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ACCELERATED PARTICLES IN AN AVF CYCLOTRON WITH A ONE OR A TWO DEE SYSTEM

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Abstract

A general analytical theory is given, describing the motion of accelerated particles in an AVF cyclotron with a one or a two Dee-system. A Hamiltonian is formulated for the horizontal motion of these particles. Especially on the first turns, where the energy gain per turn is not small with respect to the particle energy, the structure of the Dees has much influence. For high harmonic acceleration, the motion in the radial phase space may even become unstable. A coupling exists between radial and longitudinal motion. In a few examples the applicability of the formulated theory is illustrated.

1. Introduction

In the past the analysis of accelerated particles in cyclotrons has generally been carried out in two ways. In the first way, the theoretical description of non-accelerated particles is used and the acceleration is taken into account by assuming that the kinetic energy is a smooth function of the independent variable, mostly the azimuth. In the second way, numerical orbit integration codes are used. These codes normally produce a huge amount of data and the interpretation of the results is often not easy.

In this paper we give a theoretical description in which the acceleration is included from the start. The description results in a simple model, which describes the influence of the accelerating structure on the horizontal particle motion. With this model an accurate study in many different cyclotron configurations is possible. The dependent variables are quantities with a simple physical meaning: energy, phase and centre position.

For example, the model is helpful when high harmonic acceleration is intended. As the acceleration of heavy ions requires mostly high harmonic numbers, the model forms a powerful tool to investigate whether the acceleration of heavy ions can be carried out in the required manner in a certain configuration (existing or in design).

In ref. 1 an introduction to the theory which leads to the model is given. In the present paper we describe the particle motion in a slightly different manner. We will present the final model equations for a one and a two Dee system. Some examples will outline applications of the model.

2. General approach

To simplify the reasoning, we consider at first the particle motion in a homogeneous magnetic field. With induction B_o pointing in the z-direction, the Hamilton function for the particle motion in this magnetic field becomes

$$H = \frac{1}{2m}(P_x + \tfrac{1}{2}qB_o y)^2 + \frac{1}{2m}(P_y - \tfrac{1}{2}qB_o x)^2 + \frac{1}{2m}P_z^2 \qquad [1]$$

where m is mass, q is charge and P_x, P_y and P_z are the momentum variables, conjugated to x, y and z, respectively. For simplicity we will ignore the vertical motion. For convenience we scale the variables in eq. 1 by dividing the momenta by qB_o, the Hamilton function by $m\omega_o^2$, with $\omega_o = qB_o/m$, and by multiplying the time with ω_o. The dimension of the momenta is now length. The time is dimensionless.

The horizontal motion of the particle can now be separated into the motion on a circle and the motion of the centre of this circle, also called the guiding centre. The generating function of the transformation which yields this splitting is

$$G = \tilde{P}_x x + \tilde{P}_y y - \tilde{P}_x\tilde{P}_y - \tfrac{1}{2}xy$$

where

$$x = \tilde{P}_y + \tilde{x} \qquad P_x = \tfrac{1}{2}(\tilde{P}_x - \tilde{y})$$
$$y = \tilde{y} + \tilde{P}_x \qquad P_y = \tfrac{1}{2}(\tilde{P}_y - \tilde{x}) \qquad [2]$$

The new Hamilton function is

$$H = \tfrac{1}{2}\tilde{P}_x^2 + \tfrac{1}{2}\tilde{x}^2 \qquad [3]$$

The variables $\tilde{P}_y$ and $\tilde{y}$ are cyclic variables, so that they are constant. This corresponds to the fixed position of an orbit centre in a homogeneous magnetic field. We now transform the $\tilde{x}$ and $\tilde{P}_x$ coordinates into the corresponding action and angle variables and introduce a coordinate system which moves with the particle. The new variables are named E and ϕ.

$$\tilde{x} = \sqrt{2E}\cos(\phi - t) \qquad \tilde{P}_x = \sqrt{2E}\sin(\phi - t) \qquad [4]$$

The new Hamilton function is independent of E, ϕ, $\tilde{P}_y$ and $\tilde{y}$, so that all variables are constant. We will omit from now on the tilde on $\tilde{y}$ and $\tilde{P}_y$. The meaning of the variables is now:

E, ϕ : the energy (in square meter) and the phase (relative to the moving coordinate system) of the particle on the circle.
P_y, y: the x and y position of the centre of the circle.

