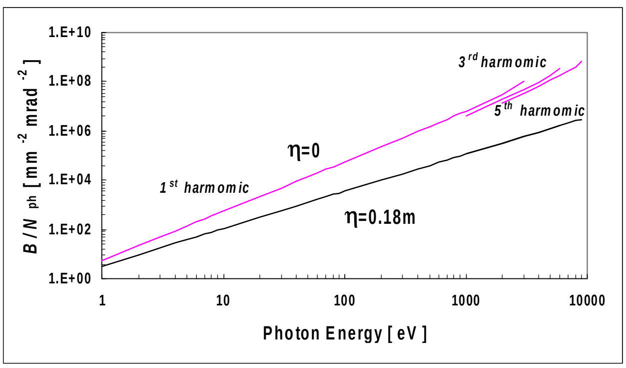

Figure 1. Normalized diffraction limited brightness without (upper curve) and with (bottom curve) dispersion.

Small emittance. For the small but finite emittance, the first term in (4) can be neglected and the optimal beta equation is given by $1 - p_u / \beta_u^2 = 0$ with the optimal beta solution $\beta_u^{opt} = \sqrt{p_u}$ independent from the emittance. This value is greater than diffraction limited beta optimum $L/2\pi$ and it increases with high photon energy (Fig.2).

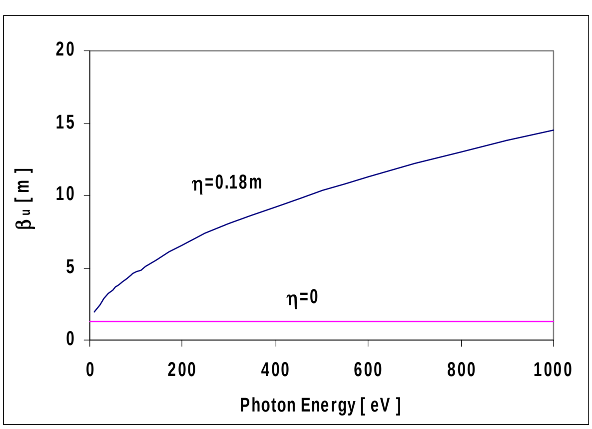

Figure 2. Optimal beta functions for the small emittances.

For the dispersion free case the optimum beta value is approximately twice greater than diffraction limited beta optimum $L/2\pi$. This effect is caused by the beam sizes variation along the undulator.

General case. For the arbitrary electron beam emittance the local optimal β_u for the fixed photon energy is given by algebraic cubic equation

$$\beta_u^3 - p_u\beta_u - q_u = 0\,, \tag{5}$$

with $q_u = L^3 \varepsilon_u / 3\lambda$. The single real positive solution of eq. (6) is given by:

$$\beta_u = \sqrt{p_u/3}\begin{cases} 2\cos(1/3 \cdot \arccos\mu), & \mu < 1 \\ \left((\mu + \tilde{\mu})^{\frac{1}{3}} + (\mu - \tilde{\mu})^{\frac{1}{3}}\right), & \mu > 1 \end{cases} \tag{6}$$

with $\mu = q_u / 2\sqrt{(p_u/3)^3}$ and $\tilde{\mu} = \sqrt{\mu^2 - 1}$. The value $F_\beta(\mu) = \beta_u / 2\sqrt{p_u/3}$ is a universal function versus dimensionless parameter μ that defines the optimum beta (Fig. 3).

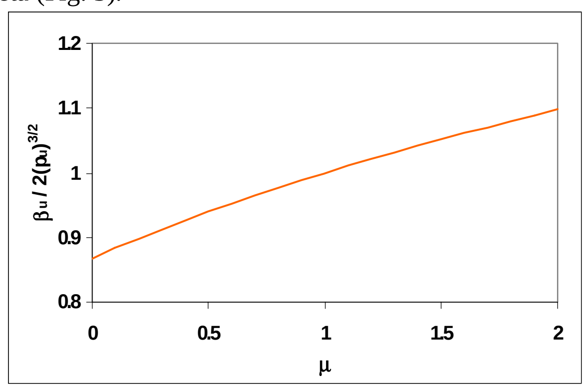

Figure 3. Optimal beta universal function $F_\beta(\mu)$.

The optimum beta function is the product of the beam emittance, dispersion at the source point and the photon wavelength. Fig.4 shows the dependence of the spectral brightness from the emitted photon energy for different horizontal beta values at the source point. For comparison are shown the CANDLE achromatic lattice ($\eta = 0$) with the horizontal emittance of 18 nm-rad and the dispersive lattice ($\eta = 0.18m$) with the horizontal emittance of 8.4 nm-rad. Dashed line gives the brightness for optimal beta values associated with each photon energy (form 6) for the dispersive lattice.

Achromatic lattice. For achromatic lattice the low beta at the source point improves the brightness. However, for the soft X-Ray region the decrease of beta function by one order of magnitude (from 10 m to 1 m) results in the brightness gain less than 10 %. For photon energy range 1-5 keV this improvement in the brightness is even smaller (less than 4%) and is negligible for energies higher than 5 keV.

Dispersive lattice. For the dispersive lattice the improvement of the brightness with small beta is visible only for the photon energies below 0.1 keV. Starting from 0.5 keV the brightness increases with larger betatron fuction, and in the energy range of higher than 5 keV, the brightness actually does not depend on the photon energy.

