MEASURING CHROMATICITY ALONG THE RAMP USING THE PLL TUNE-METER IN RHIC*

S. Tepikian, L. Ahrens, P. Cameron, C. Schultheiss

BNL, Upton, Long Island, New York

Abstract

Beam stability up the ramp requires the appropriate sign and magnitude of the chromaticity. We developed a way to measure the chromaticity using the PLL (Phase Locked Loop) tune-meter. Since, the accuracy of the PLL tune-meter with properly adjusted loop gain is better than $\approx$ 0.0001 in tune units, the radial loop needs only be changed by a small amount of $0.2mm$ at a $1Hz$ rate. Thus, we can achieve fast chromaticity measurements in $1sec$. Except during the very beginning of the ramp where there are snapback effects and the gamma changes very rapidly, we can have good chromaticity measurements along the ramp. This leads to the possibility of correcting the chromaticity during the ramp using a feedback system.

1 INTRODUCTION

To successfully ramp a beam of particles in an accelerator, the tunes and chromaticities must be controlled. This becomes harder to do in an accelerator like RHIC where the lattice optics are changing during the ramp. We propose to using the PLL tune-meter [1] to measuring the chromaticity. The tune is measured while the radial offset, hence beam momentum is varied by a sine wave function. The desire is to have a small amplitude of $\approx 0.2mm$ to the radial shift, to minimize perturbations to the beam. This small shift is possible if the measurement of the tune is of high enough accuracy (i.e. 0.0001), such as using a PLL tune-meter. This sine wave perturbation can be extracted from the tune data using linear regression [2]. From the amplitude and phase of this sine wave perturbation, both the magnitude and sign of the chromaticity can be deduced. In the next section, the theory is discussed followed by a section on implementation. Additionally, the data measured during RHIC 2001 studies, will be discussed. Finally, a conclusion is given.

2 THEORY

The measured tunes are returned as a function of time. If there are perturbations on the tune — for instance, driving the radial shift with a sine wave function — then the tune can be modeled as (for either x or y plane):

$$\nu(t) = A\sin(\omega t) + B\cos(\omega t) + Ct^2 + Dt + E$$

where ν is the tune, t is the time, A and B terms are due to chromaticity, C, D and E are the coefficients for any slow tune variations from other sources and the frequency $f = \omega/2\pi$ is set by the radial shift function. Since, ω is

fixed, the coefficients in the tune equations can be found through linear regression.

2.1 Linear Regression

The measured tune data is returned as a set of points, such as (t_k, ν_k) for k'th point (where $\nu_k = \nu(t_k)$). Choose a set of points large enough to cover one period, $T = 2\pi/\omega$, denote the number of points as N. The χ^2 function to be minimized is written as (for either x or y plane):

$$\chi^2 = \sum_{k=1}^{N} [\nu_k - A\sin(\omega t_k) - B\cos(\omega t_k) - Ct_k^2 - Dt_k - E]^2$$

The values for the coefficients, A, B, C, D and E are then found to minimize the χ^2 function. The solution is to solve:

$$\vec{v} = M \begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix}$$

where

$$\vec{v} = \sum_{k=1}^{N} \begin{pmatrix} \nu_k \sin(\omega t_k) \\ \nu_k \cos(\omega t_k) \\ \nu_k t_k^2 \\ \nu_k t_k \\ \nu_k \end{pmatrix}$$

and

$$M = \sum_{k=1}^{N} \begin{pmatrix} \sin^2(\omega t_k) & \sin(2\omega t_k)/2 & \sin(\omega t_k)t_k^2 \\ \sin(2\omega t_k)/2 & \cos^2(\omega t_k) & \cos(\omega t_k)t_k^2 \\ \sin(\omega t_k)t_k^2 & \cos(\omega t_k)t_k^2 & t_k^4 \\ \sin(\omega t_k)t_k & \cos(\omega t_k)t_k & t_k^3 \\ \sin(\omega t_k) & \cos(\omega t_k) & t_k^2 \end{pmatrix}$$

$$\begin{pmatrix} \sin(\omega t_k)t_k & \sin(\omega t_k) \\ \cos(\omega t_k)t_k & \cos(\omega t_k) \\ t_k^3 & t_k^2 \\ t_k^2 & t_k \\ t_k & 1 \end{pmatrix}$$

