TO BE π OR NOT TO BE π: THAT IS THE DILEMMA*

Stan O. Schriber, ADSR, MS H820, Los Alamos National Laboratory,
Los Alamos, NM 87545, USA

Abstract

An important aspect for a new accelerating structure is selection of the cavity mode for efficient operation. Once this selection has been made, it is important to understand structure operation from the point of view of tolerances, performance and possible constraints. We do not always use correct terminology in describing modes of resonant standing-wave cavities. This paper explains differences in terminology, as well as showing the importance of correct mode identification. In particular, the dilemma associated with when is a π-mode a π-mode will be clarified. Also, reasons for using *0*- or π/n-mode structures are given. Finally, important results determined from cavity mode spectra are given, providing correct terminology is used.

1 INTRODUCTION

Equivalent circuit theory [1-3] has been used for many years to understand and predict, in simple rf terms, the behavior of standing-wave coupled-cells. This theory has been a powerful tool in construction and operation of accelerating structures. Because of efficiency advantages for superconducting structures in accelerator complexes, the use of π-mode structures has increased significantly. Plotting passband mode resonant frequency data and determining field distributions for such π-mode structures must be done carefully in order to extract necessary information and to use this information for predictions, as described in the following sections.

The question arises: When is the π-mode a π-mode? Is it only when the fields in adjacent cells of a cavity chain are equal and of opposite sign, or is it only when field phase shifts between adjacent cells are 180^o? In reality it is neither. Real π-mode cavities with rf losses and fabrication errors along the chain have neither exactly equal fields nor 180^o phase shifts between cells. As explained below, justification for π-mode cavity operation is more complex, but simple if simple rules are followed.

2 MODES AND PHASES

Mode characteristics of coupled-cell chains are highly dependent on end-cell tuning as shown in Table 1 [4] for cavities with a possibility for 'flat-field' characteristics, i.e. lossless and errorless. 'Flat-field' is used to indicate almost equal absolute value fields in each useful cell.

Notice phase values for various modes, in particular the π-mode, as a function of end-cell termination. Also note the formula for mode field amplitudes in the chain of cells of a cavity. A simple method for determining 'phase' of a mode is the following: mode phase or number designates a particular mode in the passband, mode phase shift designates the change in phase between adjacent cells and mode phase value indicates the location of the mode on the dispersion curve. Unfortunately these three are not always the same, except for the *0*- or π-mode 'flat-field' cases. For example, the π-mode of a full-cell terminated cavity with no end-cell detuning has 180^o phase shifts cell-to-cell but is plotted with a πN/(N+1) value for the passband. The only phase shifts cell-to-cell in a lossless and errorless standing-wave cavity are 0^o, 90^o and 180^o. Table 1 gives end-cell detuning required to achieve 'flat-fields', showing efficient π/n acceleration modes.

Table 1: 'Flat-Field' Cavity Phase Values and Amplitudes

End Termination	Mode ϕ_q	End Cell Detune	Field Amplitudes	Phase Values	Mode Numbering
Full	*0*	$f_{end} = f_0\sqrt{1-k/2}$	$i_n \sim \cos[(n-1/2)\phi_q]$	$\phi_q = \frac{\pi(q-1)}{N}$	$q = 1,2,3,\cdots,N$
Full	$\pi/3$ $\pi/2$ $2\pi/3$	$f_{end} = f_0$	$i_n \sim \sin(n\phi_q)$	$\phi_q = \frac{\pi q}{(N+1)}$	$q = 1,2,3,\cdots,N$
Full	π	$f_{end} = f_0\sqrt{1+k/2}$	$i_n \sim \sin[(n-1/2)\phi_q]$	$\phi_q = \frac{\pi q}{N}$	$q = 1,2,3,\cdots,N$
Half	*0* $\pi/2$ π	$f_{end} = f_0$	$i_n \sim \cos[(n-1)\phi_q]$	$\phi_q = \pi\frac{(q-1)}{(N-1)}$	$q = 1,2,3,\cdots,N$

Details of π-mode operation, tolerances, rationale for 'flat-fields' and consequences are found in Ref. [5].

3 EXAMPLES

The following examples show consequences of correct choices for phase values in plotting passbands and hence for determining appropriate equivalent circuit parameters.

3.1 Passbands and Proper Parameter Choices

As explained, it is important to make correct choices for mode phase values and to understand the implications for fitting passband resonant frequency modes to equivalent circuit dispersion relationships. The following example is based on a 500 MHz five-cell cavity with a 2 % coupling constant. Plotted in Fig.1 are the resonant modes for four different terminations (half, full and two end-cell detuned full cases) compared to a common dispersion relationship of the singly periodic circuit that represents the cavity.

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Notice that all modes fit on the dispersion curve, as long as the proper phase values are used, even though different areas on the curve show the *0*-mode and π-mode. Fits to the passband resonant modes result in the same values for the two parameters representing the system: inter-cell first neighbor coupling constant and cell frequency. Although phase values for the π-mode of the half, full, full pi and full 0 cases vary from 144° to 180°, the mode is still called a π-mode with 180° phase shifts cell-to-cell in the cavity.