The particle motion in an AVF cyclotron is somewhat more complicated. Applying a few correcting transformations that change the variables only slightly, we may write down a Hamilton function of the above-given variables. The general form of this Hamilton function is

$$H = f(E) + \tfrac{1}{2}(\nu_r - 1)P_y^2 + \tfrac{1}{2}(\nu_r - 1)y^2 \qquad [5]$$

The derivative of the function f(E) with respect to E is equal to the relative deviation of the magnetic field from its isochronous value. ν_r is the radial oscillation frequency and is also a function of E.

The acceleration can now be included by adding to the Hamilton function in eq. 5 a potential function which describes the accelerating electric field. We will treat this for a one and a two Dee system. In ref. 2 an extensive discussion can be found.

3 The one Dee system

A schematical drawing of a one Dee system is given in fig. 1.

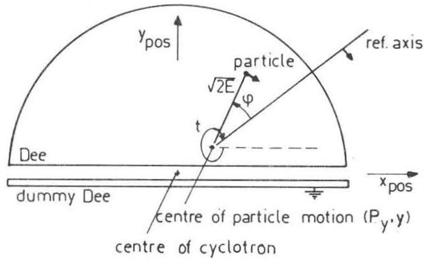

Fig. 1. A schematic drawing of a one Dee-system, in which the newly introduced variables (P_y, y for the centre motion and E, ϕ for the circle motion) are indicated.

The potential function in the original x and y variables (now called x_{pos} and y_{pos}) is

$$V = q\hat{V}\ \mathrm{He}\ (y_{pos})\ \cos h\omega_o t. \qquad [6]$$

He is the Heaviside function (equal to 1 for positive argument and equal to zero for negative argument). q is charge, $\hat{V}$ is amplitude of the accelerating voltage and h is the harmonic number of the acceleration.

As we know the character of the orbit motion, He(y_{pos}) can be written in a fourier series in time with coefficients which are functions of the dependent variables (ref 1,2, 3). By expanding these coefficients, the Hamilton function can be written in a power series in the centre position variables.

The motion can be described with respect to a central ray: the accelerated equilibrium orbit (AEO). Theoretically this orbit can be found by looking for the transformation so that all terms which are linear in one variable disappear. For the centre motion of the AEO, we find a motion which, within one turn, steps from $(P_y,y)= (-\frac{1}{2}dr,0)$ to $(\frac{1}{2}dr,0)$ and back again (dr is the turn separation per turn, see also ref. 2). For convenience we describe the centre motion with respect to the centre motion of the AEO. We do not change E and ϕ, as they are easy to understand and commonly used. Taking only the most important order into account, we arrive at the following Hamilton function.

$$H = f(E) + \frac{q\bar{V}}{\pi}\frac{\sin h\phi}{h} \qquad [7]$$

$$+\tfrac{1}{2}(\nu_r-1)(y^2+P_y^{\ 2}) - \frac{q\bar{V}}{\pi}\sin h\phi\ \frac{hy^2}{2E}$$

where proper scaling has been applied to the potential function

$$q\bar{V}= \frac{q\hat{V}}{m_o\omega^2}$$

The variable ϕ is the phase of the particle motion with respect to the centre position and is called the central position phase (CP phase when expressed in RF degrees c.q. radians). In a separate contribution to this conference, the definition and the use of the CP phase are given. (Note that due to the choice of the magnetic field, the particles move clock-wise).

A dangerous situation occurs when $q\bar{V}h\cdot\sin(h\phi)/2\pi E > \nu_r - 1$. In that case the flowlines in the centre space are hyperbolas and the motion is unstable (see ref. 3).

The extension of the gap can also be taken into account. The effect of phase focussing (ref. 4) is then clearly described (ref 2). Also the equivalent first harmonic field perturbation effect of the gap crossing resonance follows from this theory in a clear way (ref 2).

4 The two Dee system

In fig. 2 a schematic drawing of a two Dee system is given.

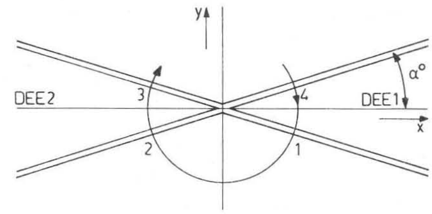

Fig. 2. A schematic drawing of a two-Dee system with half-Dee angle equal to α .

