Abstract

The interaction of external monochromatic, linearly polarized, plane electromagnetic (EM) wave with the nonlinear one-dimensional wake-wave, generated by relativistic electron bunch, moving in cold plasma, is considered. At definite conditions on parameters of plasma and electron bunch, nonlinear plasma electron density wake wave has a pronounced spikes, where density value is nearing the wave breaking limit and plasma electron velocities, being directed along the bunch velocity reach, their maximum. Presented calculations exhibit, that external EM-wave, propagating through plasma at such a conditions normal to the bunch velocity direction and with polarization along the bunch velocity can be amplified inside the spikes. EM-wave, Thomson scattered on the spikes, is also amplified. Amplification factors are obtained for the both cases at different conditions for system plasma-electron bunch - external EM-wave parameters.

It is shown that amplification factor can be larger, especially at the resonance condition, when frequency of external field is nearing to plasma frequency at spikes.

Presented results are obtained using perturbative approach, when dimensionless external EM-wave amplitude is used as a small parameter; the known exact nonlinear one-dimensional solution, obtained previously for cold plasma-relativistic electron bunch system, is taken as a zero order approximation.

Considered amplification process can serve as a physical ground for research and development of powerful klystron type amplifiers for future linear colliders.

1 INTRODUCTION

The problem of interaction of external electromagnetic wave (EM-wave) with electrons, moving in plasma, and plasma wake waves, generated by moving electrons or electron bunches, have been a subject of numerous investigations (see for example[1] and references therein).

The goal of the presented investigation is to find out the possibility and conditions, when EM wave inside the plasma with the frequency of external EM-wave and Thomson scattered EM-wave can be amplified, as compared to the external EM-wave. It is shown, that if the nonlinear plasma wake waves spikes, generated by relativistic rigid electron bunch, are taken into account as an active media, where interaction processes take place, the significant field amplification can be achieved. Wake wave spikes are occur, when nonlinear waves in plasma are nearing wave breaking limit, described for free plasma waves in[2] - [4] and at some details considered in[5] - [6] for relativistic driving bunches and underdense and overdense plasmas. In[5] - [6], in particular, the connection between wave breaking limits on plasma electron density and velocities and bunch electron parameters are obtained.

Wave breaking has followed so called "blowout" regime, introduced in[7] for PWFA process. In the frame of laser-plasma based acceleration, wave breaking regime is used for generation high current relativistic electron beams[8]. In both cases[7] - [8] the unique features of the nonlinear plasma wake waves near wave breaking limit are used for particle acceleration purposes. Results of present work indicate the possible application of plasma wake wave breaking regime for amplification EM-wave, serving as a physical ground for research and development on powerful EM-wave generators and amplifiers for future colliders and accelerators with a high acceleration gradient.

2 BASIC EQUATIONS AND OUTLINE OF THE FORMALISM

The flat rigid electron bunch, moving in cold plasma with immobile ions along z-direction with velocity \( v_0 \) in lab system, is considered. Longitudinal length of the bunch is \( d \), transverse directions of the bunch are taken infinite and problem in zero approximation (in absence of external EM-wave) is treated as one dimensional. This approach is valid for wide enough bunches, when bunch radius \( r_0 \gg \omega_p [9] \), where \( \omega_p \) is plasma frequency \( \omega_p^2 = \frac{4\pi e^2 n_0}{m} \), \( n_0 \) is plasma electron density at equilibrium.

This condition could be replaced at some sense more adjustable one, if external constant longitudinal magnetic field \( H_{||} \) is applied to the system. Then one dimensionality conditions could be \( r_0 \gg \rho_L, \Omega_L \gg \omega_p \), where \( \rho_L \) and \( \Omega_L \) are subsequently the Larmour radius and frequency for plasma electrons.

The considered cold plasma-relativistic electron bunch system is interacting with the external monochromatic, linearly polarized electromagnetic (EM) wave, propagating through plasma in x-direction and with electric vector directed along z-axis (p-polarization). The external EM-wave inside the plasma is described by:

\[
\mathcal{E}_z = \mathcal{E}_{0z} e^{-i\omega t + ik_{||} x}, \quad H_{||} = -\sqrt{c \epsilon_0} \mathcal{E}_{0z} e^{-i\omega t + ik_{||} x},
\]

where \( \epsilon \) is a dielectric constant for plasma:

\[
\epsilon \equiv \epsilon' + i\epsilon'' = 1 - \frac{\omega_p^2}{\omega} + i \frac{\sigma_p^2 \gamma \epsilon'\epsilon''}{\omega_p^2};
\]

\[
\sqrt{\epsilon} \equiv n + i\eta, \quad n = \frac{1}{\sqrt{2}} \left( \epsilon' + (\epsilon'^2 + \epsilon''^2)^{1/2} \right)^{1/2},
\]

\[
\eta = \frac{1}{\sqrt{2}} \left( \epsilon'' - (\epsilon'^2 + \epsilon''^2)^{1/2} \right)^{1/2},
\]

\[
\omega_p^2 = \frac{4\pi e^2 n_0}{m}.
\]
which reduces the set of quasi-linear differential equation in partial derivatives to the set of quasi-linear ordinary differential equations. It is still complicated enough and in order to find an analytic solution in what follows the region of $\tilde{z}$ near to $\tilde{z}_m$ is considered.

