NUMERICAL SIMULATIONS OF DYNAMIC LORENTZ DETUNING OF SC CAVITIES

G. Devanz, M. Luong, A. Mosnier, DSM/DAPNIA/SACM, CEA Saclay, 91191 Gif-sur-Yvette

Abstract

Most of high power accelerator projects rely on bulk niobium superconducting cavities technology. When pulsed operation is requiered, cavities are submitted to time varying radiation pressure proportional to the square of accelerating field. This excitation couples to the mechanical system constituted of the cavity and auxilliary components, and may excite mechanical modes at resonance. Subsequent deformation of the cavity induces a time-varying detuning. When high accelerating gradients or low beta cavities are of concern, this detuning could, if not controlled, seriously impair RF operation because of extra power required, and add constraints on the RF stabilization system to keep the beam quality high. We have studied the dynamical behavior of SC cavities under pulsed operation using a modal approach and coupling mechanical simulation to RF calculations. Stiffening, mechanical modes damping and finite stiffness of tuning system and helium vessel are included in the numerical model. Modal Lorentz detuning coefficients are extracted from these calculations on which basis a simple second order system algebraic model can be set up.

1 INTRODUCTION

Superconducting (SC) cavities technology has proven to be the best choice for high current CW machine. The number of designs of high peak power pulsed linacs relying on SC thechnology is growing, including HEP machine like TESLA [1] and high power proton linacs for neutron spallation sources like ESS [2]. Progress of SC cavities performance lead designers to rely on high accelerating gradients. Radiation pressure

$$P_{rad} = \frac{1}{4}(\mu_0 H^2 - \epsilon_0 E^2) \qquad (1)$$

depends quadratically on accelerating field E_{acc}. Its effects therefore becomes a proeminent topic when high gradients are contemplated. The consequence of P_{rad} is a mechanical deformation of the cavity and a subsequent frequency shift. This shift can usually be reduced to a non-zero minimum by means of a stiffening system. For a CW machine, the cavities can simply be detuned to compensate the static frequency shift. The Lorentz detuning coefficient K accounts for the detuning Δf through the relation $\Delta f = -|K|E_{acc}^2$. For pulsed operation the time varying detuning must be studied in order to design the RF control system [3], and evaluate the effects on beam dynamics [4]. Depending on the time structure of the RF, mechanical eigenmodes of the cavity can be excited by time varying P_{rad}. All examples shown in this paper correspond to the medium β 5-cells cavity of the 704.4 MHz SC linac proposed for ESS [2].

2 STATIC DETUNING MINIMISATION

Lorentz force detuning could be reduced by stiffening the cavity, for example designing it with a thicker wall, at the expense of tunability. As it was first proposed for TESLA cavities, a better solution is to weld rings between cells [5]. The principle is to dispose a fixed point in the cavity wall in order to balance the electric and magnetic part of the detuning. Figure 1 represents the cavity equipped with rings.

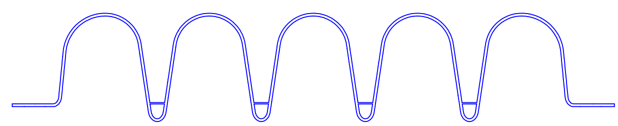

Figure 1: medium β cavity geometry

The optimisation of ring position was done for static detuning only. The cavity is modeled using the FEM mechanical code CASTEM. Since static P_{rad} tends to shrink the cavity, the boundary conditions at beam tubes should account for the finite stiffness κ_{ext} of the He tank and tuning system. Elements with a fixed stiffness included in the beam tube model simulate an external stiffness of 100 kN/mm. The radiation pressure distribution $P_{rad}(z, r)$ is obtained from Superfish calculations. The mechanical simulation consists in applying this pressure on cavity RF surface. The mesh of original cavity shape and the displacement field are used to produce two Superfish input files. This procedure ensures that both geometry descriptions are based on the same nodes and garantees the accuracy of the frequency difference between the two shapes.