The corresponding potential function for this situation, in which both Dees oscillate in phase (push-push mode), is

$$V = q\hat{V}\ \ \mathrm{He}(\mathrm{arg})\sin h.\omega_o t \qquad [8]$$

$$\text{with arg} = -(\cos\alpha\ y_{pos} + \sin\alpha\ x_{pos}).$$

$$(\cos\alpha\ y_{pos} - \sin\alpha\ x_{pos}).$$

The treatment is now analogous to the treatment in section 3 (see also ref 1,2). For the centre motion of the AEO, we can distinguish three different situations: $\nu_r = 0$, $\nu_r > 1$ and $\nu_r < 1$. For these situations the centre motion of the AEO is sketched in fig. 3.

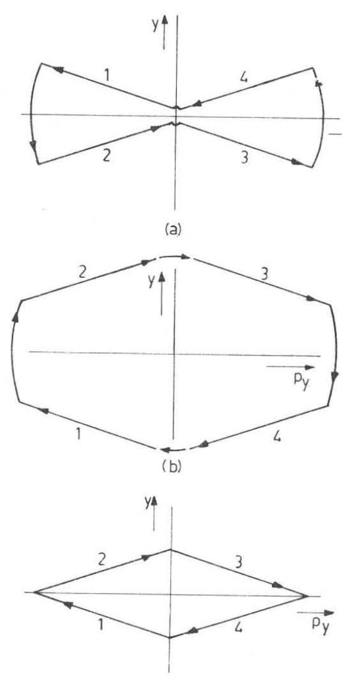

Fig. 3. The centre motion of the AEO for a radial oscillation frequency larger (a), smaller (b) and equal (c) to 1. The particle motion is clockwise. The numbers refer to the gap crossing in fig. 2.

The particle motion is described in y,P_y coordinates relative to the centre motion of the AEO and in E,ϕ coordinates, which describe the circle motion. The final Hamilton function is:

$$H = f(E) + \frac{q\bar{V}}{\pi}\; 2\,\frac{\sin h\alpha}{h}\,\sin h\phi \qquad [9]$$
$$+ \tfrac{1}{2}(\nu_r - 1)\;(y^2 + P_y^{\,2})$$
$$+ C_m\, yP_y\, \frac{\cos h\phi}{E}$$
$$+ D_m\,(\tfrac{1}{2}\, y^2 \cos^2\alpha + \tfrac{1}{2}\, P_y^{\,2}\sin^2\alpha)\,\sin h\phi/E$$

with $C_m = 2\,\frac{q\bar{V}}{\pi}\, h \cos h\alpha \cos \sin\alpha$

$D_m = -2\,\frac{q\bar{V}}{\pi}\, h \sin h\alpha$

5 Some applications

The motion of a particle can be found by integration of Hamilton's equations:

$$\frac{dy}{dt} = \frac{\partial H}{\partial P_y}\;,\quad \frac{dP_y}{dt} = -\,\frac{\partial H}{\partial y} \qquad [10]$$
$$\frac{dE}{dt} = \frac{\partial H}{\partial \phi}\;,\quad \frac{d\phi}{dt} = -\,\frac{\partial H}{\partial E}$$

Note that the particles move clock-wise. For anti-clock-wise motion, the variables y and ϕ have to change sign. The independent variable t may be replaced by the turn number and the variable ϕ may be replaced by the CP phase (ref 5). In the approach given above, the relation between ϕ and CP phase (ϕ_{cp}) is $\phi_{cp} = -h\phi$.

In ref. 3 the model is applied to the situation at the TRIUMF cyclotron. Another example of a one Dee cyclotron is the cyclotron of the Eindhoven University. Normally this cyclotron accelerates only light ions, as protons, deutrons and α-particles. Heavier ions may be used when the acceleration is performed on the third harmonic. With our model we examined the centre motion in the y,P_y space for CP phases = -50^o, -40^o, -30^o. In fig. 4 the results of model calculations after twenty turns are given. It will be clear that acceleration on the third harmonic is not without difficulties. (Remember that the centre space is equivalent to the radial phase space). Solutions to reduce the stretching of the areas, due to coupling between E, ϕ and y, P_y variables, are already mentioned in ref. 3.

