A novel short-period undulator scheme is developed. The geometry looks like helical transformation applied to an original linear undulator structure and provides a combination of fast and slow oscillations of the electron beam. The following properties of the radiation are considered: i) reduction of relative content of higher odd harmonics as the ‘slow’ helical field increases; ii) radiation enhancement as the number of helical turns increases; iii) spectral line splitting and intensity adjustability along with circular polarization of the radiation for the short-period twisted structure.

1 INTRODUCTION

Undulators with two or more frequencies are attractive from many points of view. One of them is insertion devices with special radiation characteristics.

We are considering here a somewhat new kind of two-frequency motion that could combine useful radiation properties of both linear microundulator and helical undulator. It can be implemented in a real construction as depicted in fig. 1. As it was discussed in [1] such schemes can be regarded as belonging to a new class of ‘twisted linear undulator’.

![Fig. 1. Schematic drawing of the twister undulator structure.](image)

2 UNDULATOR FIELDS

It was found in [2] the solution of Laplace’s equation for magnetic potential \( \Delta \psi=0 \) satisfying the condition of superimposed helical ‘slow’ transformation to the original linear undulator structure:

\[
B_x = (B_x \cos h_t z + B_z) \sin(k_t z),
\]

\[
B_z = (B_x \cos h_t z + B_z) \cos(k_t z),
\]

where \( k_t << h_t \) and \( h_t, r << 1 \), \( B_x \) and \( B_z \) are the amplitudes of the original linear (‘microundulator’ contribution) and ‘helical’ undulator fields with periods \( \lambda_t \) and \( \lambda_r \), respectively, \( h_t = 2\pi/\lambda_t \) and \( k_t = 2\pi/\lambda_r \).

Under these assumptions one can derive:

\[
\psi(r, j, z) = \sum_{m=1}^{\infty} \left( C_m I_m(r(mh_w + k_i)) \times \exp(\pm i(mh_w + k_i)z) \right),
\]

where \( C_o = \frac{2iB_i}{k_i} C_{ij} = i \frac{2B_w}{k_i \pm h_w} \).

3 ELECTRON TRAJECTORIES

Equations of motion for a single electron were solved assuming \( K \approx r < \gamma^2 \), where \( K = eB/k mc \).

For the twisted undulator having zero first and second integrals and \( \gamma > 1 \) one can obtain

\[
\ddot{r}(z) = \frac{K_i}{\gamma K_r} \left( \cos k_t z - \sin k_t z \right) + \sum_{m=1}^{\infty} \frac{K_w^2}{\gamma^2} \left( \cos(k_i \pm h_w z) z \right) \frac{1}{k_i \pm h_w},
\]

\[
n_t(t) = \beta_n(e^t - \frac{1}{\gamma^2} K_w^2 \sin 2h_w z(t)) \left[ \frac{1}{\gamma^2} K_w^2 + K_w^2 \sinh h_w z(t) \right],
\]

where \( \beta_n = 1 - \frac{1}{2\gamma^2} \left( 1 + K_i^2 + \frac{1}{2} (K_w^2 + K_i^2) \right) \) and \( K_w = eB_w/mc(k \pm h_w) \).

The last coupling term in (1) containing \( \sin h_w z \) can be comparable with the term containing \( \sin 2h_w z \) and does not appear for a conventional planar undulator. Hence one can expect appearance of even harmonics for on-axis view of spontaneous emission.

![Figure 2. Electron trajectory view in transverse plane.](image)

E=500 MeV, \( B_x = 10kG, \ B_z = 1kG, \lambda_x = 1cm, \lambda_z = 10cm. \)
Energy radiated per unit solid angle \((d\Omega)\) per unit frequency \((d\omega)\) per electron was investigated numerically for both zero and non-zero imposed helical ‘slow’ field \(B_t\).

We separate two components of spectral angular density of radiation for spherical frame with \(\theta\) and \(\varphi\) coordinates:

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_\theta}{d\omega d\Omega} + \frac{d^2 I_\varphi}{d\omega d\Omega}.
\]

Substitution of the equations of motion (1) into the simplified formula for \(d^2 I/d\omega d\Omega\) expressed in [3] through the product \([\mathbf{n}_j \mathbf{n}_l]\) neglecting ‘near-field’ effects gives the following general expression containing products of five sums of Bessel functions:

\[
\frac{d^2 I_{\theta,\varphi}}{d\omega d\Omega} = \frac{(e\omega c)^2}{4\pi^2 c^3} \sum_{j,k,l,m,n} \left( \frac{J_j(a_i)J_j(a_{i'})J_l(a_m)J_l(a_{m'})}{J_n(a_u)J_n(a_{u'})} \right)^2 \left( J_{j+k,l+m,n} J_{j+k,l+m,n} \right),
\]

where:

\[
\begin{align*}
F_{j,k,l,m,n} & = (-1)^{j+k+l} i^{l-j} \delta(j+l-k) \\
\cos \theta j,k,l,m,n & = \frac{\alpha_{j,k,l,m,n} \cos \phi + v_{j,k,l,m,n} \sin \phi}{u_{j,k,l,m,n} \sin \phi - v_{j,k,l,m,n} \cos \phi} \\
\left( u_{j,k,l,m,n} \right) & = \frac{1}{2} \left( S_{j+k,l,m,n} - S_{j-l,k,l,m,n} \right), \quad S_{j+k,l,m,n} = \frac{\sin x}{x}, \\
S_{j-l,k,l,m,n} & = \frac{1}{2} \left( S_{j+k,l,m,n} + S_{j-k,l,m,n} \right), \quad a_l = \frac{K_{n_1}}{\gamma} \frac{k_i}{k_i}, \\
\left( a_{n_1} \right) & = \frac{K_{n_1}}{\gamma} \frac{k_i}{k_i}, \\
\left( v_{j,k,l,m,n} \right) & = \frac{1}{2} \left( S_{j+k,l,m,n} + S_{j-k,l,m,n} \right), \quad a_{n_1} = \frac{K_{n_1}}{\gamma} \frac{k_i}{k_i}.
\end{align*}
\]

We consider below specific properties of the radiation compared to the conventional planar undulator, i.e. in the vicinity of \(\nu \approx n\). We do not consider here the radiation features corresponding to helical undulator component (i.e. when \(K \geq 1\) and \(\nu \approx nk/h_0\)) because in practice we have \(K^2 < \nu K_0\) and the influence of the ‘twisted undulator’ component is negligible.

As it can be seen from (2) the main maxima of \(d^2 I/d\omega d\Omega\) spectral distribution correspond to the frequencies \(\omega = \frac{(n h_0 \pm k_i) / c}{1 - \beta || \cos \theta} \), where \(n\) is integer.
On-axis radiation of the even harmonics predicted in the previous section has the maxima close to $B_t \sim \frac{B \lambda_c}{\lambda}$, however its intensity is negligible compared to the odd harmonics (see fig. 7).

**6 DISCUSSION**

The twisted undulator structure combines some properties of both helical and linear undulator. The main features are circular polarization, spectral line splitting, and continuous adjustability of the radiation intensity by the helical field variation. The structure can be considered also as a ‘twisted microundulator’ providing radiation with circular polarization.

Although the insertion device proposed can not be used in FELs, it is of potential interest for applied research of plasma beat waves and in solid state physics, e.g. circular dichroism.

**REFERENCES**

