OBSERVATION OF QUADRUPOLE MODE FREQUENCY
AND ITS CONNECTION WITH BEAM LOSS

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Abstract
Recent Simulation results imply that the number of particles in a high intensity synchrotron is limited by the resonance crossing of coherent mode oscillations, not by the incoherent one. In order to verify this, we measured the tune shift of quadrupole mode oscillations in HIMAC at National Institute of Radiological Sciences (NIRS) to show the connection with beam loss. In this paper, we discuss signals observed in a coating beam. We also presents the numerical calculation of envelope motion for a bunched beam.

1 INTRODUCTION
It is believed that space charge effects limit the number of particles in a high intensity synchrotron. However, the detailed mechanism of beam loss is not clear. One model tells that the incoherent tune of an individual particle is reduced by the space charge field to a major resonance line, and resonate with lattice field errors. Another model, proposed by Sacherer in 1960's [1], is that the coherent mode oscillations of a beam resonate with lattice field errors. The incoherent model is not self-consistent because it neglects the time evolution of distribution due to the space charge force.

Recent simulation results seem to support the latter model [2]. In order to verify this, we planned to measure the tune shifts of coherent mode oscillations and their connection with beam loss. We observed coherent mode signals in a coating beam in HIMAC synchrotron. To start with, we discuss the three dimensional envelope equations, where it is shown that the envelope oscillation frequencies in a bunched beam do not differ much from that in a coating beam. Then, we report our measurements.

2 ENVELOPE OSCILLATIONS IN A BUNCHEO BEAM
2.1 3D envelope equations
For a bunched beam, envelope oscillations occur in three dimensions. Sacherer studied three dimensional envelope equations and found that they depend only on rms beam size, as far as the distribution has ellipsoidal symmetry [3]. They are

\[ \frac{d^2 \tilde{x}}{d\phi^2} + \left( \nu_{0x}^2 - \frac{N_R R^2}{\beta^2 \gamma^2} \lambda_0 g(\tilde{x}, \tilde{y}, \gamma \tilde{z}) \right) \tilde{x} = \frac{(R \epsilon_x)^2}{\beta^3} \]

\[ \frac{d^2 \tilde{y}}{d\phi^2} + \left( \nu_{0y}^2 - \frac{N_R R^2}{\beta^2 \gamma^2} \lambda_0 g(\gamma \tilde{z}, \tilde{x}, \tilde{y}) \right) \tilde{y} = \frac{(R \epsilon_y)^2}{\beta^3} \]

\[ \frac{d^2 \gamma \tilde{z}}{d\phi^2} + \left( \nu_{0z}^2 - \frac{N_R R^2}{\beta^2 \gamma^2} \lambda_3 g(\gamma \tilde{z}, \tilde{x}, \tilde{y}) \right) \gamma \tilde{z} = \frac{(R \epsilon_z)^2}{\beta^3} \]  

(1)

where \( \phi \) is the azimuth along the circumference of a synchrotron, \( R \) is the ring average radius, \( \nu_{0x} \) is bare tune, and the geometric factor \( g \) is elliptic integral,

\[ g(a, b, c) = \frac{3}{2} \int_0^{\infty} ds \frac{d}{(a^2 + s)^{3/2}(b^2 + s)^{1/2}(c^2 + s)^{1/2}} \]

(2)

and \( \lambda_3 \) is about \( 1/5 \sqrt{5} \), which very weakly depends on details of distribution. In the parenthesis of the left hand side of eq.1 corresponds to a square of depressed tune \( \nu_x^2 \) when \( (\tilde{x}, \tilde{y}, \gamma \tilde{z}) \) is the matched beam size. Differentiating eq.1 and use \( \epsilon_x = \nu_x \Delta x_0 / R \), we have

\[ \frac{d^2}{d\phi^2} \Delta \tilde{x} + 4 \nu_x^2 \Delta \tilde{x} = \frac{\lambda_0 N_R R^2}{\beta^2 \gamma^2} \tilde{x}_0 \Delta g|_{x_0, y_0, z_0} \]

