BEAM-CAVITY INTERACTION SIMULATION FOR THE PSI RING-CYCLOTRON

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Abstract

Measurements of the radio-frequency spectrum on the main cavities showed an unexpected peak related to a beam-excited Higher Order Mode (HOM). The description of the effect by Maxwell's equations, using a mode-expansion of the electromagnetic fields in the cyclotron and Fourier decomposition of the particle bunches, leads to an analytical expression for the HOM-amplitudes and phases. The eigenmode solver Omega3P allowed to calculate the field basis-functions of the complex rf-structure for the first time ever. Single particle orbits in the measured static magnetic cyclotron field are then computed and provide the amplitudes and phases for the excited HOMs. The trajectory of particle distributions, containing about 100'000 macro-particles, is then simulated with additional correction for the space-charge forces and the Lorentz-forces deriving from the excited HOMs. A Particle In Cell Needle (PICN) model provides the fast space-charge correction in the horizontal direction.

MODE-EXPANSION TECHNIQUE

Starting from Maxwell's equations with potential vector $\vec{A}$ in Coulomb gauge, linear operator $\mathfrak{L}$ and reduced excitation $\vec{F}$, the equation of field evolution in time reads

$$\frac{\partial^2}{\partial t^2}\vec{A} = \mathfrak{L}\vec{A} + \vec{F}(\vec{x},t), \qquad \mathfrak{L} \equiv c^2 \triangle \tag{1}$$

$$\vec{F}(\vec{x},t) \equiv c^2\mu_0(\vec{i} - \frac{\partial}{\partial t}\varepsilon_0 \nabla\Phi) \tag{2}$$

with the Poisson equation for the electrostatic potential Φ of the charges and current density $\vec{i}$. This yields the boundary condition problem described by a linear partial differential equation of second order. The vector fields are then expanded onto a complete set of orthogonal basis-functions $\vec{\xi}_n$ for a harmonic excitation.

$$\vec{A}(\vec{x},t) = \sum_n a_n(t)\vec{\xi}_n(\vec{x}) \tag{3}$$

$$\vec{F}(\vec{x},t) = \sum_n F_n(t)\vec{\xi}_n(\vec{x}) \tag{4}$$

$$a_n = \frac{c_n}{2} + \sum_m \left[a_{nm}\cos(m\omega_0 t) + b_{nm}\sin(m\omega_0 t)\right] \tag{5}$$

$$F_n(t) = \frac{C_n}{2} + \sum_{m=1}^{\infty} \left[K_{mn}\cos(m\omega_0 t) + L_{mn}\sin(m\omega_0 t)\right] \tag{6}$$

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The basis-functions $\vec{\xi}_n$ can be chosen as set of eigenmodes $\vec{e}_n$ of the homogeneous boundary condition problem. If the cavity wall-losses are introduced into Eq. 1 as perturbation, one gets the differential equation of a damped oscillation with damping constant $r_n \equiv \omega_n/Q_n$. For the evolution of the time-functions a_n, with $\lambda_n \equiv \omega_n^2$ as eigenvalue, it yields

$$\ddot{a}_n(t) = -\lambda_n a_n(t) - r_n\dot{a}_n(t) + F_n(t) \tag{7}$$

as can be seen by comparison with the parameters of the energy conservation law.

Fourier Decomposition of the Excitation

Consideration of the space-charge variation term $\partial\nabla\Phi/\partial t$ in Eq. 2 would lead to a self-consistent description of the beam-cavity interaction and could be integrated by time-stepping. However, it is desirable to calculate the field-excitation in frequency domain and to neglect the bunch deformation in the excitation-term (rigid bunch approximation). The beam-excited cyclotron fields are calculated directly for steady-state condition this way, corresponding to infinite operation time after switching-on the beam. The effect of the excited fields onto the particle distribution can then be simulated in a subsequent step.

Calculating the projections of Eq. 1 with Eqs. 3 to 6 yield the Fourier-coefficients for the potential vector, weighted by the resonance curve of the mode

$$c_n = \frac{C_n}{\lambda_n} \tag{8}$$

$$a_{nm} = \frac{K_{nm}(\lambda_n - m^2\omega_0^2) - L_{nm}m\omega_0 r_n}{(\lambda_n - m^2\omega_0^2)^2 + m^2\omega_0^2 r_n^2} \tag{9}$$

$$b_{nm} = \frac{L_{nm}(\lambda_n - m^2\omega_0^2) + K_{nm}m\omega_0 r_n}{(\lambda_n - m^2\omega_0^2)^2 + m^2\omega_0^2 r_n^2}. \tag{10}$$

C_n, K_{nm} and L_{nm} depend on the current distribution in the excitation term in Eq. 2. It follows from Eqs. 9 and 10 that only modes with a resonance frequency close to a beam-harmonic are excited significantly.

