FEMTOSECOND ELECTRON BUNCHES FROM COLLIDING LASER PULSES IN PLASMAS

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Abstract

An injector and accelerator is analyzed that uses three collinear laser pulses in a plasma: a pump pulse, which generates a large wakefield (≥ 20 GV/m), and two counterpropagating injection pulses. When the injection pulses collide, a slow phase velocity beat wave is generated that injects electrons into the fast wakefield for acceleration. Particle tracking simulations in 1-D with injection pulse intensities near 10^{17} W/cm^2 indicate the production of high energy electrons with bunch durations as short as 3 fs, energy spreads as small as 0.3%, and densities as high as 10^{18} cm^{-3}.

1 INTRODUCTION

Plasma-based accelerators [1] may provide a compact source of high energy electrons due to their ability to sustain ultrahigh electric fields \( E_z \) on the order of \( E_0 = cm\omega_p/e \simeq n_0^{1/2}\) [cm^{-3}] V/cm, where \( \omega_p = (4\pi n_0 e^2/m)^{1/2} \) is the plasma frequency and \( n_0 \) is the plasma density. Accelerating fields of 10-100 GV/m have been generated over distances of a few mm [2-4] in both the standard [5] and self-modulated [6,7] regimes of the laser wakefield accelerator (LWFA). The characteristic scale-length of the accelerating plasma wave is the plasma wavelength \( \lambda_p = 2\pi c/\omega_p \), which is typically \( \leq 100 \) µm. Although several recent experiments [3,4] have demonstrated the self-trapping and acceleration of plasma electrons in the self-modulated LWFA, the production of electron beams with relatively low momentum spread and good pulse-to-pulse energy stability will require injection of ultrashort electron bunches into the wakefield with femtosecond timing accuracy. These requirements are beyond the current state-of-the-art performance of photo-cathode radio-frequency electron guns.

Recently an all-optical method for injecting electrons in a standard LWFA has been proposed [8]. This method (referred to as LILAC) utilizes two laser pulses which propagate either perpendicular or parallel to one another. The first pulse (the pump pulse) generates the wakefield, and the second pulse (the injection pulse) intersects the wakefield some distance behind the pump pulse. The ponderomotive force \( F_p \sim \nabla a^2 \) of the injection pulse can accelerate a fraction of the plasma electrons such that they become trapped in the wakefield, where \( a^2 \simeq 7 \times 10^{-19} \lambda^2[\mu m]/[W/cm^2] \), \( \lambda = 2\pi c/\omega \) is the laser wavelength, and \( I \) the intensity. Simulations, which were performed for ultrashort pulses at high densities (\( \lambda_p/\lambda = 10 \) and \( E_z/E_0 = 0.7 \)), indicated the production of a 10 fs, 21 MeV electron bunch with a 6% energy spread. However, high intensities (\( I > 10^{18} \) W/cm²) are required in both the pump and injection pulses (\( a \simeq 2 \)). An all optical electron injector would be a significant step in reducing the size and cost of a LWFA.

Figure 1: Profiles of the pump laser pulse \( a_0 \), the wakefield \( \phi \), and the forward \( a_1 \) injection pulse, all of which are stationary in the \( \psi = k_p(z-v_{p0}t) \) frame, and the backward injection pulse \( a_2 \), which moves to the left at \( \approx 2c \).

In the following, a colliding pulse optical injection scheme for a LWFA is proposed and analyzed that uses three short laser pulses: an intense pump pulse (denoted by subscript 0), a forward going injection pulse (subscript 1), and a backward going injection pulse (subscript 2), as shown in Fig. 1. The frequency, wavenumber, and normalized intensity are denoted by \( \omega, k \), and \( a (i = 0, 1, 2) \). Furthermore, \( \omega_1 = \omega_0, \omega_2 = \omega_0 - \Delta \omega (\Delta \omega \geq 0) \), and \( \omega_0 \gg \Delta \omega \gg \omega_p \) are assumed such that \( k_1 = k_0, k_2 \approx -k_0 \). The pump pulse generates a fast \( (v_{p0} \approx c) \) wakefield. When the injection pulses collide (some distance behind the pump) they generate a slow ponderomotive beat wave with a phase velocity \( v_{pb} \approx \Delta \omega/2k_0 \). During the time in which the two injection pulses overlap, a two-stage acceleration process can occur, i.e., the slow beat wave injects plasma electrons into the fast wakefield for acceleration to high energies. It will be shown that injection and acceleration can occur at low densities (\( \lambda_p/\lambda \approx 100 \)), thus allowing for high single-stage energy gains, with normalized injection pulse intensities of \( a_1 \sim a_2 \sim 0.2 \) (\( \sim 10^2 \) less intensity than required by the LILAC scheme). Furthermore, the colliding pulse concept offers detailed control of the injection process: the injection phase can be controlled via the position of the forward injection pulse, the beat phase velocity via \( \Delta \omega \), the injection energy via the

