FIELD SHIMMING OF COMMERCIAL CYCLOTRONS

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Abstract

Commercial Cyclotrons (for isotope production as well as for Proton Therapy) are tend to be designed with fixed Magnetic field and fixed RF frequency in order to simplify operation and reduce cost. Shimming process should provide isochronous field profile without correction by trim coils, trim rods etc. Sequence of steps (algorithm of shimming) as well as some useful expressions which might improve convergence of process are being discussed in paper.

INTRODUCTION

Procedure of Magnet preparation includes some steps:
- Computing of Sectors shape, Return Yoke, Central Plug
- Preparation of drawings - Magnet and main pieces
- Manufacturing of Magnet parts.
- Assembling of Magnet.
- Shimming of Magnetic field – field measurements, calculations to correct thickness of Sector edges or upper surface of Sectors (depends which part of Sector – side shims or top – will be cut off to fit Isochronous profile)
- Final mapping, removing of excessive first harmonic.
-Additional measurements should be provided if internal surfaces of magnet are nickel-plated.

COMPUTER SIMULATIONS OF MAGNET

Choice of Magnet shape is based on required energy range, ratio of A/Q for reference particle, value of magnetic field, dimensions, desired structure - Flutter, spiral, betatron tune etc.. Cyclotrons with Fixed Magnetic Field, fixed RF frequency are being common for commercial market. It is simplified design, robust operation. Design of Cyclotron Magnet involves 3D-TOSCA simulations. Other software might be used. Quality of magnetic field (tune diagram, RF phase motion, radial, axial beam tracking) should be checked after each set of measurements.

Magnetic field in the Median Plane might be presented as Matrix $\{B_{ij}\}$ with K points in radius and L points in azimuth. Number of harmonics used for analysis is not fixed. Difference between measured field and assembled one should not exceed few Gs. Average field $B_{av}(i)$ at radius $R(i)$ equal to $B_{av}(i)=(1/L)\sum_j B_{ij}$ with harmonic components $A_N(i)=(2/L)\sum_j B_{ij}\cos\{(2\pi/L)N(j-1)\}$ as well as $B_N(i)$. Total Field at each point $\{i,j\}$ is sum of average one and main harmonics. First harmonics and harmonics associated with off-centring of Mapping system should be included only when beam dynamics tracking is provided. Value of average field and harmonics in the intermediate points should be interpolated.

ANALYTICAL EXPRESSIONS

Static Equilibrium orbits in Cyclotron corresponding to particle with impulse $p_i=m_0c\beta\gamma=eB(r_i)r_i$ will oscillate around circle with radius r_i. For each value of energy one can estimate parameters of Equilibrium orbit (SEO). Extraction energy and final radius is fixed. For simulation purposes one may choose $R_{extr}=R(k)$ where $\boldsymbol{k}$ is last point of map. Also extraction point might be chosen where field is maximum. Value of Isochronous field B_{av} at the extraction might be determined by analytical formula

$$B_{av}(R_{extr})=\{(E_{extr})^2+2E_{extr}E_0\}^{1/2}/(3*10^{-4}R_{extr}). \qquad (1)$$

Assuming that measured field is equal to isochronous field at extraction radius plus correction factor $B_{is}(R_{extr})=B_{av}(k)+\delta$ one can analytically calculate value of isochronous field inwards for each point $i=1,K$. For that impulse $p_k=P_k/m_0c$ at final radius R_k is derived from formula (2) by Newton method [4]

$$\begin{aligned}&(Qe/m_0c)\, r\, B_{av}(r)=\\ &p\,\{1-(Qe/m_0c)^2(r/2p)^2\textstyle\sum_N (N^2-(1+p^2))^{-1}\times\\ &[(1+p^2)(1+3p^2)(A_N^2+B_N^2)/(N^2-(1+p^2))+\\ &\quad 3(A_N^2+B_N^2)+r\,d(A_N^2+B_N^2)/dr]\,\} \qquad (2)\end{aligned}$$

Here $\boldsymbol{Q}$ is charge, $\boldsymbol{c}$ -speed of light, $m_0/e=1.043977\cdot10^{-4}$.

Value of impulse p_k at final radius R_k as well as Fourier harmonics from measured data $A_n(k)$, $B_n(k)$ at final point are substituted into the expression (3)

$$\begin{aligned}&p=r\,(1+p^2)^{1/2}\Omega/c\ \{1+(Qe/m_0c)^2(r/2p)^2\\ &\quad\textstyle\sum_N N^2(A_N^2+B_N^2)(N^2-(1+p^2))^{-2}\} \qquad (3)\end{aligned}$$

and circular frequency Ω is determined. Then for fixed frequency Ω one may calculate impulse $p(i)$ by substituting of measured values of $A_N(i)$, $B_N(i)$, $R(i)$ into same expression (3). Values of $p(i)$ as well as measured values of $A_N(i)$, $B_N(i)$, $r(i)$ should be substituted into expression (2) and Isochronous field $B_{is}(i)$ will be calculated at each radius. Variation of parameter δ could help to minimize iron removing from shims.

