Abstract
The VEPP-4M electron-positron collider is now operating with the KEDR detector for the experiment of precise measurement of tau-lepton mass. In this experiment, monitoring of beam energy spread is important to know the energy spread contribution into the total systematic error. Information about the energy spread gives an opportunity to reduce the error of the tau-lepton mass measured. The energy spread measurements using several techniques are described. Width of the J/ψ and ψ' resonances measured with the KEDR detector is used as a reference.

INTRODUCTION
Magnet lattice of the VEPP-4M is reflection-symmetric about an axis, passing through the centers of the experimental and technical straight sections [1]. A feature of the VEPP-4M lattice is usage of combined-function magnets. Radiation damping of the radial beam oscillation is provided by two gradient Robinson-type wigglers installed in the technical section. Dispersion function at the wiggler locations is non-zero, \( \eta_x \neq 0 \).

We can redistribute radiation damping decrements between radial and longitudinal beam motion by change of the wigglers field, and in that way we can to vary beam energy spread \( \sigma_E \). Working value of the wiggler field is adjusted as a compromise between desirable precision of a particle mass measurement and acceptable level of luminosity.

Energy spread can be increased with two 3-pole 1 m long dipole wigglers (snakes) with a maximal field of 1.8 T, installed symmetrically in the experimental section. They are normally switched off in the low-energy operation mode to have a minimal energy spread, but we have used them for the beam energy spread measurements, as it described below.

Our aim was not only evaluation of the beam energy spread for basic modes of the VEPP-4M operation, it was also comparison of several techniques of the energy spread measurement.

Table 1: VEPP-4M operation modes.

<table>
<thead>
<tr>
<th>Name</th>
<th>( E, \text{ MeV} )</th>
<th>( I_{\text{WG}}, \text{ A} )</th>
<th>( I_{\text{SN}}, \text{ A} )</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSIS</td>
<td>1843</td>
<td>1055</td>
<td>0</td>
<td>( \psi' ) meson peak.</td>
</tr>
<tr>
<td>ZMEJ</td>
<td>1843</td>
<td>1055</td>
<td>2000</td>
<td>special mode</td>
</tr>
<tr>
<td>JPSI</td>
<td>1548</td>
<td>620</td>
<td>0</td>
<td>( J/\psi ) meson peak</td>
</tr>
</tbody>
</table>

The experiments have been done in three operation modes (see Table 1). These modes differ one from another by the energy \( E \) and energy spread \( \sigma_E \) values, and feed current values of the gradient wiggler \( I_{\text{WG}} \) and of the dipole wiggler \( I_{\text{SN}} \).

Application of several methods of the energy spread measurement to different modes of the VEPP-4M operation let us both to cross-check the measurement results and to compare the measurement techniques with relation to convenience and efficiency. Optical diagnostics system [2] was applied for measurement of the beam sizes \( \sigma_x, \sigma_y, \sigma_z \) and to obtain a spectrum of vertical betatron oscillations.

MEASUREMENT METHODS
Method I. Spectral Analysis of Chromatic Synchro-betatron Modes of Beam Oscillation
For non-synchronous particles, machine chromaticity introduces betatron frequency shift. This effect causes synchrotron sideband peaks in a spectrum of beam oscillation. Amplitudes of the central betatron peak and of the synchrotron satellites are [3]:

\[
R_m(y) = \frac{1}{y^2} \int_{-\infty}^{\infty} J_m^2(x)e^{-\frac{x^2}{y^2}}dx, \quad (1)
\]

where \( y = \left( \frac{\omega_\beta \alpha}{\omega_y} + \frac{\omega_\delta C_y}{\omega_x} \right) \delta_E \), \( m \) is the number of harmonic, \( \omega_\beta \) and \( \omega_y \) are the betatron and synchrotron frequencies correspondingly, and \( \delta_E = \sigma_E/E \) is the normalized energy spread. Determination of energy spread is based on the amplitude ratio of the synchrotron satellites to the main betatron peak.

![Figure 1: A spectrum of vertical betatron oscillations.](image-url)
presented in Figure 1. Three sideband satellites are clearly seen. Such measurements were performed for various vertical chromaticity $C_y = 5 \pm 20$. The chromaticity was changed with sextupole magnets and monitored by measurement of RF frequency depended betatron tune shift.