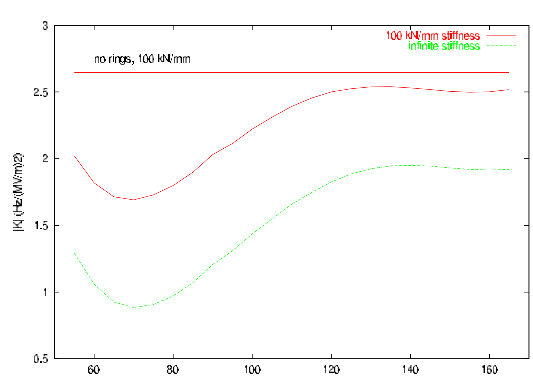

Figure 2: Ring radius optimisation

The optimisation of ring radius for the medium β 5-cells is illustrated in figure 2. For this cavity, made of 3.8 mm thick niobium, the optimum ring radius is 70 mm for a Lorentz detuning coefficient K=1.7 Hz/(MV/m)^2. The cavity stiffness κ_{cav} is kept to a low value of 1.7 kN/mm with the optimal rings, which should help operating the cavity with a low tuning force. The effect of finite κ_{ext} results in a two fold amplification of K, but no significant difference for the optimum radius location. The value of K without rings is given as a reference.

3 DYNAMIC BEHAVIOR MODEL

3.1 *Second order model*

A second order model can be set up to include the possible resonant behavior of cavity detuning Δf. Corresponding to each mechanical eigenmode of the cavity with angular frequency ω_m and quality factor Q_m, we use the equation

$$\frac{d^2\Delta f_m(t)}{dt^2}+\frac{\omega_m}{Q_m}\frac{d\Delta f_m(t)}{dt}+\omega_m^2\Delta f_m(t)=-k_m\omega_m^2E_{acc}^2(t) \quad (2)$$

to describe the contribution Δf_m of m^{th} mode to the cavity detuning. In this equation, k_m is the dynamic Lorentz coefficient of m^{th} mode. The total cavity detuning is

$$\Delta f(t)=\sum_{m=1}^{N}\Delta f_m(t), \quad (3)$$

where N is the number of mechanical modes included in the model. When all coefficients of equation 2 are known, the time dependant detuning can be computed for an arbitrary RF pulse by solving the N independant 2^{nd} order equations numerically.

3.2 *Principle of harmonic analysis*

A convenienent method to determine k_m is to modulate the radiation pressure at angular frequency ω_m in order to excite the m^{th} mode only. Substituting $E_{acc}^2(t)$ by $E_0^2\sin\omega_m t$ in equation 2 and using standard methods of linear systems analysis, one can derive the steady state of $\Delta f_m(t)$:

$$\Delta f_m(t)|_{harmonic}=E_0^2k_mQ_m\sin(\omega_m t-\pi/2) \quad (4)$$

4 NUMERICAL COMPUTATION OF K_M

4.1 *Mechanical modes*

CASTEM 2D numerical simulations have been carried out to determine the axi-symetrical mechanical modes of the cavity with optimised rings. Frequency distribution is shown in figure 3. A band structure can be observed : modes under 1 kHz correspond to axial displacements of groups of cells, last modes to cell modes. Figure 4 represents typical modes : the first one is the lowest frequency mode. The second is a result of the coupling of individual cell modes. The last one is a combination of higher order cell modes. The last two modes have been chosen since their corresponding k_m are among the highest.

