Abstract
Experimental results on transverse polarization obtained at LEP at a beam energy of 50 GeV are shown. The application of the refined orbit correction procedure known as Harmonic Spin Matching, implemented to compensate for depolarizing effects originating from orbit errors and misalignments of the machine elements, is described. Prospects and plans to improve the transverse polarization level at higher energies to extend the range of application of the direct and precise calibration with resonant depolarization are reported.

1 MOTIVATION
At LEP II the W mass should be determined with an accuracy which is 5 to 6 times smaller as the present value[1]. To reach this aim the beam energy has to be known with an accuracy of 15 MeV[1]. The energy calibrations at LEP I were based on the technique of resonant depolarization which is not directly applicable at the LEP II beam energies of about 90 GeV. Energy calibrations with the technique of resonant depolarization will be done at lower energies and the results will be extrapolated to the LEP II beam energy. The extrapolation is based on magnetic field measurements. To ensure a low extrapolation error relative comparisons of magnetic and the resonant depolarization technique calibrations have to be done at several energies. The error on the extrapolation will decrease with an increase of the difference between highest and lowest beam energy were this comparison could be performed. The lowest energy is given by the instruments for the magnetic field measurements and the highest energy by the level of polarization which allows a resonant depolarization calibration.

2 POLARIZATION MEASUREMENTS ABOVE THE Z ENERGY
It gets extremely difficult to achieve a reasonable level of polarization at energies well above the Z energy of 45 GeV (see Section 4). As a step towards higher energies where polarization can be used in order to perform an energy calibration this paper reports on a polarization measurement at 50 GeV in November, 1996 (see Fig. 1). The measurement was performed during a dedicated machine development shift. The solenoids of the experiments were switched off in order to exclude the depolarizing effects introduced by their strong longitudinal fields. For the same reason the bunch train bumps which are used during luminosity operation were deactivated. After the application of deterministic and empirical HSM (see Section 5.1) the polarization went up to ≈11%.

An attempt to measure polarization at an even higher energy around 55.3 GeV lead to a measured polarization of about 2%. This turned out to be not sufficient for the present polarimeter to perform a proper energy calibration.

Figure 1: Polarization Measurement at 50 GeV. The rise-time fit confirms ≈11% polarization measured by the LEP polarimeter.

3 POLARIZATION IN ELECTRON STORAGE RINGS
The centre of mass spin motion in electric and magnetic field is governed by the Thomas-BMT[2][3] equation:

$$\frac{d\vec{s}}{d\theta} = \vec{\Omega} \times \vec{s},$$

where \(\vec{\Omega}(\vec{B}, \vec{E}, \gamma)\) is a function of the fields and energy. \(\theta\) is the azimuthal coordinate.

In a flat ring with the conditions \(\vec{B} \parallel e_z^*, \vec{E} = \vec{0}\) and after transformation to a frame which is circulating with the beam, \(\vec{\Omega}\) becomes:

$$\vec{\Omega} = \frac{e\vec{B}}{cm\gamma} \frac{R}{2\pi} a\gamma,$$

where the first part is the relativistic cyclotron frequency, \(R\) the circumference of the ring and \(a\gamma\) the spin tune. \(a\) denotes the electron anomalous \(g\) factor.

The emission of synchrotron radiation in an electron storage ring can cause spin flip from up to down and vice versa. An asymmetry in the emission rates leads to an exponential polarization buildup with time antiparallel to the direction of the bending field:

$$P(t) = P_\infty(1 - \exp[-t/\tau_p]), \quad \tau_p^{-1} \propto \gamma^5 \int \frac{1}{\rho^3} d\theta$$
with \( \rho \) = modulus of bending radius, the equilibrium polarization \( P_{\infty} = 92.4\% \) for a flat ring and the characteristic buildup time \( \tau_p = 360 \) min for a ring like LEP at 44.74 GeV. The rise time decreases with the 5th power of the energy. So far depolarizing effects have been neglected. Due to the stochastic change of momentum by photon emission betatron oscillations are excited. Especially in the quadrupoles this leads to additional fields seen by the particle which can act on the spin. As a result spin diffusion and hence depolarization is introduced. This diffusion process is particularly strong when the precession of the spins is synchronous with orbital and energy oscillations, i.e., when the spin resonance condition:

\[
a \gamma = m + m_xQ_x + m_yQ_y + m_sQ_s
\]

is fulfilled, where \( Q_x, Q_y, Q_s \) are the horizontal, vertical and synchrotron tunes, \( m \) and \( m_x, y, s \) are integers.

