THE DEFINITION AND THE USE OF THE CENTRAL POSITION PHASE IN CYCLOTRONS

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Abstract

The high frequency phase (HF phase), normally used in cyclotrons, is replaced by the central position phase (CP phase). This CP phase gives a proper description of the behaviour of energy and phase of the particle motion and enhances the insight in the total motion (longitudinal and radial) in the median plane, especially when high harmonic acceleration is involved. The use of the CP phase is illustrated with numerical calculations.

A very important consequence for injection systems in cyclotrons is treated. Also a proper interpretation of the HF phase behaviour, measured with HF pick-up probes is given.

1. Introduction

In the description of the particle motion in cyclotrons it is customary to characterize this motion by the high frequency phase (HF phase) among other quantities. The HF phase is defined as the time of gap crossing of a particle with respect to the maximum Dee voltage over the gap. However, in the study of the orbit motion on the first few turns in the cyclotron or in the case of a very high harmonic number, funny results are obtained when a phase space area started with one HF phase is followed over several turns. An example is given in section 3, fig. 3. Especially for single turn extraction, the phase of a particle with respect to the acceleration voltage is very important. We define now a new phase, which will replace the HF phase and facilitates a study, numerical or theoretical, of the orbit motion in the central region of a cyclotron or in case of high harmonic acceleration.

2. The definition of the central position phase

The horizontal orbit motion in a homogeneous magnetic field is most conveniently described by the motion of the particle on a circle and the position of the centre of this circle. See fig. 1 and ref. 1. The four variables describing the position of the particle on the circle and the position of the centre of this circle are proper canonical variables and they can be constructed via a simple canonical coordinate transformation from the normal two position and two velocity variables [1]. The energy of a particle is determined by the radius of the circle motion. The canonical conjugated variable of the energy is the phase of the circle motion. The energy gain per turn is, in most important order, determined by the phase of the circle motion[1]. The time of gap crossing and thereby the HF phase depends on both the centre motion and circle motion. Consider for instance the situation in fig. 1, when the Dee gap lies along the x-axis. The HF phase at both gapcrossings are different.

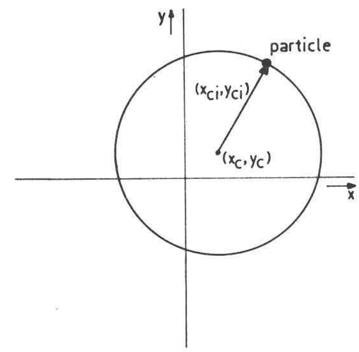

Fig.1: *The particle motion is represented by the circle motion and the centre motion*

To emphasize the difference between both phase definitions, we will call the phase of the circle motion the central position phase (CP phase), as it determines the phase of the particle motion with respect to its centre position. The relation between HF phase (ϕ_{HF}) and CP phase (ϕ_{CP}) at the moment the particle crosses the positive x-axis, is given by

$$\phi_{CP} = \phi_{HF} - h \times \arcsin(y_c/r)$$
$$\text{or } \phi_{CP} = \phi_{HF} - h \times \arcsin(p_r) \qquad [1]$$

h = harmonic number
r = radius of circle
p_r = radial momentum (mrad) at the x-axis

Concluding, there are two main reasons to prefer the CP phase above the HF phase:

1. The CP phase is independent of the centre motion and therefore unique on one turn. A measurement of the HF phase at an arbitrary azimuth does not yield a good value for the estimation of the energy gain.
2. The CP phase and the energy are a pair of canonical variables. The HF phase and the energy are not a pair of canonical variables and may therefore show an apparent coupling between, at one hand, energy and phase and, at the other hand, the centre motion. This becomes more pronounced in the case of high harmonic numbers.

3. Illustrations of the use of the CP phase

In numerical orbit integration codes it is customary to print the HF phase every time the particle crosses a gap. Performing a calculation with a centered and off-centered particle, we find, however, for the off-centered particle a HF phase that oscillates with a frequency of $\nu_r - 1$ as is sketched in fig. 2 for a special case in the VICKSI cyclotron.

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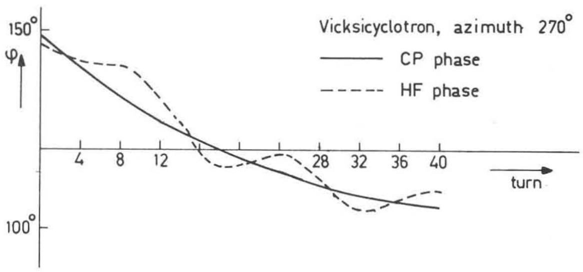

Fig. 2: *HF phase and CP phase at 270° azimuth in the VICKSI cyclotron. Δr= 40 mm at the injection radius, r= 440 mm, h= 6, oscillation amplitude = 10 mm.*

This figure demonstrates clearly the influence of the centre motion on the time of gap crossing. Especially in the case of a short bunch of about 6°, needed for single turn extraction, these effects are important. If we print instead of HF phase the CP phase, the off centered particles have the same behaviour as a function of turn number as the centered particles.