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The boundary between trapped and untrapped orbits is described by the Hamiltonian \( H_0 = -k_p^{-1}E_0 \partial \phi/\partial z \approx E_0 \gamma \). The Hamiltonian associated with Eq. (1) is given by

\[
H_0 \simeq \gamma - \beta_{pb} \left[ \gamma^2 - \gamma^2_p(\psi_b) \right]^{1/2} + \epsilon \psi_b, \tag{2}
\]

where \( \epsilon = E_z k_p / E_0 \delta k \) is constant and \( \gamma^2_p = 1 + a_1^2 + \alpha_2^2 + 2a_1 a_2 \cos \gamma \psi_b \). When \( \epsilon = 0 \), the phase space orbits are symmetric with respect to \( \psi_b \). In terms of the normalized axial momentum, the maximum and minimum points on the separatrix are given by \( u_{\text{min}} \leq \gamma p_0 \gamma_p (1 + 4\beta_0^2)^{1/2} \pm 2a_1 \gamma \psi_b \), where \( \gamma_p = (1 - \beta_{pb}^2)^{-1/2} \) and \( a_1 = \gamma \psi_b \) is assumed. When \( \epsilon \neq 0 \), the separatrix distorts into fish-shaped islands. In particular, when \( \epsilon < 0 \) (\( \epsilon > 0 \)), the “fish tail” of the separatrix opens to the right (left).

A scenario by which the beat wave leads to trapping in the plasma wave is the following. In the phase region \( -\pi/2 < \psi < 0 \), the plasma electrons are flowing backward, \( u_f = -\phi_0 \cos \psi < 0 \), and the electric field is accelerating, \( E_z/E_0 = \phi_0 \sin \psi < 0 \). Here \( \epsilon < 0 \) and the beat wave buckets open to the right. Consider an electron that is initially flowing backward and resides below the beat wave separatrix. Since the separatrix opens to the right, there exists open orbits which can take an electron from below to above the beat wave separatrix. Such an electron can acquire a sufficiently large positive velocity to allow trapping and acceleration in the plasma wave. These open phase space orbits, which provide the necessary path for electron acceleration, can exist when the phase space resides within \( -\pi/2 < \psi < 0 \).

An estimate for the threshold for injection into the wakefield can be obtained by considering the effects of the wakefield and the beat wave individually and by requiring (i) the maximum energy of the beat wave separatrix exceed the minimum energy of the wakefield separatrix, \( u_{\text{max}} \geq (\Delta \phi^{-1} - \Delta \phi)/2 \), and (ii) the minimum momentum of the beat wave separatrix be less than the plasma electron fluid momentum, \( u_{\text{min}} \simeq -\phi_0 \). These two conditions imply that the beat wave separatrix overlaps both the wakefield separatrix and the plasma fluid oscillation, thus providing a phase-space path for plasma electrons to become trapped in the wakefield. For a given wakefield amplitude \( \phi_0 \), conditions (i) and (ii) imply the optimal phase location \( \phi_0 \cos \psi \simeq 3\pi/2 - 2\phi_0 - 2\beta_{pb} \) and threshold amplitude \( \phi_1 > 3\pi/2 - 2\phi_0 + \beta_{pb} \) of the injection pulse, where \( \phi_0^2 \cos^2 \psi \ll 1, a_1^2 \ll 1, \) and \( \beta_{pb}^2 \ll 1 \) were assumed. For example, \( \phi_0 = 0.6 \) and \( \beta_{pb} = 0.05 \) imply \( \psi = -1.3 - 2\pi j \) and \( a_1 > 0.11 \).

### 3 Simulation

To further evaluate the colliding laser injection method, the motion of test particles in the combined wake and laser fields was simulated by numerically solving Eq. (1). At \( \tau = 0 \), the forward (backward) pulse profile \( a_1(\tau) \) is a half-period of a sine wave with maximum amplitude
a_{1m} (a_{2m}), centered at $\psi = \psi_1 < 0$ ($\psi_2 > 0$), with length $L_1$ ($L_2$). Test particles are loaded uniformly from $\psi = 0$ to $\psi = \psi_{\max}$ with $d\psi/d\tau = -\beta_0 \phi$ (initially at rest) and pushed from $\tau = 0$ to $\tau = \tau_{\max}$. Also, $\phi = \phi_0 [1 - \exp(-\psi^2/\pi^2)]$ for $\psi \leq 0$.