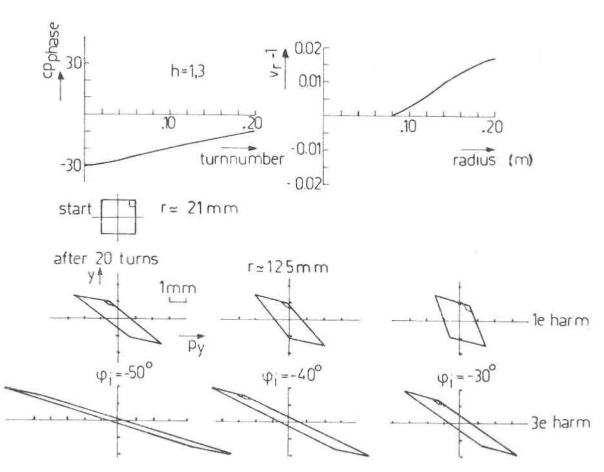

Fig. 4. Calculations for the Eindhoven cyclotron.

Not only the y,P_y motion is affected by this coupling. In fig. 5 the behaviour of an area in the P-ϕ_{CP} plane is given. The final CP phase width reduces for larger y values. The corresponding results with numerical orbit integration in a field map of the electric field (obtained with the magnetic analogue method, see ref 6) are seen in fig. 6. The agreement fails because the model calculations were not properly adapted to this situation. (Extra electric components are present that shift the y position). Two things may, however, be remarked. The effect of the coupling shows up in a similar way, and an extra bunching effect appears due to the extention of the gap. The extra bunching effect can be explained with the theory, but was not included in the model equations presented here. (A full treatment is given in ref. 2.)

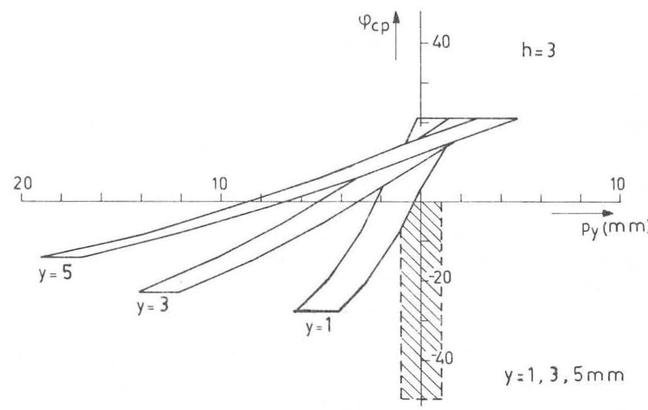

Fig. 5. The areas in $P_y\phi_{CP}$ plane after 20 turns for different y-values.

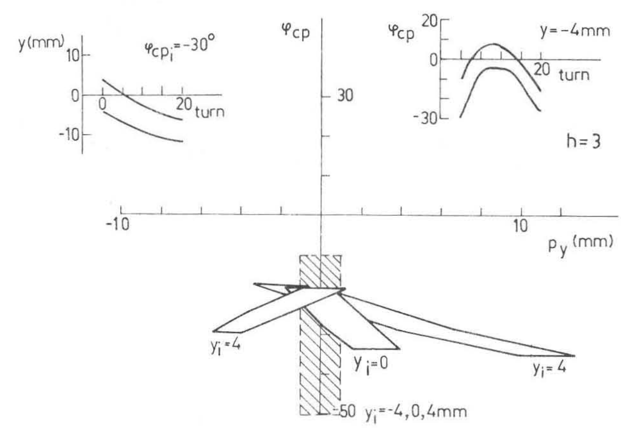

Fig. 6. Corresponding results from numerical orbit integration.

For the VICKSI cyclotron ($h_{max} = 6$) it turns out that the influence of the accelerating structure does not deform an area in centre space noticeably. Yet, by the last terms in [9], a phase dependent coupling is present, which introduces a CP phase dependent radial oscillation phase. Fig. 7 shows the oscillation phase after 50 turns as a function of the CP phase for a turn separation of 40 mm at injection radius (440 mm) and a harmonic number equal to 6. Results from numerical orbit integrations agree very well. The dotted line represents the results that are obtained when the influence of the accelerating structure is neglected.

This omission is clearly not permitted. This CP phase dependent oscillation phase is important for the centering process and in case deliberately off-centered beams are used.

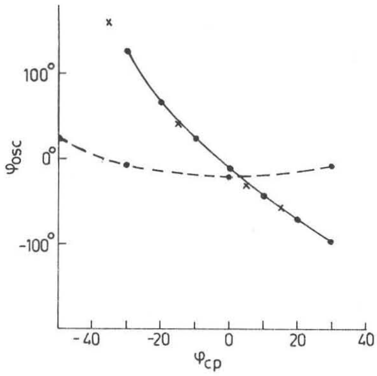

Fig. 7. The oscillation phase after 50 turns in the VICKSI cyclotron as function of CP phase.

