HIRFL-SSC TRIM COIL CURRENTS CALCULATION BY CONJUGATE GRADIENTS METHOD

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Abstract

For accelerating different kinds of ions to various energies, the HIRFL-SSC should form the corresponding isochronous magnetic field by its main coil and trim coils. Previously, there were errors in fitting the theoretical isochronous magnetic field in the small radius region, which led to some operation difficulties for ion acceleration in the inject region. After further investigation of the restrictive condition of the maximum current limitation, the trim coil currents for fitting the theoretical isochronous magnetic field were recalculated by the conjugate gradients method. Better results were obtained in the operation of HIRFL-SSC. This article introduces the procedure to calculate the trim coil currents. The calculation method of conjugate gradients is introduced and the fitting error is analysed.

THEORETICAL ISOCHRONOUS MAGNETIC FIELD

The distribution of isochronous magnetic field along the axis line of a sector is called the theoretical isochronous magnetic field. It can be calculated with Kb-Kr method.

The revolution frequency f of the particles is

$$f = \frac{q}{2\pi m\gamma} \oint_{E.O.} B ds \Big/ \oint_{E.O.} ds \qquad (1)$$

in which q, m and γ are the charge, rest mass and energy factor of the particle, B is the magnetic field intensity at the location of particle, ds is the differential arc of the equilibrium orbit ($E.O.$).

Define K_b, K_r as

$$K_b(B, R_a) = B_a \big/ \langle B \rangle ,\ K_r(B, R_a) = R_a \big/ \langle R \rangle \ , \qquad (2)$$

where B is the level of the magnetic field, R_a is the radius of the equilibrium orbit along the direction of the axis line of sector, B_a is the magnetic field of the point on the axis line of sector with radius R_a, $\langle B \rangle = \oint_{E.O.} B ds / \oint_{E.O.} ds$ is the average magnetic field along the equilibrium orbit, $\langle R \rangle = \oint_{E.O.} ds / 2\pi$ is the average radius of the equilibrium orbit.

The theoretical isochronous magnetic field is

$$B_a(R_a) = \frac{2\pi m f}{q} K_b(B, R_a) \Big/ \sqrt{1 - \left[\frac{2\pi f R_a}{K_r(B, R_a) c}\right]^2} \ , \qquad (3)$$

where c is the velocity of light.

THEORETICAL ISOCHRONOUS MAGNETIC FIELD FITTING

The HIRFL-SSC has four sectors. The isochronous magnetic field is formed by its main coil and trim coils. There are three kinds of trim coils: I type, N type and L type coils. I type coils are connected in series for all the four sectors. N type and L type coils are independent for each sector.

On the sector numbered s, using $e_{ij}(s)$ represents the efficiency of the magnetic field contribution $B_{R_i}(s)$ at the point of radius R_i on the sector axis line by the currents $I_j(s)$ of coil numbered j. It has $\sum_{j=1}^{n} e_{ij}(s) I_j(s) = B_{R_i}(s)$, $i = 1, 2, \cdots, m$, where n is the total number of coils in this sector and m is the total number of points where the theoretical isochronous magnetic field was fitted in this sector. Writing it in matrix format

$$\mathbf{E}(s)\mathbf{I}(s) = \mathbf{B}(s) \qquad (4)$$

in which

$$\mathbf{E}(s) = [\mathbf{E_I}(s) \quad \mathbf{E_{NL}}(s) \quad \mathbf{E_M}(s)],$$

$$\mathbf{I}(s) = [\mathbf{I_I} \quad \mathbf{I_{NL}}(s) \quad I_M]^T,$$

$$\mathbf{I_I} = [I_1 \quad I_2 \quad \cdots \quad I_{n_I}],$$

$$\mathbf{I_{NL}}(s) = [I_1(s) \quad I_2(s) \quad \cdots \quad I_{n_{NL}}(s)],$$

$$\mathbf{B}(s) = [B_{R_1}(s) \quad B_{R_2}(s) \quad \cdots \quad B_{R_m}(s)]^T,$$

where $\mathbf{E_I}(s)$, $\mathbf{E_{NL}}(s)$ and $\mathbf{E_M}(s)$ are the efficiency matrices of the I type coils, N and L type coils and the main coil in this sector. $\mathbf{I_I}$ and $\mathbf{I_{NL}}(s)$ are the current vectors of the I type coils and the N and L type coils. I_M is the current of the main coil. n_I and n_{NL} are the total number of I type coils and N and L type coils in this sector.