2.2 Chromaticity

After solving for the coefficients, we use only A and B to calculate the chromaticity as:

$$\xi = \pm\sqrt{A^2 + B^2}\frac{\alpha C_R}{2\pi\Delta R}$$

where C_R is the circumference, α is the momentum compaction factor and ΔR is the amplitude of the radial shift function. The sign of the chromaticity is determined from

the phase difference between $\phi = \arctan(B/A)$ and the phase from the radial shift function including any phase delays if present. Note, the phase does not have to be known accurately to get the sign. Furthermore, from the χ^2 and matrix M, the error of the chromaticity measurement can be estimated as:

$$\sigma_\xi = \frac{|A|\sigma_A + |B|\sigma_B}{\sqrt{A^2 + B^2}} \frac{\alpha C_R}{2\pi\Delta R}$$

where

$$\sigma_A = (M^{-1})_{11}\sqrt{\frac{\chi^2}{N-5}}$$

and

$$\sigma_B = (M^{-1})_{22}\sqrt{\frac{\chi^2}{N-5}}$$

Note, the terms $(M^{-1})_{11}$ and $(M^{-1})_{22}$ diagonal components are from the inverse of matrix M.

3 IMPLEMENTATION

The PLL tune data will arrive at about $70Hz$ rate or faster. Calculating the chromaticity and associated error must respond in real time. This is necessary if the chromaticity is to be part of a feedback correction system. Additionally, this may work within a tune feedback correction system. Thus, the tune data will be of two different types:

- Direct reading of the tune
- With tune feedback on, returning the error signal

In the following subsections we discuss data sliding and how to get the tunes when the tune feedback is turned on.

3.1 Sliding Data Points

In order for the chromaticity to be calculated quickly, we use a sliding data approach. The vector $\vec{v}$ and the matrix M are updated by subtracting off the oldest data point and adding in the newest data point, keeping the total number of points fixed. Hence, the most time consuming part becomes the solving for the coefficients: A, B, C, D and E. Since the matrix M is symmetric, the the matrix can be handled using cholesky decomposition [2]. The chromaticity is returned with an error status if the matrix M isn't positive definite. The data sliding is used whether or not tune feedback is on. Our plans include adding this code to the Front End Computer (FEC) of the PLL system. This will be available for the next RHIC run.

3.2 With Tune Feedback

When the tune feedback is turned on, the tune data will be extracted from the feedback error signal. This signal will be delayed and scaled from the actual tune variation. The amount of delay and scaling depends on the parameters of the feedback loop and the frequency for the radial shift sine wave function, $f = \omega/2\pi$. These parameters will be fixed and can be calculated apriori.

4 DATA ANALYSIS

Data was taken during the RHIC 2001 run to check if this method is a feasible way to measure the chromaticity during a ramp. In this test the radial shift function was set to have three $1Hz$ sine waves with an amplitude $0.2mm$ and $1sec$ boundaries at both ends. Fig. 1 shows the results in the Blue RHIC ring. The black curve shows the radial shift function, the red curve is the measured PLL tune data and the blue curve is the calculated chromaticity with the error bars. The chromaticity is only real in the regions where the radial shift function is a sine wave. When the radial shift function is constant the tune should not have a sine wave to extract, thus, leading to a calculated value of ≈ 0, which is *not* the chromaticity.
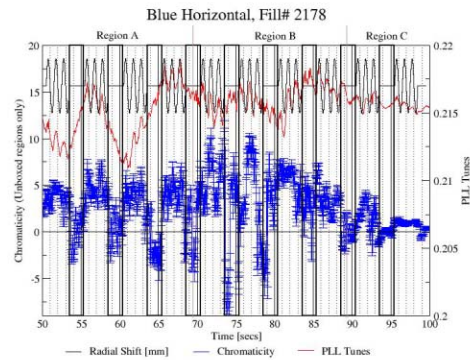

Figure 1: The graph of tunes (red, middle) fitted chromaticity (blue, bottom) and the radial shift (black, top) of the beam.

Examining Fig. 1 shows three distinct regions:

- Region A, from beginning to $70secs$
- Region B, from $70secs$ to $90secs$
- Region C, from $90secs$ to the end

In region A, we are getting good values for chromaticity ≈ 3 to 5 units. This agrees with hand calculations and our intuition about the beam dynamics. Furthermore, when the radial shift is constant, the sine wave perturbation to the tune is very small.

In region B, the PLL tune data has become quite noisy. One suspect may be beam-beam effects. At the beginning (about $50secs$) the beams at the interaction points are well separated. But in region B the separation has been reduced [3].

