PULSED MAGNET ARC DESIGNS FOR RECIRCULATING LINAC MUON ACCELERATORS*

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Abstract
Recirculating linear accelerators (RLAs) using both pulsed quadrupoles and pulsed dipoles can be used to quickly accelerate muons in the 3 – 2000 GeV range. Estimates on the requirements for the pulsed quadrupoles and dipoles are presented.

RAMPED MAGNET LIMITATIONS
For a single return arc of a muon RLA, the radius is determined by the momentum and the magnetic dipole field averaged over the arc:

\[ R_{arc} = \frac{P_{max}}{(300\text{MeV}/\text{T-m})/B_{avg}} \]

It is not feasible to ramp superconducting magnets fast enough to accelerate short-lived muons [8]; only magnets that are normally conducting might ramp fast enough. In a hybrid cell consisting of normal and superconducting magnets, the normally conducting magnets fields can swing from \(-B_n\) to \(+B_n\) while the field \(B_s\) in the superconducting magnets remains constant. The largest average field is determined by a combination of the limits on \(B_s\) and \(B_n\), the fraction of the cell filled with dipoles \(f\), and the ratio of final to initial energy.

\[ P_{max}/P_{min} = B_{max}/B_{min} = \frac{(B_s L_s + B_n L_n)}{(B_s L_s - B_n L_n)} \]

\[ x = (P_{max}/P_{min} - 1)/(P_{max}/P_{min} + 1) \]

\[ B_{avg} = f B_s (x+1)/(x(B_s/B_n) + 1) \]

Given the current state-of-the art, approximately \(B_s = 8.8\) T, \(B_n = 1.8\) T and \(f = 1\), so \(P_{max}/P_{min} \approx x\), \(B_{avg} \approx 3.0\) T and for 2 TeV, \(R_{arc} = 2.2\) km and \(L_{arc} = 16.3\) km.

3-30 GEV CONCEPT
For lower energies, it is not necessary to consider the complexities of ramped magnets at all; Bogacz et al. [1] have shown that a bisected linac could accommodate up to 7 passes, while Wang et al [2] have shown that that linac with arcs based on a NS-FFAG lattice could accommodate multiple passes through a single arc.

A simple 3-30 GeV dogbone has one small superconducting single pass arc and a much larger, multiple pass NS-FFAG arc at each end with the beam injected in the middle of the linac going to the right.

Adding pulsed quadrupoles to the linac and another pair of NS-FFAG arcs to the ends increases the number of passes to 12 and the energy to 50 GeV [2]. Ramping the NS-FFAG arcs themselves is far less advantageous, as the increased path length due to the relatively low \(B_{avg} \approx (1/3) B_{max}\) bending field of a NS-FFAG leads to excessive loss of the muons by decay.

30-2000 GEV CONCEPT
As the ratio of maximum to minimum energy increases, the proliferation of arcs leaves little alternative to ramping the magnets. The basic concept of the barbell RLA is to inject the muons of both signs into an end of the linac going in the same direction.

Since the momentum of the muons does not change while in an arc, there would be a mismatch between the momentum and the local field if the same ramping magnet apertures were used for muons of both polarities.

This problem can be avoided by using either two sets of magnets or very large acceptance NS-FFAG magnets. The latter option increases the radius and hence the muon decay loss unacceptably.

Here, droplet-shaped return arcs are used. Each teardrop arc bends outward 60°, then inward 300°, then outward 60° again for a 180° turn, for a total circumference of \((420º/360º)2\pi R\). The optics advantages of droplet arcs are described elsewhere [1].

The design of the accelerator is a compromise among many variables, the most important being the gradient and length of the linac and the average magnetic field \(B_{avg}\) of the arcs. The details of the arcs become relatively less

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![Figure 1: Conceptual design of a “dogbone” RLA for muon acceleration.](image)

![Figure 2: Conceptual design of a “barbell” RLA for muon acceleration.](image)
important as the linac gradient increases. An ILC-like 1.3GHz superconducting linac may provide as much as ~30 MeV/m of real estate gradient (including drifts, quadrupoles, etc.) averaged over the length of the linac. As a starting point, consider a 4 km linac with 123 GeV/pass (30.8 MV/m). Using two identical teardrop arcs of radius 2.2 km and $B_{\text{avg}}=$3.0 T, only 1 msec and 16 passes would be required to reach full energy and 90% of the muons survive.

The normal magnets, though, would need to ramp from -1.8 to 1.8 T in one msec, for a rate of 3700 T/s. A similar barbell using only normal magnets in the arc would have a radius of 3.7 km, require 1.7 msec to ramp, and have only 84% of the muons survive.

**RAMPED ARC MAGNETS**

D.J. Summers et al. have considered the issues concerning rapidly ramping magnets for some time and in depth and these estimates in this paper are all heavily based on that work. [3][7][11]

Since the energy stored in a magnetic field volume goes as $1/\mu$, the energy stored in the iron is small compared with that stored in the vacuum.

