# EXTRACTION BEAM ORBIT OF A 250 MeV SUPERCONDUCTING CYCLOTRON* 

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## Abstract

A superconducting cyclotron based on proton therapy facility is being developed at Huazhong university of science and technology (HUST). Due to the compact size of the main magnet, the beam orbits at the extraction region are distributed densely, which creates difficulties for beam extraction leading to severe beam loss. In order to deal with these challenges, the orbit precession method has been employed in the extraction system design. In this paper, we introduce a method of employing a first harmonic field near the $v_{r}=1$ resonance where the beam energy is about 248 MeV to adjust the amplitude of beam orbit oscillation. The optimum amplitude and phase of the first harmonic field are designed to obtain a large turn separation in the extraction region. Three different ways of generating the first harmonic field are compared for optimization.

## INTRODUCTION

As a kind of radiotherapy, proton therapy is becoming increasingly more accepted, which is preferred for most tumors due to minimal damage to healthy tissues and precise local dose control. Proton therapy is considered to be the most effective radiotherapy for cancer, with a cure rate of $80 \%$ [1].
The superconducting cyclotron HUST-SCC250 based on proton therapy facility has excellent advantages of economy and compactness, but it also complicates the electromagnetic structure. The orbit separation at the extraction region is usually smaller than the beam size, which makes the extraction efficiency very low. Resonant extraction denotes the focusing oscillation is coherently excited, thus enhancing the distance between successive turns and facilitating extraction of the beam.
In this paper, particle tracking code CYCLOPS is used to analyze the magnetic field and find the location of $v_{r}=1$ resonance, where the first harmonic covering a radial range of 2 cm is introduced to increase the turn separation [2]. In order to find the appropriate phase of the first harmonic field, the particles are tracked by CYCLONE code so that the amplitude of the field is determined to be 12 Gs, and the phase is $20^{\circ} \sim 45^{\circ}$. Finally, there are three ways to create the first harmonic field.

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## STATIC ORBIT PROPERTIES ANALYSIS

The static equilibrium orbit (SEO) characteristics of the given magnetic field maps plays an important role in the study of cyclotron extraction orbit. The level of isochronism of the field is represented by the difference of the nominal rf angular frequency $\omega_{0}$ and the revolution angular frequency of particle along the SEO $\omega$, which is given as $\left(\omega_{0} / \omega-1\right)$. Figure 1 shows the evolution of the isochronism from low energy of 5 MeV to the extracted beam energy of 252.6 MeV , which is calculated with an energy step of 0.4 MeV .


Figure 1: The isochronism parameter $\left(\omega_{0} / \omega-1\right)$ vs. energy.

The isochronism of the given magnetic, which varies around zero over the energy range, provides essential information for calculating the phase shift by the well-known phase-energy equation:

$$
\begin{equation*}
\Delta(\sin \phi)=\sin \phi_{f}-\sin \phi_{i}=\frac{2 \pi h}{\Delta E} \int_{E_{i}}^{E_{f}}\left(\frac{\omega_{0}}{\omega}-1\right) d E \tag{1}
\end{equation*}
$$

where h is the rf harmonic number, $\Delta E$ is the energy gain per turn. Figure 2 shows the results calculated with $\mathrm{h}=2$ and $\Delta E=0.4 \mathrm{MeV} /$ Turn.


Figure 2: The phase shift $\Delta(\sin \phi)$ vs. energy.
The initial phase $\phi_{\mathrm{i}}=32.8^{\circ}$ is chosen such that the integral of $\sin \phi$ over the whole energy range equals to zero, so

$$
\begin{equation*}
\left(n_{e f f}\right)^{-2}=\left|\frac{d v_{r}}{d n}\right| \tag{4}
\end{equation*}
$$

$v_{r}$ is the radial oscillation frequency, R is the orbit radius at $v_{r}=1$, and the average field $B_{0}$ is calculated at R . The radial position of particles near the extraction can be expressed as:

$$
\begin{equation*}
r=r_{0}+A \cos \varphi \tag{5}
\end{equation*}
$$

where $r_{0}$ is the radial position without harmonic field, $\varphi$ is the phase of the radial oscillation caused by the harmonic field. The turn separation at the extraction region is:

$$
\begin{align*}
\Delta r & =\Delta r_{0}+A \cos \left[\varphi+2 \pi\left(v_{r}-1\right)\right]-A \cos \varphi  \tag{6}\\
& =\Delta r_{0}+A \sin \left[2 \pi\left(v_{r}-1\right)\right]
\end{align*}
$$

Choosing the appropriate initial phase and effective duration to make $\varphi \approx \pi / 2$ at the extraction region, the first harmonic covers the range of the radial oscillation frequency $v_{r}$ from 1.2 to 0.8 , to obtain the required turn separation [3]. Using a field bump with amplitude of 12 Gs , the enough turn separation $\Delta r=5 \mathrm{~mm}$ can be obtained. Figure 3 shows the resulting turn $v_{r}$ vs. energy.


