NUMERICAL CALCULATION OF TRAPPED MODES IN TESLA CAVITIES CONSIDERING PRODUCTION TOLERANCES[*]

W.F.O. Müller, W. Koch, T. Weiland
TEMF, Technische Universität Darmstadt, Germany

Abstract

Trapped modes in TESLA cavities, which were measured at TTF, are presumed to be induced by production tolerances. While the rf properties of the perfectly shaped TESLA cavity are thoroughly investigated, the simulation of production tolerances causes a higher numerical effort. One reason is the statistical nature of these errors. Thus one has to perform series of calculations of different cavities. The other reason is the resolution of the numerical method, which has to cope with deformations in the micro meter range.

Here we present numerical tools, which allow the automatic production of different deformed cavity models for the CST MicroWave Studio™ (MWS), which handles also deformations smaller than one mesh step (by means of the Perfect Boundary Approximation). While some trapped modes are already found in the perfect geometry, the deformations induce additional ones, especially the "externally" trapped modes. But, up to a certain amount, the detuning of the higher order modes leads to better beam stability, because long range synchronous modes are eliminated. Thus we are going to simulate the whole tuning procedure to predict trapped modes, which may be harmful to the beam and to estimate tolerable limits for the production tolerances.

1 INTRODUCTION

Trapped modes, which came up the first time at the TESLA Test Facility (TTF) [1] are now confirmed by numerical calculations [2] and warm copper model measurements [3]. The Q value of some of these trapped modes exceeds 10^6 [4]. Therefore an unwanted accumulation of field energy and an influence on the beam dynamic are to be expected.

There are two different mechanisms which lead to trapped modes: The so called internally trapped ones (see Fig. 1) are modes which do not couple to the beam pipes. Caused by their field geometry they do not couple to the beam pipe wave guide modes even above the cut off frequency of the beam pipe mode. These trapped modes already exist within perfectly shaped cavities. But if we take into account the production tolerances, the resonance frequencies of different cavities are no longer the same. If one cavity is surrounded by two cavities with shifted pass bands, there are some modes at the ends of the pass bands for the inner cavity which fall into stop bands for the neighboring cavities. Thus the field energy is trapped in the inner cavity (see Fig. 2).

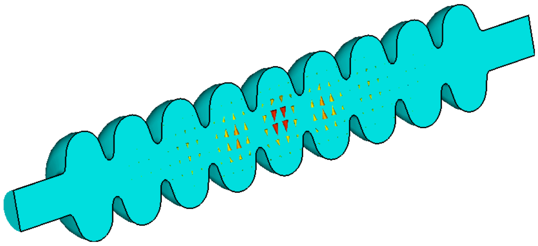

Figure 1: internally trapped mode in TESLA cavity

Due to the small group velocity and nearly no coupling from cell to cell of the trapped modes they are very sensitive to small changes in geometry which are caused by production tolerances. Thus it is necessary not only to simulate the ideal cavities with numeric tools but also to take into account the small statistical variations of the cavity walls. For these calculations it is necessary to study the production procedure first, in order to design a realistic deformation model.

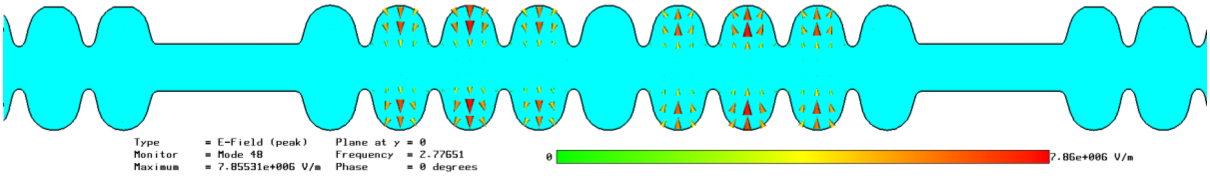

Figure 2: externally trapped mode in TESLA cavities (f=2.777 GHz)

---

[*] Work supported by BMBF under contract 01SF9983/0

2 PRODUCTION PROCEDURE

The production procedure [5] of the TESLA cavities starts from niobium sheets which were formed to half cells by deep drawing (see Fig. 3a). Only some of them are tested for mechanical length and resonance frequency; no corrections are made. Dumb-bells are assembled from two of these half cells by welding them at the iris together with a stiffening ring (Fig. 3b). Length and resonance frequency are measured and the length is corrected by trimming at the equator. Eight of these dumb-bells are assembled together with end cups and beam pipes to the complete 9-cell cavity (Fig. 3c) and the resonance frequency and field flatness of the ground mode are corrected on the tuning machine.

