BEAM-SIZE EFFECT AND PARTICLE LOSSES AT COLLIDERS

Gleb Kotkin and Valery Serbo
Novosibirsk State University, Novosibirsk, Russia

OUTLOOK:

1. Introduction

2. Qualitative description of the MD-effect

3. MD-effect and correlations of particles in a beam

4. MD-effect for KEKB and PEP-II

This report is based on the papers:
1. Introduction

The so called beam-size or **MD-effect** is a phenomenon discovered in experiments


at the **MD-1 detector** (Novosibirsk, 1981).

It was found that for ordinary bremsstrahlung, **macroscopically large impact parameters** should be taken into consideration.

These impact parameters may be **much larger** than the **transverse sizes of the interacting particle bunches**.

In that case, the standard calculations, which do not take into account this fact, will give **incorrect results**.

The detailed description of the MD-effect can be found in review

In 1980–1981 a dedicated study of the process $e^+e^- \rightarrow e^+e^-\gamma$ has been performed at the collider VEPP-4 in Novosibirsk using the detector MD-1 for an energy of the electron and positron beams $E_e = E_p = 1.8$ GeV and in a wide interval of the photon energy $E_\gamma$ from 0.5 MeV to $E_\gamma \approx E_e$.

It was observed [1] that the number of measured photons was smaller than expected. The deviation from the standard calculation reached 30% in the region of small photon energies and vanished for large energies of the photons.

A qualitative explanation of the effect was given by Yu.A. Tikhonov


He noticed that those impact parameters $\varrho$, which give an essential contribution to the standard cross section, reach values of $\varrho_m \sim 5$ cm whereas the transverse size of the bunch is $\sigma_\bot \sim 10^{-3}$ cm.

The limitation of the impact parameters to values $\varrho \lesssim \sigma_\bot$ is just the reason for the decreasing number of observed photons.
The **first calculations** of this effect have been performed in papers


using different versions of quasi–classical calculations in the region of large impact parameters.

Later on, the effect of limited impact parameters was taken into account using the single bremsstrahlung reaction for measuring the luminosity at the VEPP–4 collider and at the LEP-I collider.

A **general scheme** to calculate the **finite beam size effect** had been developed in the paper


starting from the **quantum description of collisions as an interaction of wave packets** that form bunches.
It has also been shown that similar effects have to be expected for **several other reactions:**

**bremsstrahlung for colliding $ep$–beams**


$e^+e^−$– pair production in $e^±e$ and $\gamma e$ collisions [6].

In 1995 the MD-effect was **experimentally observed** at the electron-proton collider HERA


at the level predicted in [8].
The possibility to create high-energy colliding $\mu^+\mu^-$ beams is now wildly discussed.

For several processes at such colliders a new type of beam-size effect will take place — the so called linear beam-size effect


The calculation of this effect was performed by method developed for the MD-effect in [6].

In this report we discuss two new features related to the MD-effect:

1) an account the influence of the particle correlations in the beams on the MD-effect;

2) an influence of the MD-effect on the beam losses at the existing B-factories.
2. Qualitative description of the MD-effect

We use the $ep \rightarrow ep\gamma$ process as an example.

This reaction is described by the block diagram:

\[ E_e \quad \sigma_z \quad E_p \quad \hbar q = (\hbar \omega/c, \hbar q) \]

Notations:

$\sigma_z = l$ is the longitudinal,

$\sigma_{\perp}$ ($\sigma_x$ and $\sigma_y$) is the transverse size

of the proton (positron) bunch,

$\gamma_e = E_e/(m_ec^2)$, $\gamma_p = E_p/(m_pc^2)$. 
The large impact parameters $\varrho \gtrsim \sigma_\perp$ correspond to small momentum transfer $\hbar q_\perp \sim (\hbar/\varrho) \lesssim (\hbar/\sigma_\perp)$.

In this region, the given reaction can be represented as a Compton scattering:

\[
\begin{array}{c}
\varrho \\
\hline
\hline
p \\
\hline
\hline
\omega \\
\end{array}
= \begin{array}{c}
\hline
\hline
\varrho \\
\end{array} + \begin{array}{c}
\hline
\hline
\hline
\end{array}
\]

The equivalent photons with frequency $\omega$ form a “disk” of radius $\varrho m \sim \gamma p c/\omega$.

In the reference frame connected with the collider, the equivalent photon with energy $\hbar \omega$ and the electron with energy $E_e \gg \hbar \omega$ move toward each other and perform a Compton scattering:

\[
\begin{array}{c}
\varrho m \\
\hline
p \\
\hline
\omega \\
\end{array}
\]

The main contribution to the Compton scattering is given by the region where the scattered photons fly in a direction opposite to that of the incident photons.
For such a **backward scattering**, the energy of the equivalent photon $\hbar \omega$ and the energy of the final photon $E_{\gamma}$ are related as

$$\hbar \omega \sim \frac{E_{\gamma}}{4\gamma_e^2(1 - E_{\gamma}/E_e)},$$

therefore, the radius of the “disk” is:

$$\varrho_m = \frac{\gamma_p c}{\omega} = \lambda_e 4\gamma_e \gamma_p \frac{E_e - E_{\gamma}}{E_{\gamma}}, \quad \lambda_e = \frac{\hbar}{m_e c}. \quad (2)$$

For for the PEP-II and KEKB colliders we have

$$\varrho_m \gtrsim 1 \text{ cm for } E_{\gamma} \lesssim 0.1 \text{ GeV}. \quad (3)$$

The **standard calculation** corresponds to the interaction of the photons forming the “disk” with the **unbounded flux** of electrons. However, the **particle beams at colliders** have **finite transverse beam sizes** of the order of $\sigma_{\perp} \sim 10^{-2} \text{ cm}$.