To improve signal-to-noise ratio, a digital comb filter is applied to the measured beam oscillation. The oscillation spectrum is obtained by FFT, then only the $\nu_y \pm m\nu_s$ harmonics is kept and other ones are set to zero. Inverse Fourier transform of this spectrum gives the filtered signal. For ZMEJ mode and $C_y = 18.5$, an example of the signal filtering is shown in Figure 3. The upper graph shows a measured beam oscillation, and the lower one – filtered.

In our experiments, energy spread was determined as a fit parameter, by fitting of measured beam oscillation envelope with the theoretical curve (2). This fitting was done for all the measurements performed for the method I. An example of measured oscillation envelope in comparison with the theoretical curve is presented in Figure 4. Only positive parts of the symmetrical curves are shown. Using measured values of the vertical chromaticity $C_y = 18.5$ and synchrotron tune of $\nu_s = 0.0089$, the fit gives the energy spread value of $\delta_E = 6.7 \cdot 10^{-4}$.

Analysis of all the data measured with the methods I and II gives average values of the energy spread:

- $\delta_E = (3.2 \pm 0.3) \cdot 10^{-4}$ for JPSI mode;
- $\delta_E = (4.6 \pm 0.4) \cdot 10^{-4}$ for PSIS mode;
- $\delta_E = (6.6 \pm 0.5) \cdot 10^{-4}$ for ZMEJ mode.
Method III. Current dependence of energy spread

The energy spread measurements with the methods I and II have been done with small beam current of 10÷50 mA, when collective effects are negligible. High-energy physics experiments were carried out with beam current close to the beam-beam effects threshold. Its value is of 1.5÷3.5 mA, depending on the beam energy.

For estimation of the energy spread current dependence, measurements of radial $\sigma_x$ and longitudinal $\sigma_z$ beam sizes were done. The energy spread is derived from measured radial beam size $\sigma_x$ and known values of the radial betatron and dispersion functions, $\beta_x = 620$ cm and $\eta_x = 94$ cm at the observation point. It was supposed that the Touschek effect is the main reason of the radial beam size $\sigma_x$ and energy spread $\sigma_E$ increase with the beam current [1].

![Figure 5: Radial beam size $\sigma_x$ vs. beam current $I_0$.](image)

Figure 5 shows a fit of the measured radial beam size $\sigma_x$ (dots) with the theoretical curve calculated from the energy spread growth (line). At the current higher than $I_0 = 4$ mA, the $\sigma_x(I_0)$ dependence has a threshold behavior and needs in additional studying. This threshold might be caused by microwave instability with the threshold depending on the accelerating voltage $V_{RF}$.

In our energy spread measurements by the methods I and II, reduced value of $V_{RF} = 150÷250$ kV was set to decrease the synchrotron frequency $\nu_s$ to improve the measurements resolution. High-energy physics experiments of 2002-2006 have been performed with the RF voltage of $V_{RF} \geq 400$ kV, and the instability threshold was much higher than the working currents limited by the beam-beam effects.

Measurements of the longitudinal beam size $\sigma_z$ enables us to derive the energy spread at zero current $I_0 = 0$. Current-dependent bunch lengthening $\sigma_z \sim I_0^{1/3}$ is caused by the ring broad-band longitudinal impedance of an inductive type.

DISCUSSION

One can note that measurements by all three methods are in a good agreement with data of the J/$\psi$ resonance scan at the $E = 1548 \pm 10$ MeV energy performed in 2002, with reduced current $I_{WG} = 620$ A of the gradient wiggler [6, fig7, scan IV].

All these methods are also in good agreements with the $\psi'$ meson scan at the $E = 1843 \pm 10$ MeV energy performed in 2005-2006. During the 2002 $\psi'$ meson scan [6], the wiggler current was $I_{WG} = 1155$ A and the energy spread was $\delta_E = 5.12 \cdot 10^{-4}$.

![Figure 6: Energy spread measured by three methods.](image)

Figure 6 represents collected data of the energy spread measurements for the VEPP-4M operation modes listed in Table 1.

Note that the methods I and II are perturbing in respect to the machine parameters, because to measure the energy spread with reasonable accuracy, we have to change much the chromaticity and therefore to disturb the magnet lattice.

To summarize, we should recognize that reliable measurement of the beam energy spread is a nontrivial problem. It requires careful use of several diagnostic techniques to have a possibility of cross validation of the measurement results.

REFERENCES