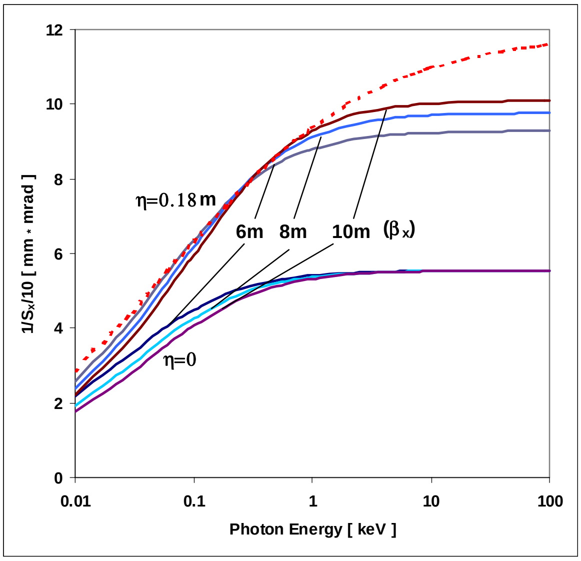

Fig.4 The brightness versus photon energy for different horizontal beta. Shown are: CANDLE achromatic lattice ($\eta = 0$), dispersive lattice ($\eta = 0.18m$) and optimal beta for dispersive lattice (dashed line).

An important feature is that with dispersive lattice that reduces the horizontal emittance to 8.4 nm-rad, the brightness is about twice larger than the brightness obtained with the achromat lattice that gives a 18 nm-rad emittance.

Fig.5 shows the spectral brightness versus of the vertical beta function for CANDLE dispersive lattice and 1% coupling. The vertical emittance is 0.084 nm-rad.

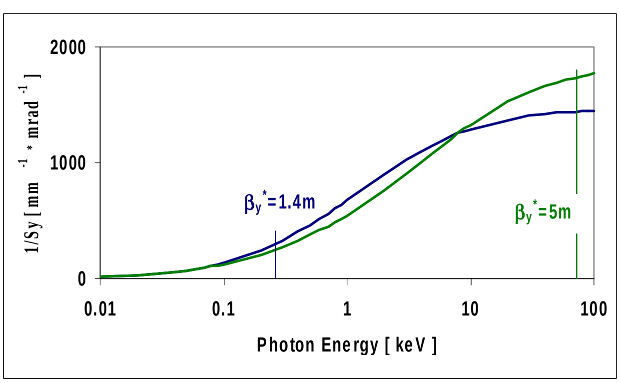

Figure 5 Brightness versus vertical betatron function.

3 COHERENCE

Spatial Coherence. The spatial coherent part of radiation is determined as the ratio of the real and diffraction limited brightness

$$N_{coh}/N_{ph} = \lambda^2 / 16\pi^2 S_x S_y \ . \qquad (7)$$

Significant coherent radiation is emitted into the forward direction from electron beams with an emittance at diffraction limited level $\varepsilon_{x,y} < \lambda/4\pi$ (Fig.6, curve 1). The dispersion presence decreases the coherent fraction of radiation (Fig.6, curve 2). The optimal beta values associated with each photon energy give the maximum coherence for given lattice (Fig. 6, curve 3).

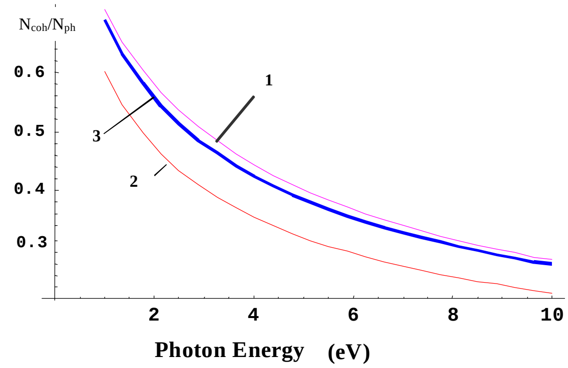

Figure 6. Spatial coherence degree

Temporal Coherence. The total radiation power $P(\omega)$ emitted by the bunch at the wavelength λ consists of the sum of incoherent and coherent parts of radiation: $p(\omega)\left[N_e + N_e^2 g^2(\sigma/\lambda)\right]$, where $p(\omega)$ is the radiation power of one particle, N_e is the number of particles in the bunch, σ is r.m.s. bunch length and $g(\sigma/\lambda) = \exp\left(-2\pi^2 \sigma^2/\lambda^2\right)$. The coherence degree is equal to:

$$\eta = N_e g^2(\sigma/\lambda) / \left(1 + N_e g^2(\sigma/\lambda)\right). \qquad (8)$$

For the CANDLE this function versus wavelength is plotted on Fig.7.

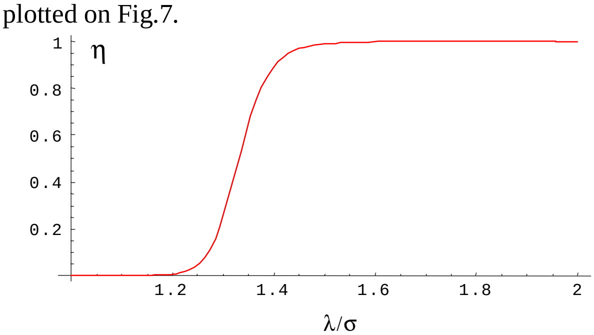

Figure 7. Temporal coherence degree vs. photon wavelength

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