Under the presence of these depolarizing effects the reachable equilibrium polarization \( P \) is given by:

\[
P = \frac{P_{\infty}}{1 + \left(\tau_p/\tau_d\right)[|a\gamma|^2]}
\]

where \( \tau_d \) denotes the characteristic time of the depolarization process and \( \tau_p/\tau_d \) depends quadratically on the beam energy in linear approximation. Figure 2 visualizes \( P \) as a function of the spin tune \( a\gamma \).

4 LIMITATIONS OF POLARIZATION AT HIGH ENERGIES

Synchrotron oscillations with large amplitudes cause a large frequency modulation of the spin tune \( a\gamma \). Similar to the mechanism of frequency modulation in RF waves, synchrotron sidebands appear in the spectrum of the spin motion. This leads to sidebands to the linear spin resonances. On the basis of a formalism introduced by Buon[6] and S. Mane[7] the strength of the sidebands can be quantified. The relevant parameter is the modulation index:

\[
\alpha = \left(\frac{a\gamma_0\sigma_e}{Q_s}\right)^2.
\]

The sideband strengths only depend on the strength of the parent resonances and on powers of \( \alpha \). For rings with large energy spreads like LEP with an relative energy spread \( \sigma_e \sim 6.5 \times 10^{-4} \) at 44.7 GeV and with \( Q_s = 0.06 \) the sideband resonances are strong up to high order (\( \alpha \approx 1.2 \)). Figure 3 visualizes the rapid change of \( \alpha \) with energy for 3 different values of \( Q_s \). It should be noted that \( \alpha \) nearly doubles between from 44.7 (\( a\gamma = 101.5 \)) to 50 GeV (\( a\gamma = 113.5 \)). The most appropriate way to fight these sidebands is the weakening of the driving linear resonances which can be performed using the method of Harmonic Spin Matching.

5 METHODS TO RESTORE POLARIZATION AT HIGH ENERGIES

5.1 Harmonic Spin Matching

The spin motion can be highly perturbed when the spin precession is in phase with energy (synchrotron) and betatron oscillations, in particular for harmonics of the perturbed orbit close to the actual spin tune. Suggested in 1985[4], Harmonic Spin Matching (HSM) is a method to minimize the spurious tilt of the spin precession axis by compensating the harmonics of the Fourier expansion of the orbit close to a specific value of the spin tune. The method relies on a lengthy procedure for the correction of the orbits, made difficult from the tiny entities of the corrections required.
only monitored by the observation of the effects on the polarization itself (Empirical HSM).

The HSM implementation described in[5] was intended to reduce the drawback caused by the very long polarization time in LEP (\(\approx 360\) min at 44.7 GeV) and consisted in deriving the amplitude of the correcting bumps for the harmonics of interest directly from the beam position information (Deterministic HSM).

In 1993 the application of this method lead to an improvement of the polarization from \(\approx 15\%\) to more than \(40\%\).[5]. Starting from this level the correcting harmonic amplitudes were varied in steps while recording the polarization level to search for an optimum. The experiment, performed at a spin tune \(a\gamma = 101.5\) corresponding to an energy of 44.7 GeV resulted in a polarization level of \(57\%\) at 55.3 GeV involving linear spin resonances plus their synchrotron sidebands derived from [8].

At higher energies with much lower equilibrium polarization (see Fig. 4) one has to improve on the Deterministic HSM just described in order to get near to the polarization maximum. Without any good initial guess it would be very difficult to optimize polarization using Empirical HSM. In fact one can improve on it by applying a “Beam Based Alignment” technique.

6 CONCLUSIONS

A measured polarization of \(\approx 11\%\) at 50 GeV has been reported. The main aim of the forthcoming studies will be to increase the maximum energy were polarization can be observed in order to reduce the energy extrapolation error at 90 GeV. Beam Based Alignment in combination with Deterministic and Empirical HSM will be used to correct the orbit properly. There are as well ongoing studies on a new dedicated polarization optics which will provide better starting conditions for polarization.

7 ACKNOWLEDGEMENTS

We would like to thank the LEP team for their continuous support of our polarization studies.

8 REFERENCES

[9] Tecker, F. et al., This Conference.

Figure 4: Estimate of the reachable polarization level at 55.3 GeV involving linear spin resonances plus their synchrotron sidebands derived from [8].

5.2 Beam Based Alignment

The technique described in[9] determines the offset between the magnetic axis of a quadrupole and its adjacent beam position monitor. The gradient of the magnet is modulated with low frequencies (a few Hz) and very small amplitudes and the effect on the beam is observed with a high sensitivity pickup. The observed movement of the orbit is minimal when the beam goes through the magnetic centre of the quadrupole. It is shown that the knowledge of these offsets together with the quadrupole alignment data brings the Deterministic HSM very near (80\%) to the maximum polarization level which can be achieved with Empirical HSM. Following this argument it was recently decided to equip all vertically focussing quadrupoles in the arcs of LEP with the necessary hardware.