A STATE-VARIABLE DESCRIPTION OF THE RHIC RF CONTROL LOOPS *

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Abstract

The beam transfer function changes during the RHIC ramp. The response of the RF control loops changes as a result. A state-variable description of the beam and the RF control loops was developed. This description was used to generate a set of feedback matrices that keeps the response of the RF control loops constant during the ramp. This paper describes the state-variable description and its use in determining the K matrices.

1 INTRODUCTION

The state-variable description of the beam was used in models of the RHIC RF phase and radial control loops. Two models were developed. A third order model was used to calculate the K matrix coefficients at 124 different values of the beam energy during the ramp. A fifth order model, that includes the radial filter, was used to simulate the response of the system using the coefficients found with the third order model. The resulting coefficients were found to be linear functions of the beam energy.

2 STATE-VARIABLE DESCRIPTIONS

2.1 The Beam

The beam state-variable description was developed from the following equations:

$$\frac{d}{dt}(\delta E) = \frac{1}{T_{rev}} eV_{rf}\left[\sin(\varphi_s + \varphi) - \sin\varphi_s\right]$$

$$\frac{d}{dt}(\varphi) = \frac{1}{T_{rev}}\left[2\pi\frac{\delta t}{T_{rf}} + \delta\omega_{rf}T_{rev}\right]$$

These equations can be simplified to [1]:

$$\frac{d}{dt}(\delta E) = \left[\frac{eV_{rf}}{T_{rev}}\cos\varphi_s\right]\varphi = A\varphi$$

$$\frac{d}{dt}\varphi = \frac{\omega_{rf}\eta}{\beta^2 E}\delta E + \delta\omega_{rf} = B\delta E + \delta\omega_{rf} = B\delta E + u$$

Choosing the states to be the phase and the delta energy gives the following description.

$$x_1 = \varphi$$
$$x_2 = \delta E$$

$$\dot{x}_1 = Bx_2 + u$$

$$\dot{x}_2 = Ax_1$$

With the following state-space equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & B \\ A & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$$

The auxiliary equations provide the relations between a change in the beam energy and the resulting change in beam radius and frequency.

$$\delta R = \frac{1}{\gamma_{tr}^2 \beta^2}\frac{R}{E}\delta E = C\delta E = Cx_2$$

$$\delta\omega_b = -\frac{\eta\omega_b}{\beta^2 E}\delta E = D\delta E = Dx_2$$

The state-space block diagram of the beam is then represented as:
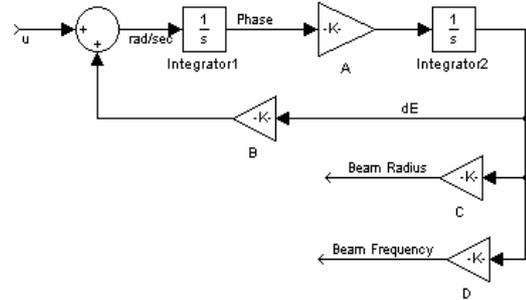
Figure 1 Block Diagram of Beam Model

The multipliers A, B, C, and D must be constants in the model. This condition is satisfied at fixed beam energies. A MatLab™ M-file was written that calculated these constants at 124 beam energies. A system matrix for each beam energy was constructed using these values. Then the K matrix at each beam energy was calculated.

2.2 System Model

The third order system that was used to calculate the K matrices has the following state-space representation:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 0 & G_R C \\ 0 & 0 & B \\ 0 & A & 0 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_{vco} \\ 0 \end{bmatrix} u + \begin{bmatrix} -1 \\ K_{vco} \\ 0 \end{bmatrix} R_o$$

$$= \overline{F}\,\overline{x} + \overline{G}u + \overline{G}_a R_o$$

$$\overline{y} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & C \end{bmatrix} \overline{x} = \overline{H}\,\overline{x}$$

with the following relations.

$$u = -K_I x_I - K_\varphi G_\varphi x_1 - K_R G_R C x_2 = -\overline{K}\begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix}$$

$$\overline{K} = \begin{bmatrix} K_I & K_\varphi G_\varphi & K_R G_R C \end{bmatrix}$$

The characteristic equation for the closed loop system is found from [2]:

$$\det\left[s\overline{I} - \left(\overline{F} - \overline{G}\,\overline{K} \right) \right] = 0$$

and is:

$$s^3 + \left(K_{vco} K_\varphi G_\varphi \right)s^2 + \left(ACK_{vco} K_R G_R - AB \right)s$$
$$+ ACK_{vco} K_I G_R$$

3 CALCULATION OF THE K MATRIX

The third order model yields an analytical expression that relates the K matrix elements and the system's closed loop pole positions. The requested poles were expanded and the coefficients of equivalent powers of the polynomials were equated. This led to these relations for the K matrix coefficients in terms of the requested poles.

$$K_\varphi = \frac{coeff(2)}{K_{vco} G_\varphi}$$

$$K_R = \frac{coeff(3) + AB}{AK_{vco} G_R C}$$

$$K_I = \frac{coeff(4)}{K_{vco} G_R AC}$$

Where the expanded pole polynomial is defined as:

$$s^3 + coeff(2)s^2 + coeff(3)s + coeff(4)$$

The MatLab program calculates the constants used in the beam state-space representation, the third order system matrices, the K matrix, the fifth order system matrices (that includes the radial filter) and the poles for the fifth order system. This procedure was repeated for 124 beam energies along the RHIC ramp, producing a K matrix at each energy. The program also initializes the workspace for Simulink™ after which a full simulation can be carried out at a fixed energy.