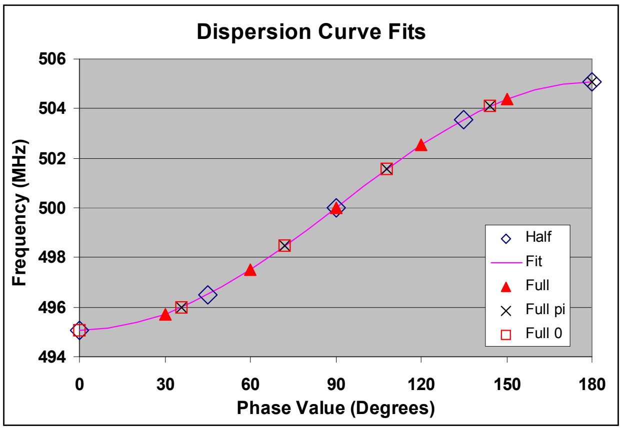

Figure 1: Dispersion curve fit to passbands from four different terminations

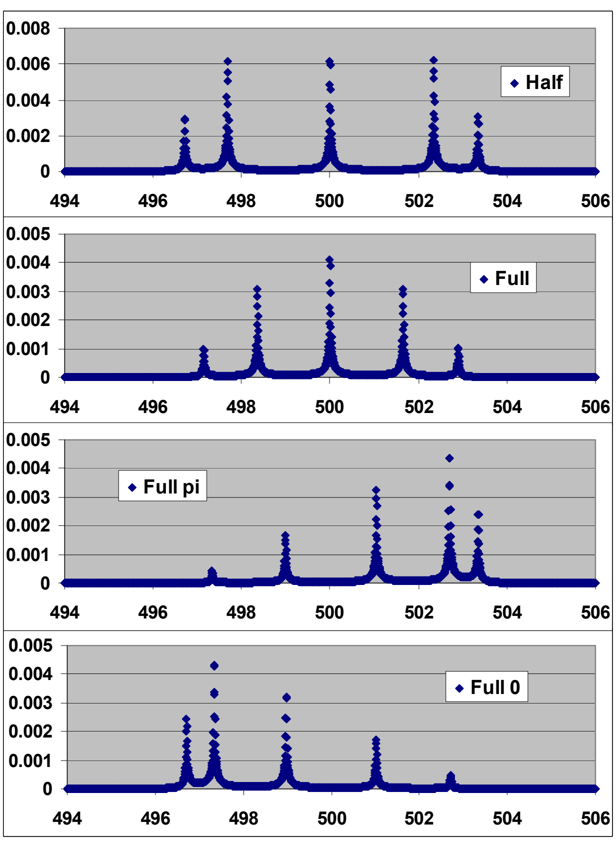

Figure 2: Typical mode spectra for four terminations.

Fig. 2 shows typical spectra for these four terminations. Spectra calculated by LOOPER [6] had maximum random errors: 0.002 % cell frequency and 2 % coupling constant. Agreement between LOOPER and measured spectra is excellent [6]. Note the telltale signs indicating the type of cavity termination from relative heights of the resonant peaks and the frequency separations between modes.

3.2 Nine-Cell Cavity Detuning

A nine-cell 500 MHz cavity with 2 % intercell coupling demonstrates end-cell detuning effects below. Details of such detuning are found in Ref. 4 and 5. Figure 3 shows the last four modes of the spectra for detuning end-cells by a fraction of the required detune value (2.544 MHz) for this geometry. Note shifts in mode separation and peak heights for a cavity with random errors as given above.

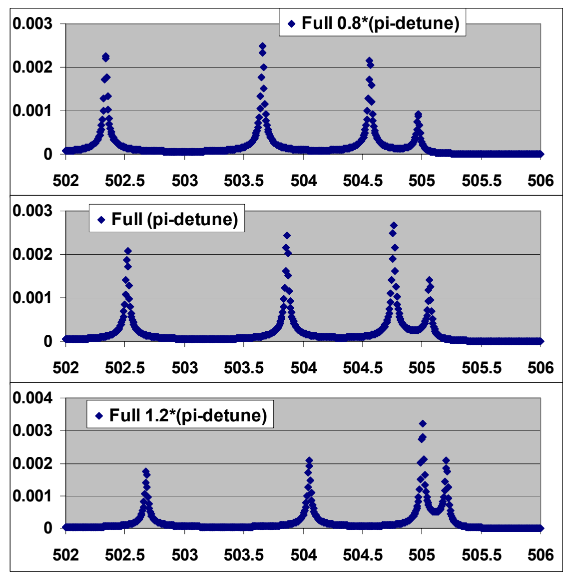

Figure 3: Last four modes for different end-cell detuning

Relative on-axis electric fields show characteristics that can be used to assist with one-step tuning [7] of π-mode cavity cells. A simple comparison of the cell-to-cell fields versus end-cell tuning can also provide a reasonable means to get 'flat-fields' in the cavity as shown in Fig. 4. Data for fractional detunes from 0.5 to 1.5 are presented for a cavity with errors as described above and a quality factor Q of 25000, a reasonable value for copper model cavities. Note that fields for the proper detune value are not entirely flat, an effect related to cell errors close to that experienced in reality.

