TRANSITION RADIATION FORMATION ZONE IN WAVEGUIDE

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Abstract

The field formation zone (time) is investigated when a charged bunch is crossing a perfectly conductive wall of a semi-infinite regular waveguide. The problem arises when two-beam acceleration mechanism is considered based on transition radiation effect in waveguide (cavity). The case of periodical train of bunches is considered, when a simple-mode regime is set at the frequency which coincides with bunches repetition rate of the beam. The features of Cherenkov radiation phenomenon are discussed, as well.

1 INTRODUCTION

The two-beam acceleration mechanism based on transition radiation (TR, relativistic klystron - RK) or on Cherenkov radiation (ChR, wake field) has been considered before in many papers. Some problems connected with the wave formation in the cavity, which is traversed by beam has been considered by us in [1,2]. Meanwhile the problem of radiation formation zone hasn't been investigated sufficiently. In this work we define and study the TR formation zone for single particle and for train of bunches and consider some properties of ChR in waveguides.

2 TRANSITION RADIATION FORMATION IN WAVEGUIDE

The transition radiation formation zone in vacuum is usually defined as [3-5]

$$Z_0 = \lambda \gamma^2 , \qquad (1)$$

where λ is the wavelength and $\gamma = \dfrac{E}{mc^2}$ is Lorenz-factor. This parameter (Z_0) becomes too large for radio wave region and high energies of the particles. At this distance the radiation field and particle's own field are diverged and may be detected independently. At distances smaller than Z_0 defined from (1) both of the fields, the radiation field and own one can not be detected separately.

Now let's consider the case, when TR is produced in a waveguide when a point-like particle q moving with the velocity v along the axis z of the waveguide is crossing the boundary of metal-vacuum in a semi-infinite waveguide. For the flux of the electromagnetic field energy through the cross-section of the waveguide one may write down the following formula in the form of

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Poynting vector flux:

$$W = 2\, q^2 v^2 \sum_n \chi_n^2 \left|\psi_n\right|^2 \times$$

$$\times \int_{\omega_{cr}}^{\infty} \frac{\omega - (\omega + \gamma_n v)\cos\left[(\omega - \gamma_n v)\dfrac{Z}{v}\right] + \gamma_n v}{\gamma_n \left(\omega^2 - \gamma_n^2 v^2\right)^2} d\omega , \qquad (2)$$

where ω is the radiation frequency, ω_{cr} is the critical frequency in the waveguide, χ_n are the eigenvalues and ψ_n are the corresponding eigenfunctions, $\gamma_n = \left(\dfrac{\omega^2}{c^2} - \chi_n^2\right)^{1/2}$ are the wave propagation constants of the considered regular waveguide. The first and the third terms in (2) are due to the TR and charged particle's own fields respectively, whereas the second one describes the interference between these two fields. In this case the wave forming length is defined from the condition that the interference between the radiation and own field becomes zero after integrating over a small frequency interval. Besides, the maximum value of the wave forming length is achieved when the group velocity $v_{grn} = \dfrac{d\omega}{d\gamma_n}$ becomes equal to particle velocity v.

Thus, it is easy to show that for the case of the waveguide the formation length is equal to

$$Z_0' = \lambda_{cr}\, \gamma , \qquad (3)$$

where $\lambda_{cr} = \dfrac{2\pi}{\chi_n}$ is the critical wavelength of the waveguide. The difference between the waveguide and free space cases is conditioned by the circumstance that TR in waveguide is effectively emitted on the main modes. Using Brillouin's conception about the wave propagation in the waveguide and introducing the angle $\vartheta = \arccos c\gamma_n/\omega$ one can show that $\lambda_{cr} = \dfrac{\lambda}{\sin\vartheta}$. Only for very high modes $\sin\vartheta \sim \vartheta \sim \gamma^{-1}$ and $Z_0 = \lambda\gamma^2$ as in the case of free space. For the main modes the distance Z_0 will be defined as $\lambda_{cr}\,\gamma$, i.e. much smaller the vacuum Z_0. This is the main distinction between the waveguide and free space cases in TR wave forming problem.

The integration (2) must be carried out over the minimal frequency interval

$$\Delta\omega \cdot \frac{Z_0}{v} \geq 2\pi \qquad \text{or} \qquad \Delta\omega \geq \frac{\omega_{cr}}{\gamma} .$$

As one can see the width of the frequency spectrum becomes narrower by γ times.

From the argument of the interference term in (2) it follows that TR field and particle own field become independent each from other if

$$\Delta t >> Z\left(\frac{1}{v} - \frac{1}{v_{ph}}\right). \qquad (4)$$

At $Z = Z_0$ this term becomes a negligible small one. In (4) the phase velocity of the wave in the waveguide is $v_{ph} = c\left(1 - \frac{c^2 \chi_n^2}{\omega^2}\right)^{-\frac{1}{2}} > c$. If detector's integration time is smaller than (4) the detector will register two different signals due to the own field and TR one. In the wave zone the radiation field and the own one are separated spatially and don't interact with each other. On the contrary, if

$$\Delta t << Z_0\left(\frac{1}{v} - \frac{1}{v_{ph}}\right) \text{ and/or the detector is placed at a}$$

distance smaller than formation zone, the full signal will be detected which consists of the TR field and the own one, as well.