Therefore, the equivalent photons from the region $\sigma_{\perp} \lesssim \varrho \lesssim \varrho_m$ **cannot interact** with the electrons from the other beam.

**This leads to the decreasing number of the observed photons.**
3. MD-effect and correlations of particles in a beam

Correlations of particle coordinates in the beams are ignored in earlier papers, since usually these correlations are small. However, more accurate measurements may be sensitive to them.

In the paper


we derived formulas which necessary to take into account quantitatively the effect of particle correlations in the spectrum of bremsstrahlung as well as in pair production.

The corresponding additional term is determined by the correlation function for the density of particles in the beam.
It should be mentioned that the same problem was considered in paper


using unrealistic assumptions.

As a result, an application of formulas obtained in this paper to the HERA experiment is ungrounded.
4. MD-effect for KEKB and PEP-II

It was realized in last years that the MD-effect in bremsstrahlung plays an important role in the beam lifetime problem.

At storage rings TRISTAN and LEP-I, the bremsstrahlung process was the dominant mechanism for the particle losses in beams. If electron loses more than 1 % of its energy, it leaves the beam.

Since the MD-effect considerably reduced the effective cross section of this process, the calculated beam lifetime in these storage rings was larger by about 25 % for TRISTAN


and by about 40 % for LEP-I


(in accordance with the experimental data) then without taken into account the MD-effect.
Usually in experiments the cross section is found as the ratio of the number of observed events per second $d\dot{N}$ to the luminosity $L$.

Also, in our case it is convenient to introduce the “observed cross section”, defined as the ratio

$$d\sigma_{\text{obs}} = \frac{d\dot{N}}{L}. \quad (4)$$

Contrary to the standard cross section $d\sigma$, the observed cross section $d\sigma_{\text{obs}}$ depends on the parameters of the colliding beams.

To indicate explicitly this dependence we introduce the “correction cross section” $d\sigma_{\text{cor}}$ as the difference between $d\sigma$ and $d\sigma_{\text{obs}}$:

$$d\sigma_{\text{obs}} = d\sigma - d\sigma_{\text{cor}}. \quad (5)$$

The relative magnitude of the MD-effect is given by the ratio

$$\delta = \frac{d\sigma_{\text{cor}}}{d\sigma}. \quad (6)$$
The correction cross section depends on the r.m.s. transverse horizontal and transverse vertical bunch sizes. In calculations we used data from Review of Particle Physics–2002 and 2006.

Besides, for the KEKB collider we have to take into account that its $e^+e^-$ beams of the length $l_e = l_p = 0.65$ cm collide to a crossing angle $2\psi = 22$ mrad. Formulae of the correction cross section for this case have been obtained in paper G.L. Kotkin, S.I. Polityko, V.G. Serbo, Sov. Yad. Fiz. 42, 925 (1985)
The calculated relative magnitude of the MD-effect:

\[
y = \frac{E_\gamma}{E_e}
\]

<table>
<thead>
<tr>
<th>(y)</th>
<th>0.001</th>
<th>0.005</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta, %) PEP-II</td>
<td>31</td>
<td>26</td>
<td>24</td>
<td>19</td>
<td>16</td>
</tr>
<tr>
<td>(\delta, %) KEKB</td>
<td>33</td>
<td>29</td>
<td>26</td>
<td>21</td>
<td>18</td>
</tr>
</tbody>
</table>

It is seen that the MD-effect considerably reduces the differential cross section.

To estimate the integrated contribution into particle losses, we should integrate the differential observed cross section from some minimal photon energy.

It is usually assumed that an electron leaves the beam when it emits the photon with the energy 10 times larger than the beam energy spread. In other words, the relative photon energy should be \(y = \frac{E_\gamma}{E_e} \geq y_{\text{min}}\) where \(y_{\text{min}} = 0.0061\) for PEP-II and \(y_{\text{min}} = 0.007\) for KEKB.
It gives:

<table>
<thead>
<tr>
<th></th>
<th>$\sigma/\sigma_{\text{obs}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEP-II</td>
<td>1.20</td>
</tr>
<tr>
<td>KEKB</td>
<td>1.23</td>
</tr>
</tbody>
</table>

The integrated standard cross section is larger than the observed cross section by about 20\%.

The number of particles, which the single electron bunch losses during a second, is equal to

$$\Delta \dot{N}_e = L \sigma_{\text{obs}}^{(e)}/n_b,$$

(7)

where $L$ is the luminosity and $n_b$ is the number of bunches.

Therefore, the partial lifetime of the electron bunch, corresponding to the bremsstrahlung process at a given luminosity, can be estimated as

$$\tau_{\text{brem}}^e = \frac{N_e}{\Delta \dot{N}_e} = \frac{N_e n_b}{L \sigma_{\text{obs}}^{(e)}}.$$  

(8)
The obtained numbers for the electron and positron beams (date from *Review of Particle Physics–2002/2006*) are:

<table>
<thead>
<tr>
<th></th>
<th>$\tau^e_{\text{brem}}$, hr</th>
<th>$\tau^p_{\text{brem}}$, hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEP-II</td>
<td>4/8.7</td>
<td>12/44</td>
</tr>
<tr>
<td>KEKB</td>
<td>8.9/6</td>
<td>14/13</td>
</tr>
</tbody>
</table>

They can be compared with the luminosity lifetime $\tau_L = 3.4$ hours for KEKB and 2.5 hours for PEP-II (from *RPP-2002*), which is some average characteristic of lifetimes for both beams.

More detailed comparison with the experimental numbers for lifetimes of beams at KEKB shows that

the bremsstrahlung process is important for the electron beam lifetime, but has rather small influence on the positron beam lifetime.