Cyclotron frequency and isochronous field might be found by well-known analytical expressions provided by Garren, Smith [1], Hagedoorn[2] etc. Standard procedure require to calculate first, second and third derivatives of main field and harmonics which are sensitive to small errors during measurements while integral equations (2), (3) should smooth possible errors. Analytical expressions are sensitive to field harmonics which are produced as result of Fourier analysis of measured field. Field is approaching to isochronous with each next mapping and values of harmonics should be close enough to real one.

FIELD BUMP

In order to improve stability of axial motion magnetic field in the centre is built up as bump of shape $B_{av}(R)= B_{av}(R_c)\ (R_c / R)^n$ providing no axial hole is designed in the Centre. Value of R_c is equal to radius with adequate magnetic focusing from Sectors $(\nu_z>0.15)$. Connection point R_c as well as field index n might be chosen such as negative shift of RF phase will not exceed 15^0 RF**.** Excessive magnetic field will shift RF phase in to negative direction

$$\varphi = ArcSin\ \{ Sin(\varphi_0) - (\pi h_{RF}\, \Omega / V_{dee}) \int_{0\to Rc} [B_{av}(r) - B_{iz}(r)]\, r\, dr\ \} \qquad (4)$$

where V_{dee} is amplitude of Dee Voltage, h_{RF} harmonic of RF, φ_0 – starting RF phase of ion. Restriction of RF phase shift to 15^0 should limit field index n to 0.01

$$n = 2\ [C - 24\ \Delta R^2 B_{iz}(R_c)]\ [C - 2\Delta R^{\,2} B_{iz}(R_c)]^{-1} \qquad (5)$$

where parameter C is determined by formula

$$C = \int_{\Delta R\to Rc} B_{iz}(r)\ r\ dr - (V_{dee}/\pi\, h_{RF}\, \Omega)\ Sin(\varphi_0) \qquad (6)$$

FIELD GRADIENT AT EXTRACTION

Value of magnetic field B_{av} at last point R_k and field one step inside might be found by solving of differential equation $\mu' = (R/B_{av}) dB_{av}/dR$ with boundary conditions $B_{av}=B_{iz}$ at points $(k\text{-}2)$ and $(k\text{-}3)$**.** Field gradient $\mu'(k\text{-}2)$ and $\mu'(k\text{-}3)$ are numerically differentiated. Field Gradient at last point is approximated by expression $\nu_R=[1+\mu'\ (k)]^{\ 1/2}$. One may choose Radial betatron frequency at extraction point as 0.96 or 0.8 etc. Field gradient at $R(k\text{-}1)$ radius is estimated by interpolation of gradients at $(k\text{-}3)$, $(k\text{-}2)$ and (k) points. Equations of motion should be solved in order to find RF phase.

MAGNET CENTRE

When centre of mapping system is not coincides with Centre of Magnet symmetry some false harmonics might appears. For Cyclotron with 4 Sector Symmetry it might be odd (3rd and 5th**)** harmonics of Fourier approximation of magnetic field. False harmonics should be separated from real one which could be result of Magnet structure etc. Field error caused by misalignment of Mapping system might be written as

$$\Delta B\ (r,\theta) \approx 2\ B_{av}(r)\ [b_3(r)/r]\ *\ \{ \delta x\ \cos 2[\theta - 2\psi(r)] - \delta y\ \sin 2[\theta - 2\psi(r)] \} \qquad (7)$$

where phase $\psi(r)= ATAN\ (B_N/A_N)$. Amplitude of False harmonics is changing as ratio *Flatter/Radius.* Phase of False harmonics is displaced with respect to main harmonic phase as $2\psi(r)$ [3].

SUMMARY OF ANALYTICAL FORMULA

- Field components $B_{iav}(i)$, $A_N(i)$, $B_N(i)$ are calculated (harmonic analysis) from measured data
- Cyclotron frequency Ω is estimated.
- Isochronous field $B_{is}(i)$ is calculated at each point.
- "Bump" is added to isochronous field in the Centre.
- Field at extraction is modified if precession extraction is used.
- First, second and third Derivatives of average isochronous field with "Bump" and "Fall off" are calculated.
- Fourier components and derivatives are found for new assembled field
- Cyclotron frequency and isochronous field at extraction radius is calculated for next iteration.