Let $R_1 = 1040mm$, $R_2 = 1060mm$, $\ldots$, $R_m = 3380mm$, so $m = 118$. In HIRFL-SSC, $n_I = 25$, $n_{NL} = 9$.

Consider the fitting of theoretical isochronous magnetic field of the four sectors as a whole. And set

$$\mathbf{I} = [\mathbf{I_I} \quad \mathbf{I_{NL}}(1) \quad \mathbf{I_{NL}}(2) \quad \mathbf{I_{NL}}(3) \quad \mathbf{I_{NL}}(4) \quad I_M]^T,$$

$$\mathbf{B} = [\mathbf{B}(1)^T \quad \mathbf{B}(2)^T \quad \mathbf{B}(3)^T \quad \mathbf{B}(4)^T]^T,$$

$$\mathbf{E} = \begin{bmatrix} \mathbf{E_I}(1) & \mathbf{E_{NL}}(1) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{E_M}(1) \\ \mathbf{E_I}(2) & \mathbf{0} & \mathbf{E_{NL}}(2) & \mathbf{0} & \mathbf{0} & \mathbf{E_M}(2) \\ \mathbf{E_I}(3) & \mathbf{0} & \mathbf{0} & \mathbf{E_{NL}}(3) & \mathbf{0} & \mathbf{E_M}(3) \\ \mathbf{E_I}(4) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{E_{NL}}(4) & \mathbf{E_M}(4) \end{bmatrix}.$$

Then

$$\mathbf{E}\mathbf{I} = \mathbf{B} \ . \qquad (5)$$

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Generally equations (5) could not be strict for any current vector $\mathbf{I}$ because there are 62 unknown variables and $118*4$ equations. So considering the least-squares solution of equations (5), introduce the residual vector $\mathbf{F}=\mathbf{EI}-\mathbf{B}$.

$$\mathbf{F}^T\mathbf{F}=(\mathbf{E}^T\mathbf{EI}-\mathbf{E}^T\mathbf{B})^T(\mathbf{E}^T\mathbf{E})^{-1}(\mathbf{E}^T\mathbf{EI}-\mathbf{E}^T\mathbf{B}) -\mathbf{B}^T\mathbf{E}(\mathbf{E}^T\mathbf{E})^{-1}\mathbf{E}^T\mathbf{B}+\mathbf{B}^T\mathbf{B} \quad . \tag{6}$$

The first term of the right side of equation (6) is nonnegative, because $(\mathbf{E}^T\mathbf{E})^{-1}$ is a positive definite matrix if $\mathbf{E}$ is a full rank matrix. The last two terms are constants. So $\mathbf{F}^T\mathbf{F}$ will be minimum if

$$\mathbf{E}^T\mathbf{EI}-\mathbf{E}^T\mathbf{B}=\mathbf{0} \quad . \tag{7}$$

Recall $\mathbf{H}=\mathbf{E}^T\mathbf{E}$, $\mathbf{b}=-\mathbf{E}^T\mathbf{B}$. Equation (7) becomes

$$\mathbf{HI}+\mathbf{b}=\mathbf{0} \quad . \tag{8}$$

Since $\mathbf{H}$ is a symmetric and positive definite matrix, equation (8) is equivalent to the minimization problem of a quadratic function $f(\mathbf{I})=\frac{1}{2}\mathbf{I}^T\mathbf{HI}+\mathbf{b}^T\mathbf{I}+c$, because $\nabla f(\mathbf{I})=\mathbf{HI}+\mathbf{b}$.