3 TR OF THE PERIODICAL TRAIN OF THE BUNCHES IN THE RADIO WAVEGUIDE

Let's consider the case, when TR is produced by a periodical train of N bunches (point-like) with distance d between neighboring bunches. For the radiation of periodical train of bunches in the waveguide the frequency distribution of the energy is described by the integrand in formula (2) multiplied by $\sin^2 N\frac{\omega d}{2v} / \sin^2 \frac{\omega d}{2v}$. Using the formula (see [3])

$$\frac{\sin^2 N\frac{\omega d}{2v}}{\sin^2 \frac{\omega d}{2v}} = \frac{2\pi N}{d}\sum_n \delta\left(\frac{\omega}{v} - \frac{2\pi}{d}n\right),$$

one may integrate the received expressions at the frequency ω within the narrow interval $\Delta\omega = \frac{2\pi v}{Nd}$ around the frequency $\omega = \frac{2\pi v}{d}$, which is equal to the frequency of the repetition rate of the bunches. The corresponding factor of the interference terms (the cosine argument) then doesn't depend on the frequency. Now, the formation length may be defined from the formula (3) and is equal to $z_0 \geq Nd$, i.e. it is much larger than the

length of the train Nd while the corresponding formation time will be $\sim Nd/v$.

Thus at the radio wave range, when the formation length of radiation becomes too large for relativistic particles, the radiation intensity measurements are very complicated, especially in the pre-wave zone and the separation of the contributions of the particle own field from that of the radiation is necessary.

4 CHERENKOV RADIATION PHENOMENON

Now let's discuss the case, when the semi-infinite waveguide is filled with dielectric medium and ChR condition is fulfilled, i.e. $\beta\sqrt{\varepsilon} > 1$. In this case the formula (2) is rewritten in the form:

$$W = 4q^2 v^2 \pi^2 \sum_n \chi_n^2 |\psi_n|^2 \times$$

$$\times \int_{\omega_{cr}}^{\infty} \frac{\sin^2\left[(\omega - \gamma_n v)\frac{Z}{2v}\right]}{\pi^2\left(\frac{\omega - \gamma_n v}{2v}\right)^2} \frac{d\omega}{\varepsilon\gamma_n(\omega + \gamma_n v)} d\omega, \qquad (5)$$

where $\gamma_n = \sqrt{\frac{\varepsilon\omega^2}{c^2} - \chi_n^2}$.

Using the formula (4) for enough large value Z one obtains:

$$\frac{\sin^2\left[(\omega - \gamma_n v)\frac{Z}{2v}\right]}{\pi^2\left(\frac{\omega - \gamma_n v}{2v}\right)^2} \rightarrow 2v\frac{Z}{\pi}\delta(\omega - \gamma_n v).$$

Then one can rewrite (5) in the form

$$W = q^2 \pi Z \sum_n \frac{|\psi_{n0}|^2}{\varepsilon(\omega_{cher\,n})}. \qquad (5a)$$

Here ω_{chern} is defined from the equation $\omega_n = \gamma_n v$.

For the case of circular waveguide with radius R

$$\psi_n = \frac{J_0(\chi_n R)}{\sqrt{\pi R^2 J_1(\chi_n R)}}.$$

Replacing in (5a) the summation over χ_n by integration when radius R tends to infinity ($\chi_n = \frac{\pi(n + 3/4)}{R}$, $\Delta\chi = \pi/R$), we receive from (5)

$$\frac{e^2}{c^2} \int_{\sqrt{\varepsilon}\beta>1} \left(1-\frac{1}{\beta^2\varepsilon}\right)\omega\,d\omega\,, \qquad (6)$$

that is the well-known formula of Tamm and Frank for Cherenkov radiation energy per a unit of length. The results (5)-(6) have been received taking into account both of the fields - radiation and particles own fields.

5 CONCLUSION

In the previous investigations devoted to two-beam acceleration method the problem of the influence of the charge particle's own field on the acceleration has not been taken into account. As it has been shown above the own field participates in the acceleration mechanism and has to be taken into account certainly. Although formation zone for cm band is much shorter than in case of longer wavelength nevertheless it is essentially larger than real longitudinal dimension of the cavity. Theoretically strict expressions (5)-(6) are received when one takes into account the own field, as well.

6 REFERENCES

[1] E.A Begloyan, E.D. Gazazyan., V.G. Kocharian and E.M. Laziev, "The Interaction of the Charged Particles Bunches and Waveguide-Cavity Structures", LINAC-94 Proceeding, Tsukuba, v.1, p.p. 230-232.

[2] E.D. Gazazyan, M.I. Ivanyan, A.D. Ter-Pogossyan., A.S. Vardanyan, "Cherenkov Radiation Mechanism in Two-Beam Acceleration Problems", EPAC-2000, Vienna, p.p. 468-470.

[3] M.L Ter-Mikaelyan, "High energy Electromagnetic Processes in Condensed Media", Wiley Interscience, New York, 1972.

[4] G.M. Garibian, Shi Yan, "X-Ray Transition Radiation", Publishing House of the Academy of Sciences of Armenian Republic, Yerevan, 1983.

[5] V.L. Ginzburg and V.N.Tsitovich, "Transition Radiation and Transition Scattering", Adam Hilger, Bristol, 1990.