Another method that can be used to determine 'flat-field' conditions is to measure cavity mode spectra as a function of end-cell detuning. This method fits measured mode spectra to equivalent circuit passband relationships using DISPER [8]. Figure 5 shows the linearity expected in the mode frequencies as a function of fractional detune. Note the decrease in frequency separation between the π-mode and its nearest neighbor as the fractional value approaches and exceeds one.

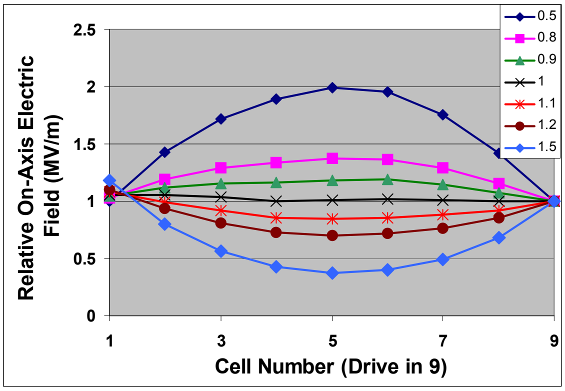

Figure 4: Relative on-axis electric field (absolute value) in the nine-cell cavity for the π-mode.

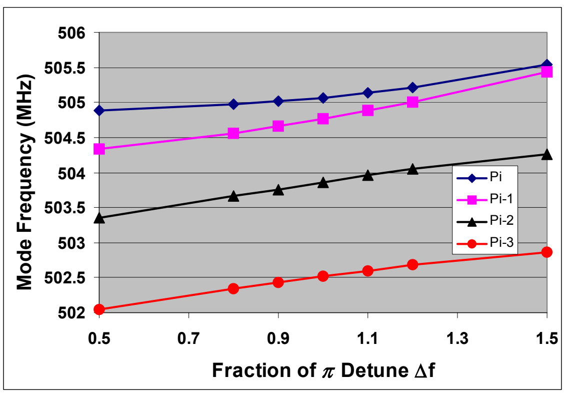

Figure 5: Mode frequency shifts for the highest four modes as a function of end-cell detuning.

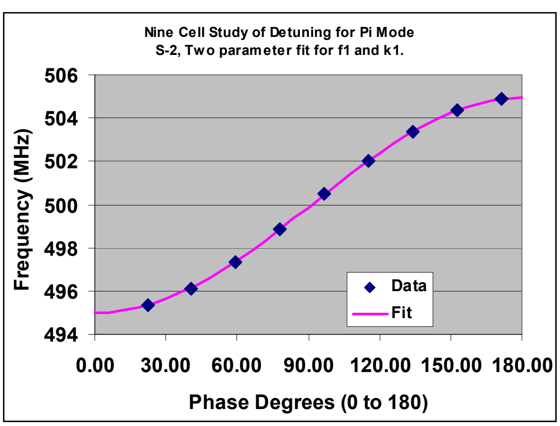

Figure 6: Results of DISPER fit to data from 0.8 detune of the end-cells.

The fit of passband mode spectra to DISPER is shown in Fig. 6 for a fractional detune of 0.8. In this example DISPER determined the best least squares fit to the singly periodic cell frequency, f1, and first neighbor coupling constant, k1, as well as the best phase value for the end mode, and the best phase separation between modes. Figure 7 shows the shift in mode phase value determined from DISPER fits to data determined for different fractional detune multipliers from 0.5 to 1.2. In this case, it clearly shows when the best detune is achieved; at the 180° mode phase value in agreement with expectations.

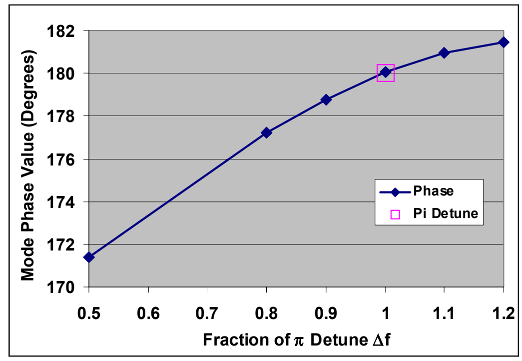

Figure 7: π-mode phase value as a function of detunes fractional multiplier.

Mode separation was 20° as expected for the best detuned case.

4 SUMMARY

Reasons for calling a π-mode the π-mode are clearly given. In addition, means for extracting usual information from spectral measurements are demonstrated.

5 REFERENCES

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