CONJUGATE GRADIENTS METHOD

It is desired to minimize the quadratic function

$$f(\mathbf{x})=\frac{1}{2}\mathbf{x}^T\mathbf{Hx}+\mathbf{b}^T\mathbf{x}+c \tag{9}$$

using the conjugate gradients method, where $\mathbf{x}\in E^n$ is the variable vector, $\mathbf{H}$ is an $n\times n$ symmetric and positive definite matrix, $\mathbf{b}$ is a n-dimensional constant vector and c is a constant scalar.

Two vectors $\mathbf{P}_i$ and $\mathbf{P}_j$ are conjugate about $\mathbf{H}$ if

$$\mathbf{P}_i^T\mathbf{HP}_j=0\ ,\quad i\neq j\ ,\quad \mathbf{P}_i^T\mathbf{HP}_i\neq 0\ ,\quad \mathbf{P}_j^T\mathbf{HP}_j\neq 0 \quad . \tag{10}$$

It is known that n vectors $\mathbf{P}_0,\mathbf{P}_1,\cdots,\mathbf{P}_{n-1}$ are linearly independent if they are conjugate about $\mathbf{H}$, so they can form a base vector set in the n-dimensional space.

Assume $\mathbf{x}^*$ is the minimum point of the quadratic function $f(\mathbf{x})$, and $\mathbf{x}^{(0)}$ is an arbitrary beginning point. Vector $\mathbf{x}^*-\mathbf{x}^{(0)}$ can be expressed by the linear combination of base vectors $\mathbf{P}_0,\mathbf{P}_1,\cdots,\mathbf{P}_{n-1}$;

$$\mathbf{x}^*=\mathbf{x}^{(0)}+\alpha_0\mathbf{P}_0+\alpha_1\mathbf{P}_1+\cdots+\alpha_{n-1}\mathbf{P}_{n-1} \quad . \tag{11}$$

The gradient vector of the quadratic function $f(\mathbf{x})$ is

$$\nabla f(\mathbf{x})=\mathbf{Hx}+\mathbf{b} \quad . \tag{12}$$

Because $\mathbf{x}^*$ is the minimum point of the quadratic function $f(\mathbf{x})$,

$$\nabla f(\mathbf{x}^*)=\mathbf{Hx}^*+\mathbf{b}=\mathbf{0} \quad . \tag{13}$$

For calculating the coefficient α_i in the equation (11), left multiply the two sides of equation (11) by $\mathbf{P}_0^T\mathbf{H}$. Utilizing equations (10), (12) and (13), we obtain

$$\alpha_0=\frac{-\mathbf{P}_0^T\nabla f(\mathbf{x}^{(0)})}{\mathbf{P}_0^T\mathbf{HP}_0} \quad . \tag{14}$$

Substituting $\mathbf{x}^{(k)}=\mathbf{x}^{(0)}+\alpha_0\mathbf{P}_0+\cdots+\alpha_{k-1}\mathbf{P}_{k-1}$ in equation (11),

$$\mathbf{x}^*=\mathbf{x}^{(k)}+\alpha_k\mathbf{P}_k+\cdots+\alpha_{n-1}\mathbf{P}_{n-1} \quad . \tag{15}$$

Left multiplying the two sides of equation (15) by $\mathbf{P}_k^T\mathbf{H}$, and utilizing equations (10), (12) and (13),

$$\alpha_k=\frac{-\mathbf{P}_k^T\nabla f(\mathbf{x}^{(k)})}{\mathbf{P}_k^T\mathbf{HP}_k} \quad . \tag{16}$$

For analysing the meaning of α_k, we search for the minimum point of the quadratic function $f(\mathbf{x})$ along the one dimension direction $\mathbf{P}_k$ from the beginning point $\mathbf{x}^{(k)}$. Assume $\lambda=\lambda^*$ is the minimum point of the function $f(\mathbf{x}^{(k)}+\lambda\mathbf{P}_k)=F(\lambda)$, then $F'(\lambda^*)=0$. Written as $\left[\nabla f(\mathbf{x}^{(k)}+\lambda^*\mathbf{P}_k)\right]^T\mathbf{P}_k=0$ with equation (12), we obtain

$$\lambda^*=\frac{-\left[\nabla f(\mathbf{x}^{(k)})\right]^T\mathbf{P}_k}{\mathbf{P}_k^T\mathbf{HP}_k} \quad . \tag{17}$$