CORRECTION OF ISOCHRONOUS FIELD BY PERIOD

To correct isochronous field calculated by analytical formula following steps might be provided.

- Magnetic field $B(r,\theta)$ is assembled from analytical isochronous field $B_{iz,}$ Bump in the Centre and field in the extraction region as well as from Fourier components.
- Equilibrium orbits are calculated by numerical integration of equations of motion in assembled field $B(r,\theta)$ and Periods of rotation of particles at equilibrium orbits $T_\mathbf{i}$ are estimated.
- Corrections to assembled field are being estimated which minimize deviations of rotation periods from isochronous one ***(2π)*** .
- Fourier analysis of new field is provided and Equilibrium orbits are tracked in new field.

Iterations are being repeated until difference between period of rotation and isochronous period will be less than 10^{-5} .

EQUILIBRIUM ORBITS

Equilibrium orbits are found as periodic solutions $(r(0)=r(2\pi /N)$ *and* $p_r(0)=p_r(2\pi /N))$ of two equations of radial motion and two equations for small deviations.

$$dr/d\theta = rp_r(1-p_r^{\,2})^{-1/2} \qquad (8)$$

$$dp_r/d\theta = (1-p_r^{\,2})^{1/2} - 3\cdot 10^{-4} Qr\ B(r,\theta)\{E(E+2E_0)\}^{-1/2} \qquad (9)$$

$$dx/d\theta = rp_x\ (1-p_r^{\,2})^{-3/2} + xp_r\ (1-p_r^{\,2})^{-1/2} \qquad (10)$$

$$dp_x/d\theta = -p_r p_x(1-p_r^{\,2})^{-1/2} - 3*10^{-4} Q\, x\ \{B(r,\theta)+r\partial B(r,\theta)/\partial r\}\ \{E(E+2E_0)\}^{-1/2} \qquad (11)$$

with starting conditions for radius [4]

$$R_i = (P_i / B_i)\{1+\textstyle\sum_n a_n(r_i)[n^2-(1+p_i^{\,2})]^{-1} - 0.25\ (1+p_i^{\,2})^{-1} B_i^{-2}\ *\ \sum_n [n^2-(1+p_i^{\,2})]^{-1} [3+(1+p_i^{\,2})(1+3p_i^{\,2})(n^2-(1+p_i^{\,2}))^{-1} + r_i d/dr_i]\ [a_n(r_i)^2 + b_n(r_i)^2]\} \qquad (12)$$

and starting conditions for momentum

$$p_{ri}=P_i\sum_n b_n(r_i)n(n^2-1-p_i^2)^{-1}\{[B_i^2+B_i\sum_n b_n(r_i)^* (n^2-1-p_i^2)^{-1}]+[\sum_n b_n(r_i)n(n^2-1-p_i^2)^{-1}]^2\}^{-1/2} \quad (13)$$

Cyclotron units are $p_i=P_i/m_0c$, $r_i=R_i/R_\infty$, $B_i=B_i/B_0$ $B_0=m_0c/eR_\infty$, $R_\infty=c/\Omega$. Equations of small deviations are integrated for initial conditions $x_1(0)=\delta r(0)$, $P_{x1}(0)=0$ and second time - for $x_2(0)=0$, $P_{x2}(0)=\delta P_r(0)$. δ is small parameter. Value of $x(0)$ and $p_x(0)$ are found from solving of system of two linear equations

$$x(0)[x_1(2\pi/N)-1]+p_x(0)x_2(2\pi/N)=r(0^0)-r(2\pi/N) \quad (14)$$

$$x(0)P_{x1}(2\pi/N)+p_x(0)[P_{x2}(2\pi/N)-1]=p_r(0^0)-p_r(2\pi/N) \quad (15)$$

Equations (8,9,10,11) are integrated with new initial conditions $r^*(0)=r(0)+x(0)$ and $p_r{}^*(0)=p_r(0)+p_x(0)$. Procedure is repeated until condition of periodicity $|r(0^0)-r(2\pi/N)|+|p_r(0^0)-p_r(2\pi/N)|\le 10^{-6}$ will be satisfied. Measured values of $B_{av}(r_i)$, $a_n(r_i)$, $b_n(r_i)$, $dB_{av}(r_i)/dr$, $da_n(r_i)/dr$, $db_n(r_i)/dr$ are interpolated on each integration step

FIELD CORRECTIONS

Equilibrium orbits have been integrated in the measured magnetic field. Corrections of Periods of rotation of E.O. ΔT_i with respect to isochronous period should be calculated. ΔT_i are calculated for each measured point except "Bump" in the Centre and "Precession" at the edge. Period of rotation might be expressed as integral over one period of magnetic structure

$$T(r_i)=N\int_{2\pi/N}\{[1+p(r_i)^2]^{1/2}[p(r_i)^2-p_r(r_i)^2]^{-1/2}r_i(\theta)\}d\theta \quad (16)$$