One particle bunch flying with velocity $\vec{v}$ can be approximated as a one dimensional wave with transversal density function. If, for a simplified description of the beam-profile, the density-function f is supposed to be a normalized, periodic train of Gaussians of width σ with repetition frequency ω_0, one has

$$f(t,z) = \frac{\omega_0}{2\pi v} + \frac{\omega_0}{\pi v}\sum_{m=1}^{\infty} e^{-\frac{(m\omega_0\sigma)^2}{2v^2}} \cos\left(m\frac{\omega_0}{v}z\right)\cos\left(m\omega_0 t\right) + \frac{\omega_0}{\pi v}\sum_{m=1}^{\infty} e^{-\frac{(m\omega_0\sigma)^2}{2v^2}} \sin\left(m\frac{\omega_0}{v}z\right)\sin\left(m\omega_0 t\right). \quad (11)$$

The action of the beams to cyclotron fields is then the superposition of all the excitation-terms provided by trains of Gaussians at different velocities and positions. For the determination of these parameters, a single-particle trajectory is calculated from injection to extraction. This yields, after scaling of f to the DC-value of the beam current and projection of the particle distributions onto the modes, the coefficients C_n, K_{nm} and L_{nm} in Eq. 6. It results that only modes with electric field components in beam-propagation direction can be excited by the beam and that the amplitudes are proportional to the beam current.

EIGENMODE CALCULATION

For the calculation of the basis-function $\vec{e}_n$ (resp. $\vec{\xi}_n$) in Eq. 3, it is required to calculate a set of eigenmodes. The cyclotron-geometry has no exact symmetry in azimuthal- or vertical-direction and the lower cut-off frequency of 56.4MHz for the beam-slot opening in the main cavities causes propagation of eigenmodes around the cyclotron-structure. As consequence, the eigenmodes must be solved for the entire cyclotron-structure.

The cyclotron-geometry is simplified in order to get a reasonable problem size. It is intricate to calculate the eigenmodes in such a complex structure, where the modes are tightly clustered. SLAC's parallel Exact Shift Invert Lanczos (ESIL)-solver in Omega3P [1, 2] was chosen for this purpose because it guarantees safe numerical-convergence to the desired eigenmodes. Omega3P uses unstructured tetrahedral elements for the discretization of the simulation-volume. This mesh can be generated with the CUBIT [3] program. It is possible to generate a mesh with 1.2 million elements and 246 thousand nodes only. Using curved second order elements, the average edge-length of 6.4cm yields an upper frequency-limit of 700MHz at least. Omega3P was run on the IBM-SP4 of the *Swiss National Supercomputing Centre* [4] to find 280 eigenmodes with a resonance-frequency close to a beam-harmonic. The typical solution-time for 20 modes with 6.4 million degrees of freedom was about 45 minutes using 32 CPUs and required a total memory of about 120GB. The modes in a cyclotron-structure can be classed into three groups. *Cavity-modes* have their field-energy localized in one of the cavities. These modes could also be calculated in a rf-model of the corresponding cavity only. *Vacuum-chamber modes* have most of the field-energy in the vacuum-chamber and almost no field-energy in the cavities. All other modes are *mixed modes* where energy is located in the vacuum-chamber *and* in the cavities. A total of 44 cavity-modes are found, among these evidently also the fundamental modes at 51.04MHz for the four main-cavities and at 150.52MHz for the flat-topping cavity. The vacuum-chamber modes have a vertical electric field distribution, a relatively low quality-factor and appear at relatively low-frequencies only. A total of 18 modes are calculated for this class. Their vertical electric field distribution suggests that their interaction with the beam must be relatively small. Measurements of the lowest two vacuum-chamber modes confirmed the simulation results.

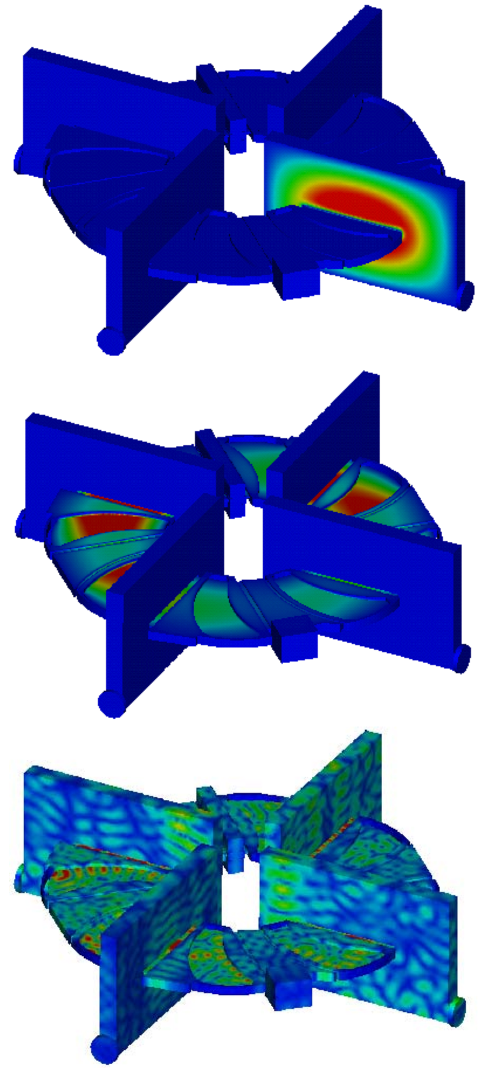

Figure 1: Example of a fundamental cavity- (top), vacuum-chamber (middle) and mixed-mode (bottom)

The mixed-modes are the most numerous ones with a total of 218 modes. They appear at higher frequencies and are tightly clustered. Because of the smaller wavelength, their field-pattern gets very dense. Therefore the beam-interaction terms compensate and become less important. These modes probably couple to the vacuum-port and therefore get slightly detuned when the vacuum-shutter is opened.