Comparing (17) with the equation (16), it can be concluded that: if the n vectors $\mathbf{P}_0,\mathbf{P}_1,\cdots,\mathbf{P}_{n-1}$ are conjugate about $\mathbf{H}$, to calculate the minimum point $\mathbf{x}^*$ of the quadratic function $f(\mathbf{x})$, start from an arbitrary beginning point $\mathbf{x}^{(0)}$, search for the minimum point in one dimension along the direction of n vectors $\mathbf{P}_0,\mathbf{P}_1,\cdots,\mathbf{P}_{n-1}$ sequentially, and get the value of α_k from equation (16). We can obtain the minimum point $\mathbf{x}^*$ from equation (11).

The n vectors $\mathbf{P}_1,\mathbf{P}_2,\cdots,\mathbf{P}_n$, which are conjugate about $\mathbf{H}$, can be formed from an arbitrary set of base vectors $\mathbf{g}_1,\mathbf{g}_2,\cdots,\mathbf{g}_n$ in the n-dimensional space.

Let $\mathbf{P}_1=\mathbf{g}_1$. $\mathbf{P}_2=\mathbf{g}_2+\alpha_1^{(2)}\mathbf{P}_1$. From equation (10) we obtain $\alpha_1^{(2)}=\dfrac{-\mathbf{P}_1^T\mathbf{Hg}_2}{\mathbf{P}_1^T\mathbf{HP}_1}$. Assume that $\mathbf{P}_1,\mathbf{P}_2,\cdots,\mathbf{P}_k$ have been conjugate about $\mathbf{H}$. Let $\mathbf{P}_{k+1}=\mathbf{g}_{k+1}+\sum_{r=1}^{k}\alpha_r^{(k+1)}\mathbf{P}_r$, and also from equation (10) obtain $\alpha_i^{(k+1)}=\dfrac{-\mathbf{P}_i^T\mathbf{Hg}_{k+1}}{\mathbf{P}_i^T\mathbf{HP}_i}$, $i=1,2,\cdots,k$.

Restrictive Conditions

No restrictive conditions have been considered for vector $\mathbf{x}$ in the conjugate gradients method. In fact, the current of each coil has its limitation. In HIRFL-SSC, the restrictive conditions on the main coil current I_M and trim coil currents I_{TC} are

$$\left|I_M\right|\leq 4000A\ ,\quad \left|I_{TC}\right|\leq 240A \quad . \tag{18}$$

If some currents of the solution are outside these restrictive conditions, each time set one of these currents as a known value that is most near its original quantity within the condition (18). Recalculate in the $n-1$ dimensional space, until the solution is in the 'multi-dimensional box' (18). If more than one current is out of bounds, the order of setting the currents will affect the accuracy of the fitting results.

RESULTS AND DISCUSSION

For the particle $^{13}C^{6+}-60MeV/u$, the fitting results of the theoretical isochronous magnetic fields of the four sectors are shown in Figure 1-4. Fitting errors are about $\pm10Gs$ at the radius from $1320mm$ to $1520mm$ for all the four sectors. Fitting errors are less than $\pm5Gs$ on all the other places except at the small radius region of the 4th sector. The fitting errors are $39Gs$ and $-23Gs$ at the radius from $1040mm$ to $1080mm$ of the 4th sector, caused by the sharp change of the theoretical isochronous magnetic fields in the small radius region, as shown in Figure 4. From equation (3), the distribution of $K_b(B,R_a)$ for all the four sectors in the small radius region is shown in Figure 5. The reason for the sharp change of $K_b(B,R_a)$ in the small radius region on the 4th sector is the magnetic perturbation caused by the last injection magnetic element near the small radius region.

