SUPER-B LER DYNAMIC APERTURE STUDY AND OPTIMIZATION

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Abstract

A project of the SuperB Factory in Italy with the crab-waist collision scheme and extremely large luminosity [1, 2] addresses new challenges to the nonlinear beam dynamics study. Among these challenges are: low emittance lattice requiring strong sextupoles for chromatic correction, sub-mm vertical betatron function at the IP, crab sextupoles placed at both sides from the IP, etc. In this report we describe the results of the DA limiting sources analysis and optimization of the arrangement of the IR and Crab sextupoles and octupoles for the Low Energy Ring (LER).

INTRODUCTION

A recently proposed crab waist scheme of beam-beam collisions [3, 4] provides a substantial luminosity increase without the bunch length reduction and with moderate beam currents. However, this approach requires several specific conditions (atypical for ordinary colliders), which send the challenge to the nonlinear beam dynamics study and dynamic aperture optimization. Among them are:

- Sub-millimetre vertical beta at IP $\beta_y^*$, producing large chromatic aberration of the beta functions in the first quadrupole and, consequently, downstream in the IR section according to

$$\frac{1}{\beta_y} \frac{d\beta_y}{d\sigma} = \frac{L}{\beta_y^*},$$

where $L$ is the distance from the IP to the first quad. To increase the bandwidth, strong sextupoles in the IR region are required.

- Horizontal emittance of nanometre order of magnitude requiring strong focusing arcs with high chromaticity and powerful sextupoles to correct it.

- Large crossing angle producing in the fringe field of the first quadrupoles a sextupole component, which, together with the high betas, can degrade the dynamic aperture significantly.

- Cross talk between the beam-beam and the magnetic nonlinearities providing the particle distribution tail growth and the beam lifetime reduction. All these issues are discussed in the paper with emphasising on the IR sextupole optimisation. In the future, when the lattice design is completed (including the damping wigglers section, polarization insertion, etc.), knowing of the main DA limiting factors will facilitate the general optimization of the SuperB LER dynamic aperture.

LER OPTICS MODEL

The LER lattice dated February 5th, 2009 was given to us by Marica Biagini. Table 1 summarizes the relevant parameters of the lattice while the optical functions of the IR region are depicted in Fig.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $E$</td>
<td>4 GeV</td>
</tr>
<tr>
<td>Circumference, $L$</td>
<td>1.692 km</td>
</tr>
<tr>
<td>Compaction factor, $\alpha$</td>
<td>3.19x10^{-4}</td>
</tr>
<tr>
<td>Emittance, $\epsilon_x$</td>
<td>2.83 nm</td>
</tr>
<tr>
<td>Energy spread, $\sigma_E$</td>
<td>8.2x10^{-4}</td>
</tr>
<tr>
<td>Betatron tune, $\nu_x/\nu_z$</td>
<td>43.57/21.59</td>
</tr>
<tr>
<td>Natural chromaticity, $\xi_x/\xi_z$</td>
<td>-135/-560</td>
</tr>
<tr>
<td>Crossing angle</td>
<td>60 mrad</td>
</tr>
<tr>
<td>$\beta^*$, $h/v$</td>
<td>35/0.22 mm</td>
</tr>
<tr>
<td>Sigma$^*$, $h/v$</td>
<td>246/13.7 $\mu$m</td>
</tr>
</tbody>
</table>

Figure 1: Lattice functions and sextupole arrangement of the LER interaction region (the IP at the left side is the reflection symmetry point of the plot).

INTERACTION REGION

There are several families of the sextupole magnets in the LER IR: in the vertical beta chromatic section ($SY0$ and $SY4$ in Fig.1), in the horizontal chromatic section ($SX0$ and $SX4$), the Crab sextupoles and the sextupoles to control the energy dynamic aperture. As here we discuss the on-energy aperture, the latter are not shown in Fig.1.

To cancel the sextupole geometric aberration, a pair of the beta-chromatic magnets ($SY0/SY4$ and $SX0/SX4$) is separated by $-I$ transform. This cancellation works well for the kick sextupoles, but the finite sextupole length generates aberrations of high orders [5], which deteriorate the dynamic aperture. In our case the strongest effect comes from the vertical IR chromatic sextupoles because of their high strength and beta-function (Fig.2).
Figure 2: DA reduction due to the finite length of the IR chromatic sextupoles.