By the time the beam reaches region C, the PLL tunemeter isn't following the tune as well as in region A. Two possible explanations have been advanced for the observed behavior, both involving reduced PLL loop gain, which would reduce loop bandwidth from the nominal $2Hz$ to considerably less than the $1Hz$ of the radial modulation, and result in the lowpass filtered quality of the data. The gain includes both the electronics and the beam transfer function. The BTF portion can be reduced in many ways.
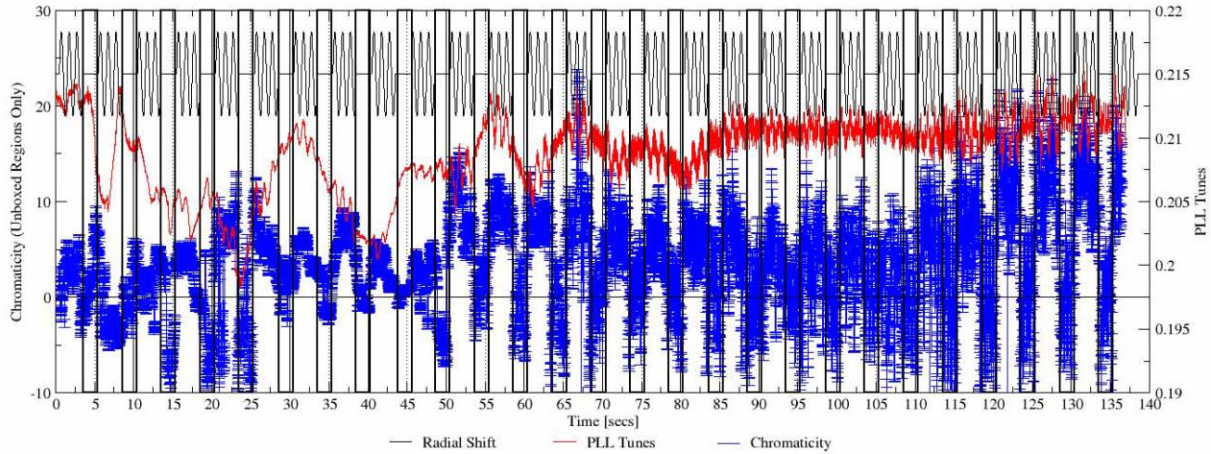
Figure 2: Similar to Fig. 1 but taken at a earlier time and including the entire ramp.

One is that large chromaticity can broaden the betatron line and reduce the number of particles excited resonately. In this case one would expect to see the signal of large chromaticity for some time before the bandwidth reduction becomes obvious, which we did not. Another possibility, perhaps more likely, is that incorrect phase information pulled the loop to the edge of the betatron line, again reducing the number of particles excited and the loop gain. The source of error might have been incorrect phase compensation during ramping, leakage from the opposite plane due to large chromaticity, close tune approach, or coupling, or some other cause which is not yet understood.

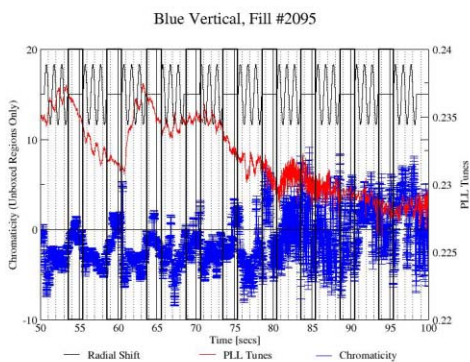Fig. 2 shows another situation but with the full ramp of data.

Figure 3: Data Taken at the same time as Fig. 2 but for the vertical tune.

Finally Fig. 3 is interesting since it showed us that the chromaticity was negative in the vertical plane. Once this was fixed, the beam stability improved. This shows that we can get the sign of the chromaticity as well as the magnitude.

5 CONCLUSION

A method was presented of using the PLL tune-meter for measuring the chromaticity. In this method, a radial shift is applied to induce a perturbation on the tunes. From the size of the tune perturbation, the chromaticity can be calculated. Furthermore, the radial tune shift required is small to minimize the destructiveness of this type of measurement.

This method was tested during the RHIC 2001 run. For part of the ramp, we were able to deduce the chromaticity and its corresponding sign with this method. This method fails when the tune data is too noisy or the PLL system loses the lock. If the PLL tune-meter system fails due to the chromaticity being too large, we can use the schottky cavity to give us an estimate of the chromaticity for a first pass correction. Then the PLL tune-meter system should work and can be used to refine the chromaticity.

During the next RHIC run, we will have an improved PLL system which will be more reliable. Additionally, this code will become part of the PLL tune-meter system. Finally, when the system is completely functional, we plan to employ a feedback system to correct the chromaticity on the ramp.

6 ACKNOWLEDGMENTS

7 REFERENCES

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