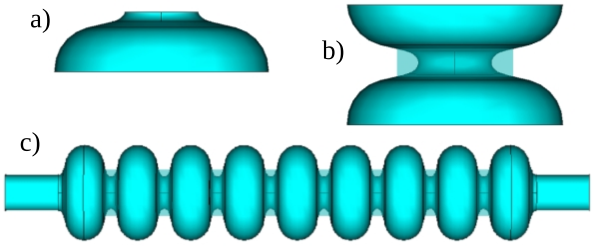

Figure 3: half cell (a), dumb-bell with stiffening ring (b) and complete 9-cell cavity (c)

3 SIMULATION OF PRODUCTION TOLERANCES

Due to the statistical nature of production tolerances it is necessary to calculate a variety of different deformed cavities. Therefore we wrote a program which automatically produces sets of different cavities by varying some parameters of the half cells and combines them to 9-cell cavities (Fig. 4).

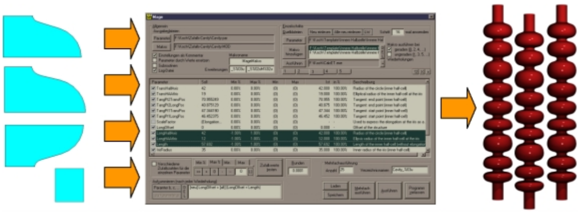

Figure 4: deformation and assembling program

The program MaGe takes the MWS macros of ideal shaped cavity parts (beam pipes and half cells) as input and generates macros with systematic or random variation of selected parameters. These parts are combined to complete 9-cell cavities. An other program, called Queue, sends all these macros one after the other to MWS for the calculation of the resonance frequencies and stores the results in separate files for further evaluation.

4 SIMULATIONS

4.1 Deformation Methods

We first tested some simple deformation methods for the half cells and improved them to fit better to physical deformation and to measurement results for the frequency shift caused by stretching. Four methods are presented in Fig. 5. The first one (a) is a simple linear stretching of the cell which is simple to implement but not quite realistic. The second one (b) keeps the length of the two dimensional contour constant: a stretching in longitudinal direction causes a reduction in radius. The third one (c) takes the stabilization of the stiffening ring into account. And the last one (d) is a linear variation of the deformation proportional to the radius.

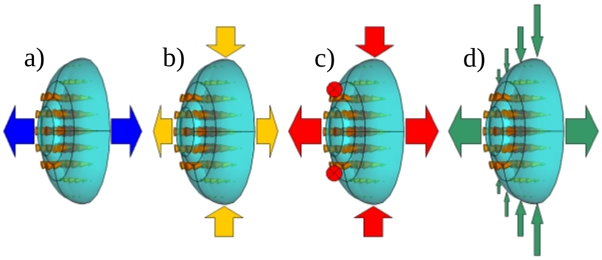

Figure 5: deformation methods for half-cells

In table 1 the calculated frequency shift caused by a change in length of 1 mm for a half cell for these four different deformation methods are shown. These values compare to the measured value of 5.67 MHz/mm. Method (a) differs not only in a factor of ten but also has the wrong sign. Method (c) with the stiffening ring shows roughly the realistic behaviour.

Table 1: simulation results

Fig. 5	deformation method	detuning (MHz / mm)
a	elongation in longitudinal dir.	-0.57
b	constant contour length	9.27
c	same as b with stiffening ring	7.85
d	radial variation of deformation	9.94

4.2 Lorentz Force Calculation

To get a more realistic model of the mechanical deformation we used the results of a Lorentz force calculation [6]. Fig. 6 shows the shape of one cell as blue line for the regular cavity and as black line for the contracted one. This deformation was used as basis for a parametric model in MicroWave Studio. The calculated frequency shift is shown in table 2; it is a factor of two bigger than the measured value.