Expression for Period is integrated together with system of equations (8-11). Average radius for each E.O. might be found as integral

$$r_{av}(i)=4\int_{0\to 2\pi/N}\{r(\theta)p_i[p_i^2-p_{ri}^2]^{-1/2}\}d\theta \quad (17)$$

Average radii are not equal to values of $r(i)=i\Delta r$ where field was measured. Interpolation is applied - with radius **r** as argument and deviation of period ΔT as function**.** Corrections to periods for radii where field was measured is converted to Field corrections $\Delta B(r_i)=B_{iz}(r_i)-B_{map}(r_i)$ as

$$\Delta B(r_i)/B_{map}(r_i)=-[\Delta T(r_i)/T(r_i)][\gamma(r_i)]^2 = -[\Delta T(r_i)/T(r_i)][1+E(r_i)/E_0]^2 \quad (18)$$

Profile of Cuts of Sector shims is based on corrections of field determined by formula (18).

RF PHASE MOTION

RF Phase shift per turn is proportional to

$$d\Phi/dn=T_i-T_0=-2\pi(\Delta B_i/B_i)[1-r^2(1+\sigma)^2]^{3/2}[1+\sigma]^{-1} \quad (19)$$

Energy gain per turn might be presented as

$$dE/dn=W_0\cos\Phi=(2QN_{dee}V_{dee}/m_0c^2)\sin(h_{RF}\Delta\theta/2)\cos\Phi \quad (20)$$

where Q is charge, $\Delta\theta$ is angular width of Dee, Optimum RF phase $\Phi=0$ when particle is crossing middle of the Dee. When Dee edges are spiralled and particle receives different energy gain at entrance and exit of the Dee the particle will receive maximum energy gain per turn at isochronous RF phase Φ_{iz} **.** In general Φ might be defined as RF phase wrt to Φ_{iz}. RF phase regardless of definition is represents deviation of magnetic field from isochronous one. Total shift of RF phase Φ is integral of

$$\Phi(R_i)=ASIN\{\sin\Phi_0+(\pi h_{RF}/W_0)^*\int_{0\to R}(\Delta B(r)/B(r))\,r[1+\sigma(r)+r\sigma'(r)]\,dr\} \quad (21)$$

Total RF phase shift Φ is calculated as sum of phase shifts at intervals $\Delta R_i=R_i-R_{i-1}$ To estimate shift at each interval energy gain $\Delta E_i=E_i-E_{i-1}$ is divided on few subintervals. Intermediate periods corresponding to intermediate energies are determined for each subinterval by double three point interpolation of Lagrange.

BETATRON TUNE

Radial betatron frequency ν_R is found from Equations of small oscillations [3]

$$\nu_R(E_i)=(N/2\pi)ACOS\{0.5[x_1(2\pi/N)+P_{x2}(2\pi/N)]\} \quad (22)$$

Two equations of small axial oscillations should be integrated two times as part of system (8-11) in order to find axial betatron frequency $\nu_z(E_i)$

$$dZ/d\theta=R\,P_z\,(P^2-P_R^2-P_z^2)^{-1/2} \quad (23)$$

$$dP_z/d\theta=Z\{R\partial B(R,\theta)/\partial R-P_R(P^2-P_R^2-P_z^2)^{-1/2}x\ \partial B(R,\theta)/\partial\theta\} \quad (24)$$

Initial conditions $Z_1(0)=\delta z$, $P_{Z1}(0)=0$ applied for the first time while for second run $Z_2(0)=0$ and $P_{Z2}(0)=\delta p_z$ are used. Axial betatron frequency is found by

$$\nu_Z(E_i)=(N/2\pi)ACOS\{0.5[Z_1(90^0)+P_{Z2}(90^0)]\} \quad (25)$$

CONCLUSION

Shimming procedure is well developed and general expressions are well known. Algorithm described in the paper might be used to simplify magnet simulations. Supplement formulas might benefit if one would like to check and correct iteration process during mapping. Authors should thanks to Dr.A.Val'kov (KINR, Ukraine) for principal description of basic expressions.

REFERENCES

[1] Garren, Smith, "Report UCRL" (1959)
[2] Hagedoorn, NYM,18,19, (1962).
[3] A.Val'kov."Calculations of magnetic field parameters of Isochronous cyclotron U-240" KINR (1972) 24p.
[4] A.Val'kov. "Program Rezhim". KINR (1992) 32 p.