\[ \frac{d^2}{d\phi^2} \Delta \tilde{y} + 4 \nu_y^2 \Delta \tilde{y} = \frac{\lambda_0 N_R R^2}{\beta^2 \gamma^2} \tilde{y}_0 \Delta g|_{y_0, z_0, x_0} \]

\[ \frac{d^2}{d\phi^2} \gamma \Delta \tilde{z} + 4 \nu_z^2 \gamma \Delta \tilde{z} = \frac{\lambda_0 N_R R^2}{\beta^2 \gamma^2} \gamma \tilde{z}_0 \Delta g|_{\gamma \tilde{z}_0, x_0, y_0} \]

(3)

where the subscript “0” denotes matched solutions. The equations above can not be solved analytically because of elliptic integral. One way to solve them is to take some approximations [4] and substitute arithmetic function for \( g \). Here, we employ numerical integration.

2.2 Numerical solution
We solved eigen equations of eq.3 numerically, for a given focusing force, emittance and beam intensity. It is expected that, in the limit of long bunch and low synchrotron frequency, the behavior of envelope motion will be similar to that of a coating beam.

In solving eq.3, matched beam size \( (\tilde{x}_0, \tilde{y}_0, \gamma \tilde{z}_0) \) was calculated by eq.1 with \( d^2 / d\phi^2 = 0 \) and elliptic integral
is evaluated with a numerical package available [5]. We chose parameters in Table 1. In order to study bunch length dependence, we varied longitudinal focusing force, and adjusted beam intensity simultaneously so that transverse incoherent tune remains constant.

At first, we show in Fig. 1 the eigen frequencies of envelope oscillations, together with the one calculated by a 2D formula.

\[ \nu_{\pm} = \frac{x_0 + \rho_0}{2y_0} \nu_x \pm \sqrt{\left( \frac{2\nu_x^2}{\nu_y^2} - 2\nu_y^2 \right)^2 - \left( \nu_x \Delta \nu_y \right)^2} \]

\[ = 7.01^2, 6.39^2. \] (4)

In each region except \( \gamma z_0 \approx 250 \text{mm} \), two of calculated modes agree with the values estimated by eq.4. Eigen vectors, shown in Fig. 2, prove that these modes are pure horizontal, vertical and longitudinal one. At around \( \nu_{y_0} \approx \nu_{x_0}, \) the oscillations form two coupled modes, i.e. breathing(even) mode with higher frequency and quadrupole(odd) mode with lower frequency. This behavior is similar to the coupled phenomena of a single particle.

In conclusion, the coupling of transverse and longitudinal envelope oscillations due to space charge force seems negligible for a synchrotron where the synchrotron oscillation frequency is much less than betatron oscillation. More detailed discussion could be found in Ref. [6]

Table 1: Parameters used in envelope mode calculation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle</td>
<td>6MeV/u He(^{2+}) (( \gamma = 1.006 ))</td>
</tr>
<tr>
<td>Bare tune</td>
<td>( (\nu_{x_0}, \nu_{y_0}) = (3.6, 3.3) )</td>
</tr>
<tr>
<td>( \nu_{y_0} )</td>
<td>( 0.00978 ) @ ( \gamma z = 5000 \text{mm} )</td>
</tr>
<tr>
<td>100% emittance</td>
<td>( e = (10, 10, 55000) ) ( \pi \text{mm-mrad} )</td>
</tr>
<tr>
<td>Beam intensity</td>
<td>( 1 \times 10^{11} \text{ppp} ) @ ( \gamma z = 5000 \text{mm} )</td>
</tr>
</tbody>
</table>

Figure 1: Bunch length dependence of eigen mode frequencies with the parameters of the Table 1.

Figure 2: Amplitude ratio of eigen modes in x(solid line), y(broken line) and z(dot line). Mode 1, 2 and 3 is of the highest, middle and the lowest frequency.