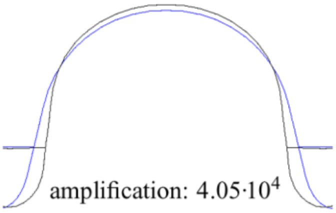

Figure 6: deformation by Lorentz forces

4.3 Perturbation Calculation

In order to cross-check these results we made a perturbation calculation for the accelerating mode in a half cell. The TM010-π-Mode was calculated with electric boundary at the equator plane and magnetic boundary at the iris opening. Then the electric and magnetic field energy at the displaced wall is calculated by volume integration and added up with the right sign (red and blue regions in Fig. 7):

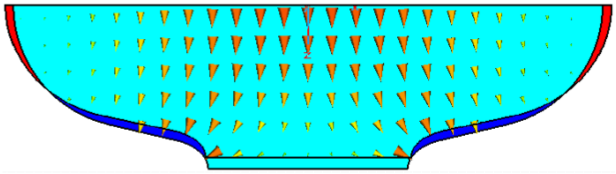

Figure 7: perturbation calculation for half-cell

The energy of the displaced field in relation to the total energy stored in the electromagnetic fields leads to the frequency shift by Slater's formula:

$$\frac{f-f_0}{f_0} = \frac{\int_{\Delta V} (\mu|H_0|^2 - \varepsilon|E_0|^2)dV}{\int_{V} (\mu|H_0|^2 + \varepsilon|E_0|^2)dV} \tag{1}$$

This volume integration for a curved object in a cartesian grid is not very precise but confirms the MWS results sufficiently (see table 2). There is still a factor of two between the calculations and the measurement results. But that's just for the simple reason that the deformation simulation we used for our model is made for the Lorentz force detuning not for stretching the whole cavity. Due to the stiffening rings the deformation is quite similar but it is not the same. So it is necessary to perform a numerical simulation of the mechanical deformation of a TESLA cavity with stiffening rings.

Table 2: simulation results

Detuning	Half Cell (MHz / mm)	9-Cell Cavity (kHz / mm)
MicroWave Studio	12.65	703
Perturbation Calc.	13.41	745
Measurement	5.67	315

4.4 Mechanical Deformation Calculation

In collaboration with K. Rothemund, Univ. Rostock, we are planning a deformation calculation with PERMAS for a TESLA cavity under pressure in longitudinal direction to resemble the effect of the *in situ* tuning mechanism. The setup for the calculation is shown in Fig. 8.

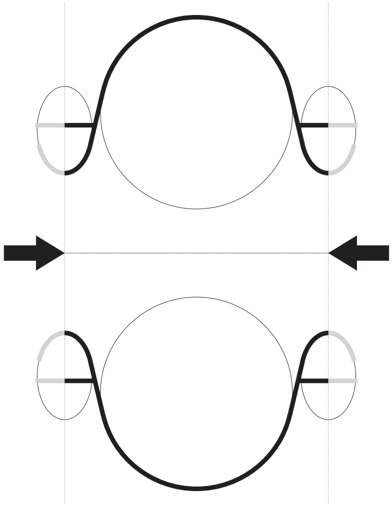

Figure 8: simulation of deformation

5 REFERENCES

[1] N. Baboi, M. Dohlus, C. Magne, A. Mosnier, O. Napoly, H.-W. Glock, "Investigation of a High-Q Dipole Mode at the TESLA Cavities"; Proceedings of the 7th European Particle Accelerator Conference, pp. 1107, Vienna, 2000

[2] R. Schuhmann, T. Weiland: "Rigorous Analysis of Trapped Modes in Accelerating Cavities"; Physical Review Special Topics – Accelerators and Beams, Vol. 3, 122002, 2000

[3] F. Marhauser, P. Hülsmann, H. Klein: "Trapped Modes in TESLA Cavities"; Proceedings of the 1999 Particle Accelerator Conference, New York, 1999

[4] W.F.O. Müller, R. Schuhmann, T. Weiland: "Calculating Trapped Modes in TESLA Cavities with Time and Frequency Domain Methods"; Proceedings of the Particle Accelerator Conference 2001, pp. 273, Chicago, 2001

[5] G. Kreps, D. Proch, J. Sekutowicz: "Half-Cell and Dumb-Bell Frequency Testing for the Correction of the TESLA Cavity Length"; Proceedings of the 9th Workshop on RF Superconductivity, pp. 499, Santa Fe, New Mexico, USA, 1999

[6] H. Gassot et al.: "Mechanical Stiffening of SRF niobium cavities by thermal sprayed coating"; Proceedings of the 9th Workshop on RF Superconductivity, pp. 472, Santa Fe, New Mexico, USA, 1999