The oscillations in the HF phase, which are strongly correlated to the orbit centre position, may not be seen as a coupling between longitudinal and radial phase space. The oscillating character only appears because the HF phase is not a proper canonical variable of the motion. The CP phase, instead, is a convenient canonical variable which is conjugated to the energy (see ref. 1).

A study of the motion in radial phase space, especially for large harmonic numbers, will lead to misleading results if a set of particles is started on the same HF phase, with different starting conditions in centre coordinates. These particles will have different CP phases (see [1]). Therefore they will gain per turn a different amount of energy according to their CP phase, and consequently particles with different energies are compared. An example is shown in fig. 3 in which an area is started at the injection radius of the VICKSI cyclotron with the same HF phase. The harmonic number is 6. The results after different turns are given. Funny oscillations in surface appear, even so strong that at larger radii the orientation of the area changes. If we repeat this calculation with particles that all have the same initial CP phase, the phase space does not show the funny oscillations and has a constant surface when the momentum p_r is expressed in mrad (see fig. 4). The coupling with the E-ϕ_{CP} space has no significant influence as concluded also in ref. 1. In a calculation for the Louvain-la-Neuve cyclotron the difference between CP phase and HF phase in case of a p_r equal to 0.05 at 70 mm radius (h = 3) is as much as 12°.

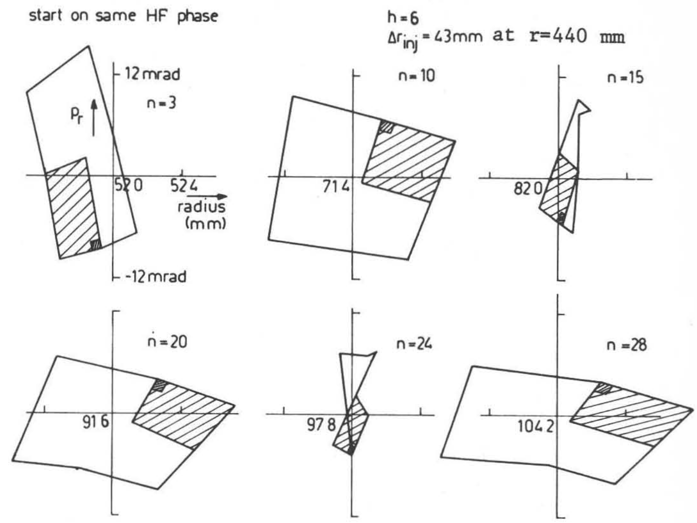

Fig. 3: *An area in radial phase space followed after several turns. The oscillations are due to different energies because of different CP phase.*

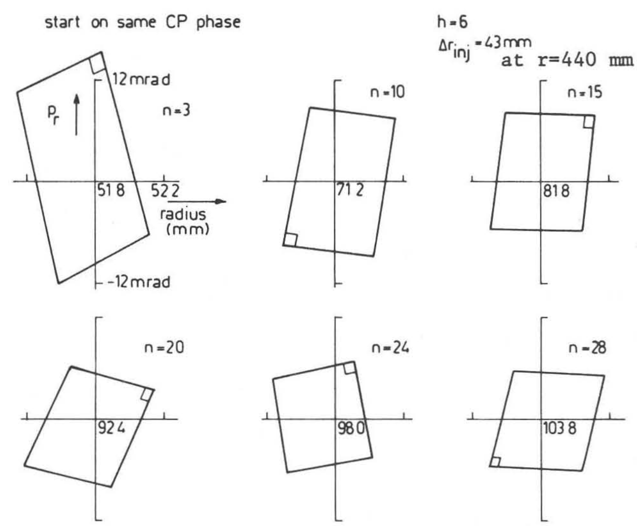

Fig. 4: *An area in radial phase space started at same CP phase. A proper behaviour of the beam is seen.*

3.1. The accelerated equilibrium orbit (AEO)

The HF phase of the AEO will generally not be equal to its CP phase, due to a similar effect as described above. The relation between the HF phase, measured when a particle passes the mid-valley line of a Dee-free valley in the VICKSI cyclotron, and the CP phase can be estimated when examining the centre motion of the AEO, sketched in ref. 1, fig. 3.

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At this valley line, the orbit centre is displaced about $\frac{1}{4}\Delta r$ (Δr is turn separation) perpendicular to the valley line. Consequently p_r equals $\frac{1}{4}\frac{\Delta r}{r}$ or $\frac{1}{8}\ \Delta E/E$. The difference between the two phases in the case of a turn separation of 40 mm (r_{inj} = 440 mm) and h=6 is, according to [1], equal to about 8^o at injection and nearly 0^o at extraction. This difference as a function of radius is sketched in fig. 5.