For the proposed Low Emittance Muon Collider, the emittances of the muon beams are small, so the apertures may also be small. Assuming the muons have normalized emittances of 2.1 mm-mrad, beta functions < 20 m, at 30+123=153 GeV the aperture needs to be

$$10 \sigma_\perp = 10 \sqrt{\left(\epsilon_\perp N / \beta \gamma\right) \beta_y} = 1.7 \text{ mm.}$$

To begin, consider an arc made only of normal magnets ($B_{\text{avg}}$=1.8 T); the stored energy in the dipoles is

$$W = 1/(2\mu) \int B^2 \, dV = ½ L I^2 = 3.7 \text{ J/m}$$

$$dW/dt = 2.2 \text{ kW}$$

The current through the magnet depends on the number of turns $N_{\text{turns}}$ (here taken to be 1):

$$I = B h / \mu_0 N_{\text{turns}} = 2400 \text{ A}$$

$$L = 2W/I^2 = 1.3 \mu \text{ H}$$

$$V = -L \, dl/dt = 4.5 \text{ kV}$$

This voltage is under the limitof 5 kV recommended by Ken Bourkland [11]. The arcs here are 27.15 km long (the circumference of the LHC!), so each requires 60 MW of ramp power. That leads to the suggestion that a resonant circuit could be considered. Roughly, the frequency ought to be $\sim 150$ Hz (4x the ramp time), so for a simple LC circuit:

$$C = 1 / (2 \pi L f^2) = 5.4 \text{ F/m}$$

A 3rd order component would probably need be superimposed to maintain a more linear ramp to match the energy gain of the linac; alternatively, the phasing of the linac could be modified during the ramp to make the energy gain match the magnets at some sacrifice in muon survival.

Using a laminated dipole design very similar to that proposed by D.J.Summers[7], the core losses for 12 mil 3% silicon steel can be described as [9]:

$$\text{Watts/kg} = 1.49E-3 \cdot f^{-1.55} B^{1.71} \sim 10 \text{ W/kg.}$$

Assuming the magnet is about 2”x1.5” overall gives about 13kg/m of steel, or about 130 W/m from core losses. Ohmic heating of the copper conductors is about 800 W/m.

Figure 3: Alternating layers of a laminated normal ramped arc dipole magnet [7].

It is reasonable to have the two normal conducting magnets share the return yoke as in the LHC, reducing the size and cost of the two arcs. If the muons travel the same direction around the arc, the fields are equal and opposite.

A hybrid normal/superconducting magnet allows a higher average magnetic field; however, the rapidly changing field from the normal magnets must not penetrate the superconducting magnets. Due to the very small apertures, however, the required gaps have little impact on the overall length of the cell. The range of the momentum of the beams determines the maximum average field that can be made by the hybrid.

In this case, it may become advantageous to divide the momentum range between two arcs. Any advantage is greatly reduced as the energy/pass of the linac is increased.

For the hybrid case with a single arc at each end, $B_{\text{avg}}=3$ T, so the total ramp time is about 1 ms. The normal magnets swing from $-+ 1.8$ T, so the frequency is about 500 Hz, and the superconducting magnets remain unchanged. While the stored energy in the normal magnets is the same as before, the ramping power and voltages are about 3.3x as high and the core losses are about 6.5x as high. The cell length is dominated by the normal dipole ($L_n=4.74 L_s$), so averaging over the whole arc the power/length by is only decreased $\sim 17\%$.
RAMPED QUAD MAGNETS

Both the linac and a single arc in a barbell will require very similar ramped quadrupole magnets.

At 30 GeV, the aperture needs to be also ~3 mm and the gradient ~10T/m for a 1 m quadrupole, while for 2000 GeV the aperture only needs to be ~0.5 mm and the gradient needs to be about ~300 T/m. Again the energy stored in the yokes is small with respect to that in the vacuum since the energy density goes as $1/\mu$.

As the beam energy is increased for a given beta function, the required aperture decreases and the gradient increases such that the stored energy would remain about the same, corresponding to about 0.2 J/m of stored energy. However, since the quadrupoles can’t change aperture as the energy increases, the stored energy increases by the square of the gradient. By 2 TeV, the stored energy is 1.5 J/m and the power 1.5 kW. With 3% silicon steel return yoke, the core losses would be only ~80W/m at 2000 GeV.

As in the case of the dipoles, a resonant circuit to move the energy in and out of the field is a possibility.

It ought to be noted that while the arc quadrupole gradients do scale with the average $B\cdot dl$ of a cell, they do not scale with the normal components of a hybrid cell, making a combination design problematic.

The Fermilab Booster is upgrading to ramped combination corrector magnets as shown in Figure 5 [4]. These have about ~6x the stored energy/length of RLA quadrupoles and can switch full quadrupole field polarity in 1 ms.

Also, the HCX fusion experiments use pulsed quadrupoles that store about 840 times the energy of the RLA quadrupoles, but are used only for very low duty cycles (Figure 6) [5].

CONCLUSION

While challenging, ramped quadrupole, dipole, and combination magnets do not appear unrealistic. Much work remains to be done to model these devices and the optics of the whole accelerator system in detail.

REFERENCES

[1] G. Wang et al., this Conference