ANALYSIS OF MODES

In order to get an estimate of the beam-cavity interaction in the ring-cyclotron, the longitudinal and transverse ”gap”-voltages are calculated. The relevant gap for the particles means in this case the trajectory from cyclotron injection- to extraction-location. Using the method developed in the previous sections, the beam-excited cyclotron fields can be calculated for steady-state condition with the contribution of all the 1320 particle bunches (220 in radial- and 6 of each set in azimuthal-direction).

Only modes with a difference in resonance frequency to the beam-harmonic of less than 1.5MHz are considered, because other modes get heavily damped according to Eqs. 9 and 10.

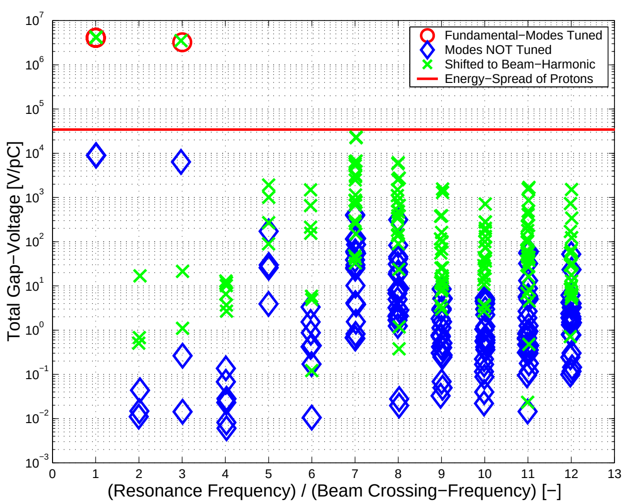

Figure 2: Interaction in longitudinal direction.

Fig. 2 shows the spectrum of the beam-excited modes. The diamond symbols indicate the results found for the modes with the calculated resonance frequencies. Because of the simplifications in the simulated rf-structure, there is an uncertainty from calculated to the real resonance frequency and damping-factors. Therefore the upper bounds of the mode-excitation are indicated with cross symbols, obtained by shifting the resonance frequencies to exactly the closest beam-harmonic.

The beam-loading of the fundamental cavity-modes are known from input rf-power measurements on the directional couplers of the cavities with and without beam. A small difference from measured to simulated gap-voltage in the fundamental modes can be explained by the uncertainty of the coupling factor to the final-stage amplifier. Comparing the mode-amplitudes with the measured energy-spread of the particles in the cyclotron, it can be concluded that no mode can be excited heavily enough to yield a relatively large additional energy-spread.

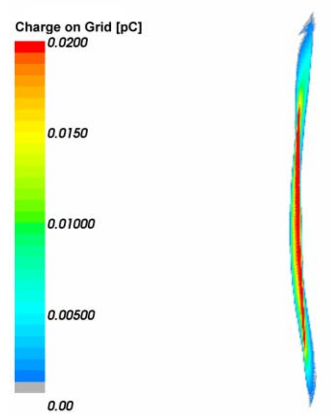

Figure 3: Top-view on Space-charge distribution after 214 turns. Average-current is 1.8mA and 30 HOM are selected.

EFFECT ON PARTICLE-BUNCHES

A Particle In Cell Needle (PICN) model [5, 6] combined with particle tracking was developed in order to get information of the influence of the excited fields back on the particle distributions. The tracking results of one bunch for time-steps of $0.1ns$ is shown in Fig. 3. Comparing the beam shape of this simulation with a HOMless one indicates that the effect of the HOMs onto the beam quality is relatively small for this range of beam currents.

CONCLUSIONS

The simulations confirm that the beam-cavity interactions should have no big influence on the operation of the machine. This result is expected from the machine-performance and from the HOM-measurements where a linear dependence of HOM-signal with beam current is observed. On the other hand, these results validate the perturbation approach chosen for the calculation of the excitation and its effect on the particle distribution. Improvement of the model can be made by adding more details to the geometry, taking into account the effect of the air pressure on the surface curvature and simulating the wall materials more accurately.

REFERENCES

[1] L. Q. Lee et al., “Solving Large Sparse Linear Systems in End-to-End Accelerator Structure Simulations”, SLAC-PUB-10320.

[2] Z. Li et al., “High Performance Computing in Accelerating Structure Design and Analysis”, Proc. of ICAP 2004.

[3] http://cubit.sandia.gov

[4] http://www.cscs.ch

[5] S. Koscielnak and S. Adam, “Simulation of Space-Charge Dominated Beam Dynamics in a Isochronous AVF Cyclotron”, IEEE-Proc. of PAC 1993.

[6] http://www.fftw.org