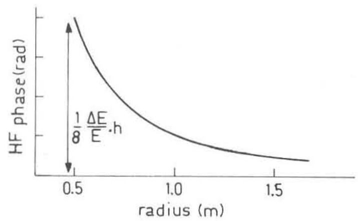

Fig. 5: *The expected HF phase at 270° azimuth in case of a perfect isochronous field.*

Since the CP phase is the proper canonical variable conjugated to the energy, it will be clear that in programs that calculate isochronous fields, the HF phase curve as given in fig. 5 should be the goal. Values of CP phase equal to zero yield maximum energy gain per turn and for other CP phases this energy gain is proportional to $\cos\phi_{CP}$. In cyclotrons with non-interceptive HF phase measuring equipment the phase probes are in general positioned on one azimuth. From the signals the HF phase is calculated. Consequently the desired phase curve must be a curve similar to the $\Delta E/E$ curve in fig. 5.

In the calculations for Ganil in France these curves were also found. In one of the cyclotrons the harmonic number is 14 and HF phases of nearly 30° are reported for maximum energy gain [3].

In [2] some experimental illustrations of the above mentioned phase behaviour, obtained at the VICKSI cyclotron with the HF phase measuring equipment, is given.

3.2. Injection lines

Although we have indicated already how to adjust the magnetic field to obtain a proper isochronous field, we still have to assure that particles, injected in a very short bunch (about 6°), all start at the same CP phase If no attention is paid to it, an enlargement up to 10° can be expected in the VICKSI cyclotron when h=6, due to variations in initial p_r.

Let us examine an injection line in which no acceleration occurs and in which we assume magnets with a horizontal symmetry plane. The transfer matrix for the horizontal and longitudinal motion in such a system has the general form:

$$M = \begin{pmatrix} M_{11} & M_{12} & 0 & M_{14} \\ M_{21} & M_{22} & 0 & M_{24} \\ M_{31} & M_{32} & 1 & M_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{corresponding vector :} \quad \begin{pmatrix} x \\ x' \\ l \\ l' \end{pmatrix} \qquad [2]$$

x = horizontal displacement perpendicular to the optical axis in mm

l = displacement along optical axis with respect to central particle

' = differentiation with respect to the l direction in mrad.

The symplectic condition for M is

$$M^T S M = S \qquad S = \begin{pmatrix} 0 & 1 & . & . \\ -1 & 0 & . & . \\ . & . & 0 & 1 \\ . & . & -1 & 0 \end{pmatrix} \qquad [3]$$

With this condition, it follows that

$$\begin{pmatrix} M_{31} \\ M_{32} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} \begin{pmatrix} -M_{24} \\ M_{14} \end{pmatrix} \qquad [4]$$

Let us now consider an injection line from an arbitrary point in the beam guiding system between pre-accelerator and cyclotron up to a certain point on the first turn in the cyclotron. At this point the HF phase, expressed in the final coordinates of the matrix notation, is:

$$\phi_{HF} \simeq l_f \frac{h}{r} + \phi_{HF}, \text{ central particle} \qquad [5]$$

h = harmonic number

r = average radius in the cyclotron belonging to the injection energy

f = final

A particle with positive l_f has a more positive HF phase than the central particle. The CP phase can also be expressed in this way, using [1]

$$\phi_{CP} = \phi_{HF} - h.\arcsin\ (p_r)$$

or

$$\phi_{CP} \simeq \phi_{\text{central particle}} + l_f \frac{h}{r} - \dot{x}_f \cdot h \qquad [6]$$

We can express the final l_f in the initial coordinates (index i)

$$l_f = l_i + M_{34} l_i' + M_{31} x_i + M_{32} \dot{x}_i \qquad [7]$$

with [4] this becomes

$$l_f = l_i + M_{34} l_i' - M_{24} x_f + M_{14} \dot{x}_f \qquad [8]$$

When the beam is bunched before the considered system, there exists a relation between l_i and l_i' such that the sum of the first two terms on the rightside equals zero. To achieve the same CP phase at the final position for all particles, there has to be a correlation between l_f and x_f, indicated by [6]. This correlation is achieved when $M_{14} = r$ and $M_{24} = 0$. Hinderer first formulated these demands. He observed that these demands are equivalent to dispersion matching of the injection line.[4]

More illustrations of the use of the CP phase in theoretical studies of longitudinal and radial coupling can be found in another contribution to this conference[1].

4 Conclusions

The CP phase, the canonical conjugated variable of the energy, is the proper substitute for the HF phase when orbit motion in the central region or high harmonic acceleration has to be studied, numerically or theoretically. In the use of phase probes it helps to understand the measured phase behaviour.

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ref. 1 W.M. Schulte and H.L. Hagedoorn. "Accelerated Particles in an AVF cyclotron with a one or two Dee system". 8th Int.conf.on cycl., Bloomington.

ref. 2 W.M. Schulte "The theory of accelerated particles in AVF cyclotrons", Thesis, University of Technology, Eindhoven 1978.

ref. 3 A. Chabert e.a., "Etude théorique et numérique de l'acceleration des particles dans les Css du Ganil", Ganil internal report 77-005.

ref. 4 G. Hinderer, European Cyclotron Progress Meeting, Milan 1976

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