To validate the analytical predictions for the trapping thresholds, a “near threshold” case was simulated with $\omega_1/\omega_p = 100$, $\omega_2/\omega_p = 90$, and $\phi_0 = 0.6$, which for $\lambda_1 = 2\pi c/\omega_1 = 1 \mu m$ implies $n_0 \simeq 10^{17} \text{cm}^{-3}$ and $E_z = 0.6 E_0 \approx 19 \text{GV/m}$. Also, $a_{1m} = a_{2m} = 0.3 (1.2 \times 10^{17} \text{W/cm}^2)$, $L_1 = L_2 = \lambda_p/8$ (42 fs), $\psi_1 = -13.6$ and $\psi_2 = 21.4$ (chosen so the beat wave and test particles overlap). After a propagation distance of $\tau_{\max} = 300$ (0.48 cm), trapped electrons were observed with a bunch length $L_b = 6.3 \mu m$ (21 fs) with 60% of the electrons are contained within 66 MeV ±8%. The fraction $f_{tr}$ of those particles which encounter the beat wave that become trapped was $f_{tr} \approx 30\%$. A numerical optimization of the parameters was also performed to determined the trapping threshold. The optimal phase for injection (which minimizes the value of $a_{1m}$ required for trapping) was found to be $\psi_1 = -13.8$, in good agreement with theory. Furthermore, trapping was observed for $a_{1m} > 0.17$, somewhat higher than the analytical prediction (0.11). Additional simulations indicate that trapping occurs when the center of the $L_1 = \lambda_p/8$, $a_{1m} = 0.3$ pulse is located within $-14.2 \leq \psi_1 \leq -13.5$. This implies that the forward pulse must be synchronized to the wake with an accuracy < 37 fs, which is not a serious constraint and can be relaxed somewhat by using a longer forward pulse.

More dramatic results can be obtained by moving the position of the injection pulse slightly forward and increasing both duration and amplitude of the injection pulses (in comparison to the previous “near threshold” example). As an example, a simulation was performed with $\psi_1 = -12.6$, $a_{1m} = a_{2m} = 0.5$, $\phi_0 = 0.7$, $L_1 = L_2/4 = \lambda_p/4$, $\omega_1/\omega_p = 100$, and $\omega_2/\omega_p = 85$ ($\lambda_1 = 0.85 \mu m$, $\lambda_2 = 1 \mu m$, and $\lambda_p = 85 \mu m$). After a distance of $\tau_{\max} = 100$ (0.14 cm), the results are quite dramatic: a bunch duration of 2.9 fs was obtained due to natural compression provided by the axial electric field, with a mean energy of 27 MeV and a standard deviation in energy of 0.32%. The trapping fraction is $f_{tr} \approx 19\%$ and the bunch density is $n_b \approx 1.8 \times 10^{18} \text{cm}^{-3}$. Furthermore, in this run, the trapped electron are injected into and remain within a phase region of the wakefield that is both accelerating and focusing.

The bunch density is $n_b \approx f_{tr} n_0 L_z/L_b$, where $L_z \approx (L_1 + L_2)/2$ is the length of plasma that encounters the overlapping pulses. Assuming that the 1-D results hold for a pump laser of radius $r_0$ implies a total number of trapped electrons $N_b \approx f_{tr} n_0 L_z \pi r_0^2$, e.g., $N_b \approx 7.7 \times 10^9$ for Fig. 4 with $r_0 = 40 \mu m$. Note that $N_b$ can be increased by increasing $n_0$, $r_0$, $a_{1m}$ (via $f_{tr}$) and, in particular, $L_z$ by increasing the duration of the backward pulse $L_2$. The ratio of $N_b$ to the theoretical beam loading limit $N_0$ [10] is $N_b/N_0 = f_{tr} k_p L_z E_0/E_z$, which can easily approach unity. For $N_b$ near $N_0$, however, space-charge effects become important and a self-consistent simulation is required.

In summary, a method has been proposed and analyzed for injecting plasma electrons into a large wakefield using two colliding laser pulses. Simulations of test electrons in prescribed 1-D fields indicate the production of relativistic ($\geq 25$ MeV) electrons with bunch durations as short as 3 fs, energy spreads as small as 0.3%, and densities as high as $10^{18} \text{cm}^{-3}$.

4 ACKNOWLEDGEMENTS

The authors acknowledge the assistance of C. Schroeder and P.B. Lee. This work was supported by the Department of Energy and the Office of Naval Research.

5 REFERENCES