In this region, approximately[6]

$$
\beta_0^{(1)} \approx \beta_m + \frac{1}{2} \left( \frac{d^2 \beta_0^{(1)}}{dz^2} \right)_{z_m} (\tilde{z} - \tilde{z}_m)^2 = \beta_m - \frac{n_m (m - 1)}{2 \beta_m (1 + \rho_m^2)^{3/2}} (\tilde{z} - \tilde{z}_m)
$$

$$
|\tilde{z} - \tilde{z}_m| \ll \left( \frac{2 \beta_0^{(1)} m_0^2}{n_m^2} \right)^{1/2} \sim \frac{\beta^{3/2}_m}{n_m},
$$

where $n_m, \rho_m$ are the plasma electron density and momenta at wake wave spike region (6) at $z = \tilde{z}_m$. Supposing also that quantities $1 - \beta_0^{(1)} = \frac{n_m}{\rho_m}, 1 - \beta_m = \frac{n_m}{\rho_m}$ are small enough it is possible essentially simplify the obtained set of equations and then reduce it to one first order equation for $\rho_0^{(1)},$ particular solution of which in the region (6) is not difficult to obtain.

### 3 Estimates for Amplification Factors

From approximate expression for $\rho_0^{(1)}$, obtained by above mentioned way, it is possible to estimate the amplitudes of internal electric field $E_{x, \tilde{z}, 0}^{(1)}$

$$
E_{x, \tilde{z}, 0}^{(1)} \approx -2 \mathbf{E}_{\tilde{z}, 0} \left( 1 - \frac{n_m \omega_p}{\rho_m \omega_p} \right),
$$

$$
E_{x, \tilde{z}, 0}^{(1)} \approx -\frac{\mathbf{E}_{\tilde{z}, 0} \cdot \omega_p}{2 \sqrt{\varepsilon} \omega_p} \times
$$

$$
\left[ \frac{2 n_m \left( \frac{\omega_p}{\omega_0} \right)^2}{\rho_p \omega_0} + \frac{2 n_m \omega_p}{\rho_p \omega_0} - 1 \right].
$$

When $\frac{n_m \omega_p}{\rho_p \omega_0} \gg 1, \frac{\omega_p}{\omega_0} \ll 1$ i.e.

$$
\beta_0 - \beta_m \ll \frac{\omega_p}{\omega_0} \gamma_0^{-1}, \frac{\omega_p^2}{\omega_0^2} > n_m \gg \frac{\omega_p}{\omega_0} \gamma_0, \gamma_0 \ll \frac{\omega_p}{\omega_0} \quad (8)
$$

the field amplification factors

$$
K_{x, \tilde{z}} = \left| E_{x, \tilde{z}, 0}^{(1)} / E_{x, \tilde{z}, 0} \right|, K_z \approx \frac{2 n_m \omega_p}{\rho_p \omega_0} \gg 1,
$$

$$
K_x = \sqrt{\varepsilon} \left( \frac{n_m \omega_p}{\rho_p \omega_0} \right), n_m \left( \frac{\omega_p}{\omega_0} \right)^2 < \frac{1}{\sqrt{\varepsilon}} \times
$$

$$
\times \left( \frac{n_m \omega_p}{\rho_p \omega_0} \right), K_x \gg 1; \quad (9)
$$

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The plasma current density, generated by external EM-wave in considered system, in the first approximation is given by
\[ j_x^{(1)} = n_0 (\beta_x^{(1)}), j_z^{(1)} = n_0 (\beta_z^{(1)} + n_0 (\beta_z^{(0)})^{(0)} \] (10)
and it can be shown that at the condition (8)
\[ j_z^{(1)} \approx \frac{2i}{\sqrt{c}} \left( \frac{m_e \omega_p}{p_m \omega_0} \right)^2 \mathcal{E}_z^{(1)} , j_z^{(1)} \approx -2i \left( \frac{m_e \omega_p}{p_m \omega_0} \right)^2 \mathcal{E}_z^{(0)} \] (11)
and corresponding amplification factors for the intensity of EM wave Thomson scattered from the plasma wake wave spikes are:
\[ K_{rad,x} = \frac{1}{W_0} \frac{dW_{x,z}}{d\omega} \sim \left| \mathcal{E}_x^{(1)} \right|^2 | \mathcal{E}_z^{(0)} | \] (12)
where \( \frac{dW_{x,z}}{d\omega} \) is radiated energy flux in unit solid angle per second from unit volume of plasma wake wave spike, and \( W_0 \) is the incident energy flux of the external EM-wave on unit area of plasma wake wave spike cross section, normal to direction of EM-wave propagation, per second. From (11) and (12) it follows that amplification factors can be large enough at the conditions (8).
\[ K_{rad,x} \sim 4 \left( \frac{n_0 \omega_p}{p_m \omega_0} \right)^4 \gg 1 \] (13)
Further increase of amplification factors \( K_x \) and \( K_{rad,x} \) (9, 11, 12) can be realized at resonance condition \( \omega_0 = \omega_p \) when \( \epsilon' \rightarrow 0 \). Then (see (2))
\[ \left( \frac{1}{\sqrt{\epsilon}} \right)^{-1} \sim \left( \frac{\omega_0}{\nu_{x,ff}} \right)^{1/2} , \omega_0 \gg \nu_{x,ff} \] (14)
and corresponding increase of \( K_x \) and \( K_{rad,x} \) from (9, 12) can be easily noticed. Resonance increase due to factors (14) for \( K_x, K_{rad,x} \) can take place independently from conditions (8).

4 CONCLUSION

The obtained analytical results demonstrate that at certain conditions on cold plasma - relativistic electron driving bunch - external EM wave parameters (8), essentially large amplifications are existed of electric field inside the plasma (9, 14) as well as of intensity of Thomson scattered on plasma spikes EM-wave (13,14). Presented results could be used in research and development of powerful klystron type amplifiers and generators of high frequency EM - waves for future linear colliders.

The presented estimates have at some extent qualitative character and must be complemented by more quantitative calculations, presumably by computer simulations.

5 REFERENCES