Figure 2 Simulink™ Model used to simulate the RHIC RF Loop

4 K MATRIX AS A FUNCTION OF Βρ

The individual elements of the K matrices yields the following graphs:
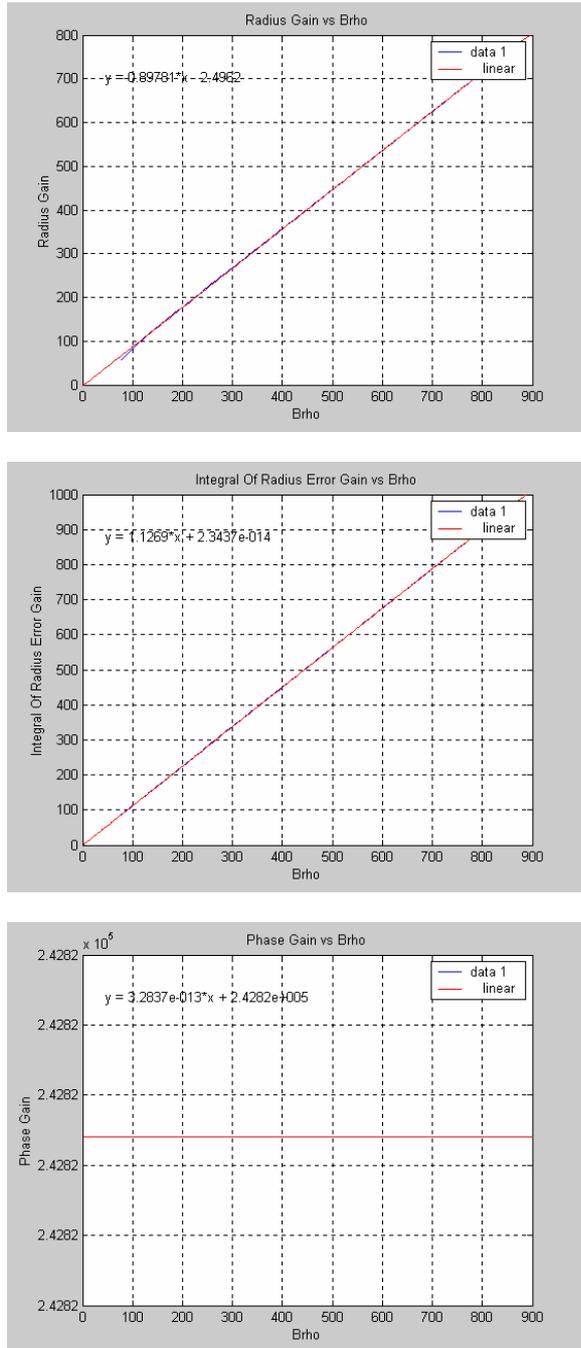
Figure 3 Graphs of the K Matrix Elements as functions of βρ.

5 RESULTS

Table 1 shows the calculated pole positions for a system using a fixed K matrix and a system using a varying K matrix. A measure of the variation in the pole position is the change in the natural frequency and the change in the damping factor. Three of the poles are real and remain real. The other two poles are due to the radial filter. The percent change in the natural frequency of the poles of the system with a varying K matrix is much smaller than the uncorrected system, especially for the low frequency poles.

Table 1 Pole Positions for systems with Fixed and Varying K Matrices.

	Poles at Injection (rad/sec)	Poles at Flattop (rad/sec)	Percent Change ω_n
Fixed K Matrix	-0.1225	-0.5157	321
	-312.5	-6.396	-98
	-4812	-5128	6.6
	-6437+6429i	-6432+6432i	-0.01
	-6437-6429i	-6432-6432i	-0.01
Varying K Matrix	-0.1225	-0.1225	-0.0
	-313.1	-329.8	5.3
	-4800	-4519	-5.9
	-6437+6429i	-6570+6350i	0.7
	-6437-6429i	-6570-6350i	0.7

Figure 2 shows the model that was used to simulate the RHIC RF loop. For a 1mm radial step input, the model predicts a beam frequency rise-time of eight milliseconds. The same rise-time was measured for a radial step input into the RHIC RF system.

6 CONCLUSIONS

The graphs show an excellent linear fit to the calculated K matrices. As expected the phase gain is independent of the beam energy.

The varying K matrix stabilizes the closed loop pole positions. The K matrix affects the three low frequency poles the most, this is to be expected since these poles are related to the phase, radius, and integrated error of the radius loops. The other two poles are related to the radial filter and move due to the increasing control effort required as the beam becomes "stiffer".

7 REFERENCES

[1] J. M. Brennan, "RF Beam Control for the AGS Booster," September 26, 1994, pages 7-8.
[2] G. F. Franklin et al., "Feedback Control of Dynamic Systems," Third Edition, Addison-Wesley, page 494.