Figure 3: The $v_{r}$ vs. energy.
Figure 3 implies that the $v_{r}=1$ resonance occurs at $\mathrm{E}=247.8 \mathrm{MeV}$ so that the first harmonic field should distribute between $\mathrm{E}=240 \mathrm{MeV}$ and 251 MeV . Assuming the
phase search range of the first harmonic is $0 \sim 2 \pi$, Fig. 4 shows the evolution of the radial phase space from $\mathrm{E}=240$ MeV to 251 MeV with harmonic field phase of $0^{\circ}, 90^{\circ}$, $180^{\circ}$ and $270^{\circ}$ when the amplitude of the field is 12 Gs .


Figure 4: The phase diagrams ( 12 Gs $\theta_{0}=0^{\circ} \sim 270^{\circ}$ ). Phase space motion with precession near the extraction region.

For the harmonic magnetic field with phase of $90^{\circ} \sim 270^{\circ}$ the orbit is staggered and compressed near the extraction, which cannot satisfy the purpose of increasing the turn separation at the extraction region. A set of phase diagrams of beam radial motion was calculated with field phase of $20^{\circ}$, $45^{\circ}$ and $70^{\circ}$, as shown in Fig 5.


Figure 5: The phase diagrams of the extracted beam under different phase. The turn separation of more than 4.7 mm can be obtained in the phase range between $20^{\circ} \sim 45^{\circ}$.

As the beam accelerates through $v_{r}=1$ resonance, there is a slowly growth in orbit displacement up to 4.7 mm for the first harmonic field with phase of $20^{\circ} \sim 45^{\circ}$, which is beneficial to beam extraction. With appropriate harmonic field phase of $\theta_{0}$ the precession extraction will yield a large space between orbits at the entrance angular position of the electrostatic deflector (ESD) $\theta=182^{\circ}$. For the harmonic field with amplitude of 12 Gs and with phase of $45^{\circ}$, the single particle acceleration orbit is obtained as shown in the right portion of Fig. 6. Compared with the
orbit without harmonic field in the left portion of Fig. 6, the turn separation increases obviously.


Figure 6: Left: the acceleration orbit without harmonic field. Right: the acceleration orbit When the amplitude of the harmonic field is 12 Gs and the phase is $45^{\circ}$. The turn separation increases obviously.

## THE WAY OF INTRODUCING FIRST HARMONIC

The generation of the first harmonic can generally be achieved by two methods: (1) In active method, the magnetic field is introduced by trim coils. (2) In passive method, the trim-rods or magnet shimming is employed to generate a disturbance field by changing the structure of the magnet at the extraction region.

## Trim Coil

Trim coils distributed on the hills of the main magnet are employed at the extraction region to create the desirable field. By independently controlling the current of the four coils, the continuous and adjustable amplitude and phase of the first harmonic field can be obtained. The ampere turn of coils should be calculated by 150 to ensure that the amplitude of the field reaches 12 Gs [2].

## Trim-rod

Compared with the trim coil, the magnetic field generated by the trim-rods which are located on the central line of the magnetic pole is more predictable and stable. In order to adjust the amplitude of the first harmonic field from 0 to 10 Gs , the depth of trim-rod should exceed 15 cm [4]. Due to the wider radial distribution of the magnetic field generated by the trim-rod, its side effect on the isochronal field is inevitable. Therefore, it is necessary to re-shimming the magnet after determining the position of the rest positions of trim-rods.

## Magnet Shimming

Trim coil and trim-rod are always avoided in superconducting cyclotron accelerating single particle species. The beam extraction of this kind of cyclotron is mainly achieved by radial and axial pole shaping.

The magnet shimming is an iterative process, which consists of two steps: (1) Determining the field error (2) Predicting the modification of the magnet pole shape according to the field error in step (1) [5].

For the second step, the modification of the pole shape can be achieved by three methods: (1) The methods based on analysis or experience; (2) Linear transformation of magnetic field errors using hard-edge approximation; (3) Matrix method including non-linear edge field effect. The hard-edge approximation can be used to transform field errors into shape changes, thus a series of studies discussed the magnet model and the effect of edge cutting on the local magnetic field. A. Papash et al. proposed a shimming algorithm based on a set of analytic formulas [6]. W. Kleeven et al. introduced a more sophisticated solution to relate pole shaping with a large computational matrix containing the effect of isochronous field errors and harmonic field effects [7].

## CONCLUSION

This paper introduces the design of beam orbit on the extraction of 250 MeV superconducting cyclotron. Utilizing several codes, such as MATLAB, CYCLOPS and CYCLONE, to track the orbit, an optimized first harmonic with the phase of $20^{\circ} \sim 45^{\circ}$ is selected, which contributes a turn separation of 4.7 mm . Three ways of generating the first harmonic are introduced, which are trim coil, trim-rod, magnet shimming, respectively.

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