3 MEASUREMENTS IN HIMAC

3.1 The HIMAC tune shifts

We chose a flatbase operation (no acceleration) with a He\(^{2+}\) beam to obtain the largest tune shift. Machine parameters are same as Table 1 except horizontal emittance of 264\( \pi \text{mm-mrad} \) and maximum beam intensity of \( 1 \times 10^{11} \text{ppp} \). That is due to multi-turn injection. The incoherent tune shift of vertical oscillation is calculated as \(-0.0446/10^{11} \text{ppp}\). The coherent ones are \(-0.60216/10^{11} \text{ppp} \) for dipole mode and \(-0.0590/10^{11} \text{ppp} \) for quadrupole mode.

3.2 Quadrupole monitor

Among coherent mode oscillations, the tune shifts of quadrupole mode is dominant. Therefore, we installed a quadrupole pickup monitor in HIMAC in the summer of 1998. It is an electrostatic type pickup with four electrodes(Fig.3). The amplitude of quadrupole mode can be seen in a quadrupole channel output, which is subtracting the sum of vertical two electrodes signal from that of horizontal. The difference between opposite two electrodes are used to observe dipole mode oscillations. The estimated magnitude of output signal for a 6MeV/u He\(^{2+}\) beam is around 15mV/10\(^{11} \text{ppp} \) in each channel, and a quadrupole component is about the order of \( (x/a)^2 \) lower than that, where \( a \) is the radius of pickup electrodes.

3.3 Spectrum and tune shifts

We observed beam spectrum of a coasting beam of \( 3 \times 10^{10} \text{ppp} \) with a real time spectrum analyzer. We used RFQ[7] in order to excite quadrupole mode oscillations. Fig.4 shows the beam spectrogram around 4 times of revo-
olution frequency. Quadrupole mode and dipole mode oscillations in horizontal and vertical space can be seen. Some of them are shifting in frequency as the beam current decreases at \( \approx 1000 \text{msec} \) after injection.

We estimated the space charge tune shift of each mode by plotting frequency versus beam current seen by a slow current transformer. Fitted value of tune shifts for quadrupole mode is about \(-0.082/10^{11}\) ppp, which is not in good agreement with theoretical value. A part of the discrepancy can be attributed to uncertainty of emittance. We are continuing measurements at other operation points.

First, we solved 3D envelope equations numerically, and found that the coupling between transverse and longitudinal motion due to space charge force is negligible in synchrotrons. Therefore, the mode frequencies in a coasting beam are fairly good approximations of that in a bunched one except one longitudinal mode.

Secondly, we presented our experiment in HIMAC synchrotron. We identified the coherent mode signals. The tune shift is not so good agreement with the expected value, at this moment. However, a part of the discrepancy can be attributed to uncertainty of emittance. We are carrying on the experiments to reveal the connection between tune shifts and beam loss.

Finally we show in Fig.6, where the operating point is moving gradually by varying defocusing field. As the current of defocusing magnet goes down, the quadrupole mode frequency approaches to revolution frequency. At 2500msec after beam injection, it finally meets \(2\nu_q=7.0\) resonance line and beam disappears rapidly. The tune shift of coherent quadrupole mode oscillations at the beam current just before the resonance is so small that it is difficult to determine if the resonance occurs by coherent mode oscillation or incoherent one.

4 SUMMARY

First, we solved 3D envelope equations numerically, and found that the coupling between transverse and longitudinal motion due to space charge force is negligible in synchrotrons. Therefore, the mode frequencies in a coasting beam are fairly good approximations of that in a bunched one except one longitudinal mode.

Secondly, we presented our experiment in HIMAC synchrotron. We identified the coherent mode signals. The tune shift is not so good agreement with the expected value, at this moment. However, a part of the discrepancy can be attributed to uncertainty of emittance. We are carrying on the experiments to reveal the connection between tune shifts and beam loss.

5 ACKNOWLEDGEMENT

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6 REFERENCES