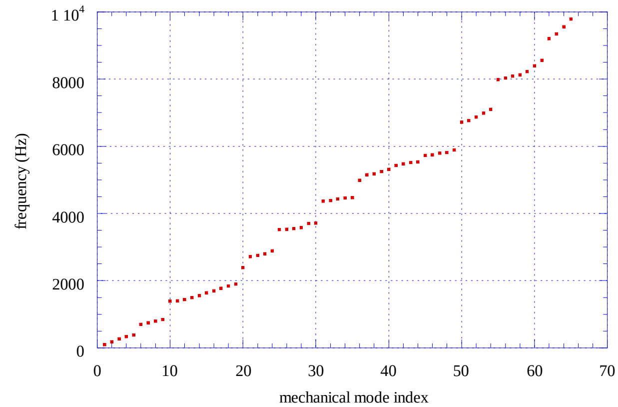

Figure 3: Eigenfrequencies of medium β cavity

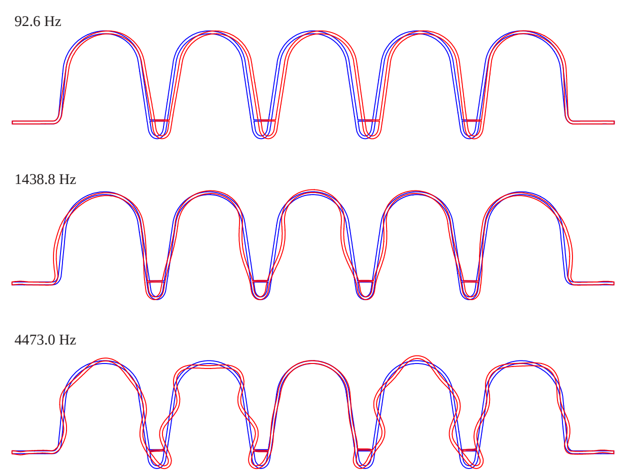

Figure 4: Three typical mechanical modes. Red countour is the deformed cavity shape (amplified)

4.2 *Harmonic response of the cavity*

The general method employed in time domain simulations is to generate a modal basis out of N mechanical modes. A damping coefficient is assigned to each mode. Once the radiation pressure distribution is projected on this modal basis, the time domain problem is solved as a system of N differential equations, therefore needing much less computing power than direct method (time domain FEM calculation). Since our goal is to excite modes on resonance, direct method with linear materials would be inefficient, since modes practically have no width. The modal calculation allows one to choose the mode width such that modes are separated although wide enough to be driven at resonance. Time domain simulations have been carried out with N=65 modes in the modal basis, corresponding to a maximum frequency of 10 kHz. Each mode is excited harmonically in turn, and a set of deformed shapes in steady state regime are sampled inside a mode period. The RF frequency of each of these shapes is computed with Superfish. The value of k_m is derived from the fit of frequency versus time data to equation 4. The first 55 coefficients have been computed and are shown in figure 5. For mechanical modes with frequencies above 8 kHz, coefficients are decreasing to values of the order of 2 % of strongest k_m.

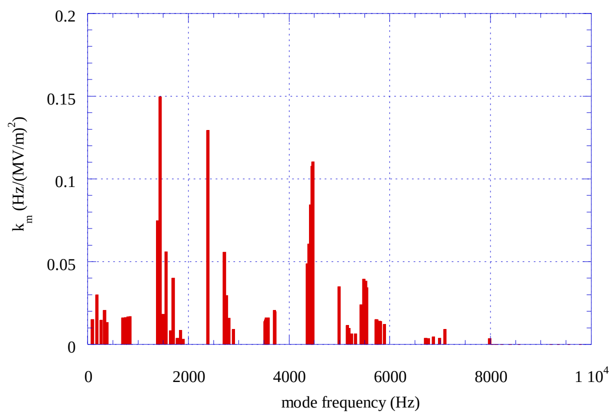

Figure 5: First 55 k_m of medium β cavity

4.3 *Step response reconstruction*

The cavity response to a unit step of radiation pressure is modeled by setting $E^2_{acc}(t) = E^2_0\Theta(t)$ where $\Theta(t)$ is the Heaviside function and $E_0 = 1MV/m$ in equation2. The steady state is then found to be $\lim_{t\to\infty} \Delta f_m(t) = k_m$ for m^{th} mode. The total detuning is thus simply $\Delta_f = \sum_{m=1}^{N} k_m$ when steady state has been reached. Using the 55 k_m values computed for the medium β cavity, $\sum_{m=1}^{N} k_m$ = 1.65 Hz/(MV/m)^2, which is to be compared to the $|K|$ = 1.7 Hz/(MV/m)^2 coefficient given by static detuning calculation. This indicates that a sufficient number of modes have been taken into account in order to reproduce the static behaviour successfully.