If we represent a solution of the sextupole equations
\[
\frac{d^2x}{ds^2} = -k_z(s)(x^2 - z^2) \quad \text{and} \quad \frac{d^2z}{ds^2} = k_z(s)xz
\]
as Taylor series
\[
x(s) = \sum_{n=0}^{\infty} \frac{a_n}{n!} s^n \quad \text{and} \quad z(s) = \sum_{n=0}^{\infty} \frac{b_n}{n!} s^n,
\]
a recurrent sequence of the aberration coefficients for the map through the $-I$ sextupole pair with the length $L$ each can be found. Quadratic (sextupole) terms are cancelled exactly and the first non-zero term belongs to the 3rd order (octupole-like) aberration:

\[
x = -x_0 - p_{x0} \cdot L - \frac{1}{12} k_z^2 \cdot L^3 \left( x_0^3 + x_0 \cdot z_0^2 \right) + O(L^4)
\]
\[
p_x = -p_{x0} - \frac{1}{6} k_z^2 \cdot L^3 \left( x_0^3 + x_0 \cdot z_0^2 \right) + O(L^4)
\]

and the same for the vertical plane. The map shows that (i) the 3rd order aberration strength grows quadratically with the sextupole strength and (ii) it seems reasonable to try to correct this aberration with the help of the octupole magnets. However, as the aberration term differs from the exact octupole $\sim x^3 - 3x \cdot z^2$, one should not expect perfect cancellation.

Figure 3: Finite length sextupoles DA (black) and its recover by correction octupoles (green) and sextupoles (blue).

In another approach, correction sextupoles instead of octupoles are used to solve this problem. Each chromatic sextupole is coupled with very weak correction sextupole also shown in Fig.2 ($SY1/SY3$ and $SX1/SX3$). These sextupoles with the strength of $-1.5\%$ of the main ones drastically increase the on-energy dynamic aperture. The effect of the correction sextupole for the vertical chromatic section in comparison with the octupoles is shown in Fig.3. The explanation of this effect is not clear yet and the work is in progress, but the same approach can be used to tune the dynamic aperture reduced by the finite length Crab sextupoles. Fig.4 demonstrates the dynamic aperture of the kick-like and finite length Crab sextupoles before and after compensation by the weak (1% in strength) correction sextupoles.

Figure 4: DA aperture due to the kick (blue) and finite length (red) Crab sextupoles and that recovered by the correction sextupoles (green).

The final DA caused by the IR sextupoles before and after its increase by the correction sextupoles is shown in Fig.5.

Figure 5: IR on-energy DA before and after correction.

Figure 6: DA tune scan before (upper row) and after (lower row) correction. The color indicates the maximum DA size.
The IR dynamic aperture as a function of the betatron tune before and after correction is shown in Fig. 6. Note that due to the nonlinear detuning, the real resonance lines at the plot do not correspond well to those found from \( m_1 \nu_z + m_2 \nu_y = n \).

**OTHER POSSIBLE SOURCES OF THE LER DA LIMITATION**

An intrinsic feature of the Crab Waist machines is the large crossing angle at the IP so the beam goes through the QD0 at an angle to the quadrupole end faces (Fig. 7). In this case the sextupole term is produced in the quadrupole fringe field according to [6]

\[
(ml)_{1,2} = 2k_1 \cdot \tan \alpha_{1,2}
\]

where \( k_1 \) is the normalized quadrupole gradient. In our case the integrated strength of the sextupole term at the quadrupole edge is \((ml) \sim 10^{-4} \text{ cm}^2\).

Figure 7: With the crossing angle at the IP the beam passes the quadrupole entrance and exit at some angle.

The final design of the QD0 quadrupole is not finished yet but the preliminary estimation shows that the sextupole term in the quadrupole fringe field, together with high beta functions (especially the vertical one), may degrade the dynamic aperture significantly (Fig. 8).

Figure 8: DA reduction due to the sextupole component in the quadrupole fringe field.

Another potential source of the nonlinear beam dynamics degradation is the common action of the beam-beam and the magnetic nonlinearities. Usually it is not a problem for colliders because the magnetic nonlinearities (mainly sextupoles) reduce the aperture at the large amplitudes where the beam-beam force falls down to zero. But for the crab waist machines with the low dynamic aperture and high strength of the beam-beam perturbation, a cross-talk between them can deteriorate the collider performance [7].

Figure 9: SuperB beam population distribution with the beam-beam effects for the linear (left) and nonlinear (right) lattice.

Figure 9 shows the transverse beam intensity distribution for colliding beams in the amplitude space with the nonlinear lattice elements switched-off and on. One can see that the magnetic nonlinearities provide the vertical beam tail growth above the dynamic aperture. The simulation shows that the luminosity beam lifetime for this case reduces to \( \tau \sim 4 \text{ min} \).

**CONCLUSIONS**

In the paper we have indicated the potential sources of the nonlinear beam dynamics problems for the SuperB LER. Of course, the list of such sources is not complete and others sources (arc chromatic sextupoles, wigglers, magnets field imperfections, etc.) should be carefully studied and corrected in future.

**REFERENCES**

[2] M. Biagini et al, MO 3RAC04, this Conference