SINGLE BUNCH STABILITY IN PEP-II LER *

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Abstract

The note describes results of studies of the single bunch stability in the low energy ring (LER) of the PEP-II B-factory[?]. Simulations describe the potential well distortion (PWD) obtained by numerical solution of the Haisinski equation[?] and results on the beam stability obtained with the code TRISIM[?]. Both longitudinal and transverse wake fields are taken into account.

1 INTRODUCTION

Preliminary estimates[?] indicate that single bunch in the LER of the PEP-II B-factory has to be stable, both longitudinally and transversely, at the maximum design bunch current 1.8 mA (beam current 3A). However, realistic wakes of the machine has been constructed only recently using results of the extensive numerical simulations of the vacuum components of the ring[?]. Additional to that, the code TRISIM[?], a simulation program for single-bunch collective effects written by one of the authors (G. S.), became recently available. This allows us to study beam stability in a more reliable way than it is possible analytically.

In the beginning, we discuss shortly how the wake fields responsible for the effects under consideration have been constructed. Then we present results on the PWD and, later, results on the beam stability obtained with the code TRISIM. The input parameters are based on the nominal parameters of the LER: energy 3.105 GeV, revolution period 7.336 μs, rf voltage 5.1 MeV, RF frequency 476 MHz, SR energy loss 0.75 MEV/turn, momentum compaction α = 1.23×10⁻³, average beta function β₀ = 14.5 m, betatron tune ν₀ = 36.642, horizontal emittance εₓ = 65.58 nm, coupling εₓ/εₑ = 0.03, damping times τₑ = 30.2 ms, τ₀ = 61.1 ms. The nominal bunch length of 33.1 ps and the rms energy spread of 2.394 MeV were used to generate the initial distribution of a bunch.

2 WAKE FIELDS

The longitudinal wake field of a point-like charge \( W_{l}^{S} (z) \) was constructed[?] from the combination of a narrow-band impedances of the higher order modes of the RF cavities and 290 BPMs, and of the broad-band impedances of the resistive walls, high-frequency tail of the RF cavities impedance, and the impedance of the vacuum components of the ring. Parameters of the HOMs were taken from the results of the measurements[?]. The conservative scenario of 8 cavities in the ring was taken for calculations. Because of the large number of different vacuum components

\[ Z(\omega) = -\frac{i\omega L}{c_{0}^{2}(1 - i\omega\alpha/c_{0})^{3/2}}, \quad (1) \]

which is pure inductive at small frequencies \( \omega\alpha/c_{0} << 1 \) and rolls-off as \( \omega^{-1/2} \) at high frequencies. The corresponding wake at \( z > 0 \) is

\[ W_{l}^{S}(z) = \frac{L}{\sqrt{\pi z^{2}a}}(1 - \frac{2z}{a})e^{-z/a}, \quad (2) \]

and is equal to zero at \( z < 0 \). In the limit \( a \to 0 \) it corresponds to the usual inductive wake \( W_{l}(z) = \frac{L\delta(z)}{\partial z} \). The roll-off parameter \( a \) defines the loss factor. Dependence of the loss factor on the rms \( \sigma \) of a collimator, with the wake of a typical component of the vacuum chamber, was compared with the numerical code ABCI. Dependence is in good agreement with that predicted by Eq. 1. Agreement is much better than that for \( Q = 1 \) model.

The transverse wake \( W_{l}^{T}(z) \) was calculated from measured dipole modes of the RF cavities, impedance of the resistive walls, and the impedance of the vacuum components of the ring. The later was defined from the Wenzel-Panofsky theorem assuming that the dipole longitudinal wake \( W_{l}^{T=1}(z) \) is related to the monopole longitudinal wake, \( W_{l}^{T=1} \approx 2W_{l}^{m=0}/b^{2} \) where \( b \) is the average beam pipe radius. This assumption was confirmed by ABCI.

The wake of the triangular bunch \( W_{l}^{T}(z) \), which is needed as input for TRISIM, was calculated then by convolution of the \( W_{l}^{S}(z) \) with the density of a bunch \( \rho(x) \). Fig. 1 shows the dependence of the position of the bunch centroid \( x > \), the rms bunch length \( \sigma \), and the energy spread \( \delta > \) in MeV vs bunch current in mA. Each point corresponds to one run of TRISIM. Fluctuations increase at large currents. The growth of the energy spread is noticeable above 1 mA indicating onset of the microwave instability.