5 EXTERNAL STIFFNESS

The influence of κ_{ext} on static K can be approximated the following way: the loaded cavity exerts a force on the tuner represented by stiffness κ_{ext}. The variation of cavity length Δl_{ext} is thus proportional to P_{rad}/κ_{ext}. If $\kappa_{ext} \gg \kappa_{cav}$, extra detuning is $\Delta f_{ext} = \Delta l_{ext} df/dl$. In this condition, the static coefficient K can be expressed as the sum of κ_{ext} dependent and independent detunings:

$$|K(\kappa_{ext})| = |K_\infty| + \frac{F_z}{\kappa_{ext}}\frac{df}{dl} \qquad (5)$$

where K_∞ is static K computed for an infinitely stiff tuner, and F_z the axial component of the force at beam tube end due to radiation pressure for E_{acc} = 1 MV/m.

In dynamic analysis, modes should behave in distinct ways. Modes whose displacement field is low at beam tube, such as high frequency cell modes, should respond to radiation pressure independently of κ_{ext}. In contrast, cavity modes of the first band involve large displacements at cavity ends : taking lower values of κ_{ext} should greatly enhance the k_m coefficients of this particular modes. This is illustrated on figure 6 where the first 40 computed k_ms for κ_{ext} equal to 20 kN/mm and 100 kN/mm can be compared: k_m of the first two passbands increase dramatically for the lower value of κ_{ext}. This clearly favors a stiffer tuning system for pulsed operation.

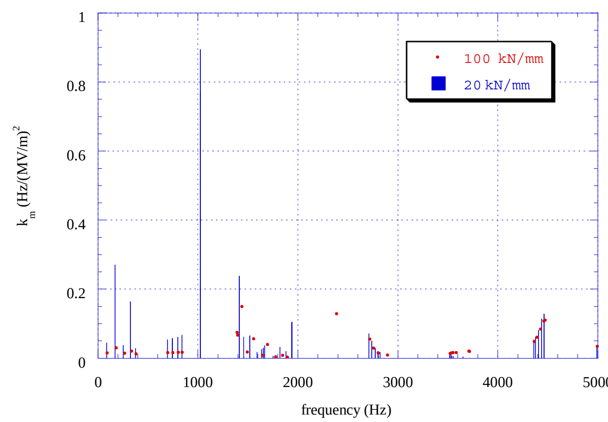

Figure 6: Influence of κ_{ext} on the first 40 k_m

6 CONCLUSION

We evaluated the dynamic Lorentz coefficients using a combination of mechanical and RF computations. The method of modal analysis with N=65 modes was found to be adequate for mechanical calculations. The k_m coefficients can be included in a second order model of the time dependant detuning of the cavity. This model allows one to determine the frequency shift induced by an arbitrary RF pulse. The k_m coefficients of the optimised medium β cavity were computed for a stiff and a looser tuning system. Choosing a stiffer tuner improves dramatically the dynamic detuning behaviour of the cavity.

7 REFERENCES

[1] TESLA Technical Design Report,DESY, 2001

[2] ESS Technical Report, Volume III, 2002

[3] M. Luong et al., Minimizing RF Power Requirement and Improving Amplitude/Phase Control for High Gradient Superconducting Cavities, these proceedings

[4] A. Mosnier, Control of SCRF Cavities in High Power Proton Linacs, these proceedings

[5] A. Matheisen, H. B. Peters, A. Mosnier, Mechanical properties of the 1.3 GHz TESLA Cavity,Proc. of the XVth Int. Conf on High Energy Accelerators, Hamburg, July 20-24, 1992