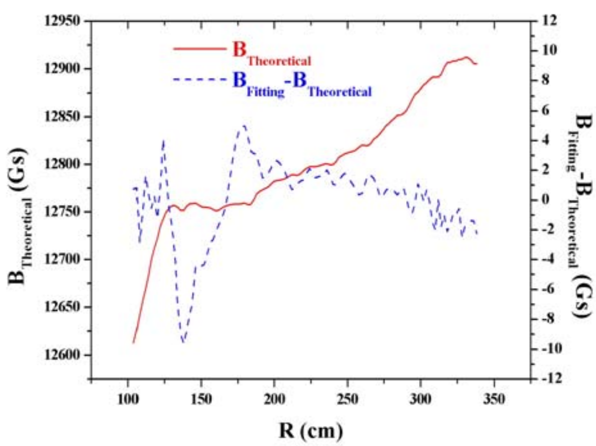

Figure 1. Fitting result of sector one

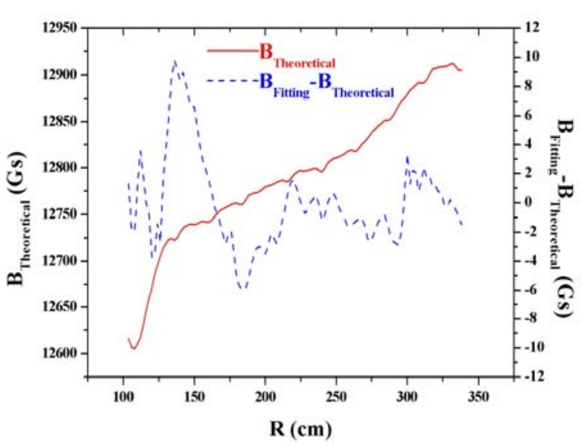

Figure 2. Fitting result of sector two

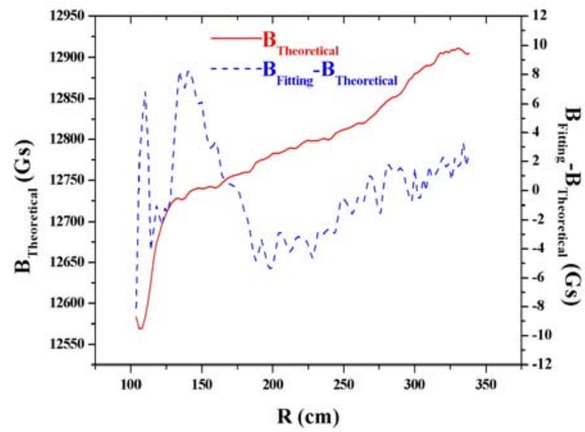

Figure 3. Fitting result of sector three

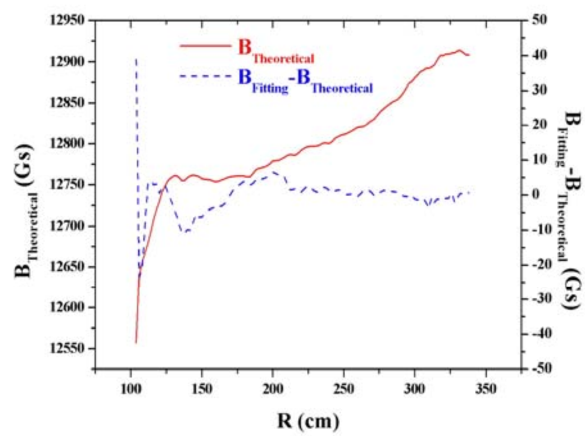

Figure 4. Fitting result of sector four

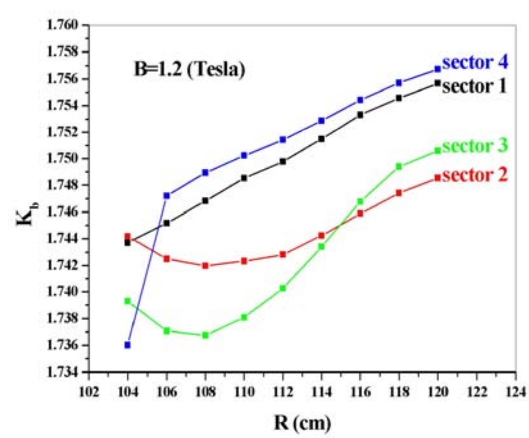

Figure 5. The distribution of K_b

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