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The bunch lengthening is clear visible, but even in the last high current case (total current 6.2 A, bunch current 3.75 mA) the potential well does not show a second minimum. At the maximum nominal machine current of 3 A bunch lengthening is small, less than 10%.

Haiinski solution defines the action variable \( J = \frac{1}{2} \sqrt{\frac{2}{\omega(0)}} \int \frac{dx}{2\pi} \sqrt{2[H - U(x)]} \) and the synchrotron frequency \( \omega(0) \). Fig. 2 depicts the dependence of the synchrotron frequency on the action \( J \) of oscillations for various bunch currents. At small amplitudes, the frequency \( \omega(0) \) decreases with \( N_B \) while the frequency spread increases and is about 20% at 3 A.

\[
J = \frac{1}{2} \sqrt{\frac{2}{\omega(0)}} \int \frac{dx}{2\pi} \sqrt{2[H - U(x)]}
\]

4 BUNCH STABILITY

The bunch stability was studied with the code TRISIM. The typical results of TRISIM are shown in Fig. 3 at the bunch current \( I_b = 1.8 \) mA (total current 3 A). Both longitudinal and transverse wakes were taken into account. No feedback system was turned on. The chromaticity of the ring \( \xi = (1/\nu_p) d\nu_p/d\delta \) was set to \( \xi = 0.05 \).

The bunch is stable: no particles were lost after \( 2 \times 10^4 \) revolutions (about \( 5\tau_E \)) but the bunch centroid oscillates with irregular and large amplitude variations which reaches sometimes 40 ps. The rms bunch length increases at this current to 34.22 ± 1.0 ps, or by 8%, and the energy spread grows to 2.54 MeV. The spectrum of the longitudinal motion has a large peak at the synchrotron frequency \( \nu_s = 0.034 \) and a smaller peak at \( 2\nu_s \). The spectrum of the transverse motion indicates a large betatron peak at the fractional betatron tune 0.64 = 1 − 0.363. At the positive chromaticity there are synchrotron sidebands with the amplitude strongly dependent on \( \xi \).

Fig. 4 shows bunch profiles in \((y,z)\) plane at each 5-th revolution after few damping times at 2 mA bunch current. The oscillations of the bunch centroid with synchrotron period are quite noticeable, but the distortion of the bunch remains relatively small.

These results have been checked and confirmed with the number of macro-particles increased up to \( 10^4 \). The instability occurs as a result of increasing longitudinal oscillations of the bunch centroid rather than due to oscillations of the bunch shape. Tracking with large aperture shows recurrences in the bunch blow-up: at the 2 mA bunch current, bunch slowly blows up during 8000-9000 turns, then shrinks in 100-200 turns to a small size, and blows up again. The process repeats itself several times during 40000 turns.

Bunch remains stable for positive chromaticity up to very high currents but, at 12.5 mA, becomes unstable indicating the onset of the (strong) head-tail instability. The situation is drastically different with a negative chromaticity. At the zero or negative chromaticity and \( I_b = 1.8 \) mA bunch is unstable.
5 DISCUSSION

The growth of the energy spread, indicating the onset of the microwave instability, is noticeable at the bunch current above 1 mA. The growth of the energy spread and the rms bunch length is relatively small and not much different from the PWD results up to the bunch current 1.8 mA. However, the amplitude of the longitudinal bunch centroid oscillations increases and, at $I_b = 2$ mA, bunch is already unstable, rms becomes very large, and particles get lost. It is interesting, that $\omega(I)$ dependence at this current becomes flat at small amplitudes $I$. This might indicate that microwave instability starts when $d\omega/dI = 0$. It would be interesting to understand whether the longitudinal feedback system can increase the threshold of the microwave instability. The bunch remains transversely stable up to the $I_b = 10$ mA at the zero or positive chromaticity. This corresponds to the previous estimates of the threshold of the head-tail instability. However, at the negative chromaticity the (weak) head-tail threshold is very low: bunch is unstable at $I_b = 1.8$ mA and $\xi = -0.05$.

6 REFERENCES