In ref. 2 similar results are given for other turn separations (i.e., Dee voltages). In the VICKSI Cyclotron the coupling effects will deform an area in centre space noticeably when h = 8 or higher.

More drastic effects are seen in cyclotrons with an internal source. On the first turns the energy is very small and E appears in the denominator in [7] and [9]. As example we give results of calculations in which a situation was assumed similar to the situation in the cyclotron in Louvain-la-Neuve. The radial oscillation frequency is close to unity and a magnetic cone gives a phase slip over 10^o for h = 1 and 30^o for h = 3. In ref. 2 more detailed information is given. We followed an area in centre space over 20 turns for the harmonic numbers 1,2 and 3. The result is given in fig. 8. It will be obvious that without further action no proper acceleration is possible on the 3rd harmonic. Numerical orbit calculations (see ref. 2) and experiments (ref. 8) confirm this conclusion.

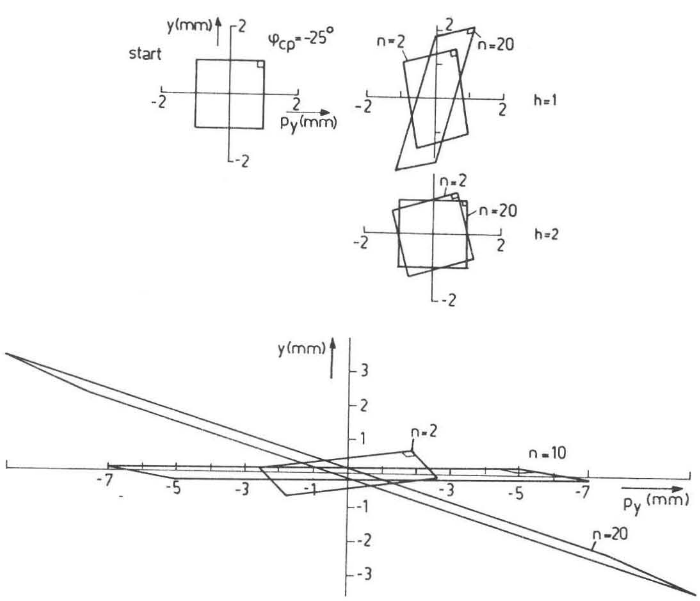

Fig. 8. The areas in centre space for h = 1, 2 and 3 in the cyclotron of Louvain-la-Neuve.

Studying the Hamilton function, the responsible term is the C_m-term in [9]. A most direct compensation of this term can be achieved with a second harmonic magnetic field component (ref. 3). However, a rather high excitation must be applied before a reasonable reduction is possible. (The phase slipping method as suggested by Bolduc, ref. 7, does not function here because the C_m term is hardly phase dependent. In this case the phase would have to be shifted to a value of more than 45^o. This is not realistic because of the vertical focussing.)

6 Conclusions

With the model described above, an accurate investigation of the motion of accelerated particles is possible. In case of high harmonic acceleration, a drastic influence of the accelerating structure on an area in centre space may appear, especially in cyclotrons with an internal source. Generally the agreement between model calculations and extensive numerical orbit integrations is excellent. More information can be found in ref. 2.

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** DISCUSSION **

H. BLOSSER: You mentioned a third harmonic difficulty in your Eindhoven cyclotron. This apparently does not occur in the SIN injector which is very similar. Have you checked this with your formalism?

W. SCHULTE: The formalism can be adapted easily to the situation in the SIN injector. I did not do this. However, the designers of this injector spent a lot of time optimizing the central region. Their central phase lies to more positive values than the (ϕ = -50°, -30°) which I showed. In their case the effect is less pronounced.

G. DUTTO: It seems to me that this effect is quite phase dependent, so for a narrow phase acceptance it is not too serious.

W. SCHULTE: The effect is indeed phase dependent. However, for a large or a small phase width, the effect on the phase space will be the same. The central phase is mostly determined by the vertical focussing. In a one Dee system the phase slipping as applied at TRIUMF will help to reduce the stretching. In a two Dee system phase slipping is less promising and it is more difficult to compensate the effects.