NEUTRINO INDUCED RADIATION AT MUON COLLIDERS *

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Abstract

Intense highly collimated neutrino beams are created from muon decays at high-energy muon colliders causing significant radiation problems even at very large distances from the collider ring. A newly developed weighted neutrino interaction generator permits detailed Monte Carlo simulations of the interactions of neutrinos (and of their progeny) to be performed using the MARS code. Dose distributions in a human tissue-equivalent phantom (TEP) are calculated when irradiated with neutrino beams (100 MeV–10 TeV). Results are obtained for a bare TEP, one embedded in several shielding materials and for a TEP located at various distances behind a shield. The distance from the collider ring (up to 60 km) at which recommended annual dose limits can be met is calculated for 0.5, 1, 2, 3 and 4 TeV muon colliders. The possibility to mitigate the problem via beam wobbling is investigated.

1 INTRODUCTION

As pointed out by King [1] neutrinos from muon decay may cause significant radiation problems at large distances from the collider ring. Dose at a given location grows with muon energy roughly as $E^3$ due to the increase with energy of the neutrino cross section, of total energy deposited, and of the collimation of the decay neutrinos—each responsible for a factor of $E$. From simple geometry, dose is expected to decline with radial distance as $R^{-2}$ and it is estimated that for a 2+2 TeV collider the Fermilab off-site annual dose limit of 10 mrem is reached only after some 34 km [2]. Detailed Monte Carlo calculations [3] confirm the great importance of this problem for high-energy muon colliders. In these studies a weighted neutrino interaction generator is developed and incorporated in the MARS [4] code. Ref. [1] uses the `equilibrium assumption' which deals with a human TEP embedded in an essentially infinite tissue-equivalent medium. By contrast ref. [5] assumes the TEP is surrounded everywhere by a vacuum. These rather sweeping assumptions bear greatly on the maximum dose encountered within the TEP. In a more realistic situation the TEP may be embedded in, e.g., soil, concrete, steel, lead—or placed in an evacuated region and then embedded into a material medium. Each of these geometries affects neutrino interaction probabilities and subsequent shower development in different ways. This is investigated here for mono-energetic neutrinos as well as for those produced by a muon collider.

2 NEUTRINO INTERACTION MODEL

The model represents energy and angle of the particles—$e^\pm$, $\mu^\pm$, and hadrons—emanating from a simulated interaction. These particles, and the showers initiated by them, are then further processed by the MARS transport algorithms in the usual way. The four types of neutrinos are distinguished throughout: $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$, $\bar{\nu}_e$. The model identifies the following types of neutrino interactions for $\nu_\mu$ $(\overline{\nu}_\mu)$ and similarly for $\nu_e$ $(\overline{\nu}_e)$: $\nu_\mu N \rightarrow \mu^+ N$, $\nu_\mu N \rightarrow \mu^- X$, $\nu_\mu p \rightarrow \mu^+ n$, $\nu_\mu p \rightarrow \mu^- p$, $\nu_\mu n \rightarrow \mu^+ N$, $\nu_\mu n \rightarrow \mu^- N$, $\nu_\mu e^- \rightarrow \nu_\mu e^-$, $\nu_\mu e^- \rightarrow \nu_\mu e^-$, $\nu_\mu A \rightarrow \nu_\mu A$. Total and differential cross sections for all these processes are taken from the literature. The corresponding sampling algorithms are developed and implemented into the MARS [4] code. For example, for the first reaction—corresponding to charged current deep inelastic neutrino interactions—total cross sections are assumed to be $6.7 \times 10^{-39} E_\nu \text{ cm}^2$ per nucleon $(E_\nu$ in GeV) for neutrinos and half of that for antineutrinos. The differential cross section is taken from [7] as

$$\frac{d\sigma}{dx dy} = \frac{G^2 x s}{2\pi} \left( Q(x) + (1-y)^2 \overline{Q}(x) \right)$$

where $x = -q^2 / 2 M$, with $q$ the momentum transfer, $M$ the nucleon mass, and $\nu$ the energy loss of the neutrino in the lab, $y = \nu / E_\nu$, $G$ is the Fermi coupling constant, $s$ is the total energy in the center of mass, and $Q(x)$, $\overline{Q}(x)$ represent the quark, antiquark momentum distributions inside the nucleon. Both $xQ(x)$ and $x\overline{Q}(x)$ are taken from experiment in numerical form. For antineutrinos the roles of $Q(x)$, $\overline{Q}(x)$ in Eq. 1 are interchanged. Once the direction and momentum of the lepton is decided in the Monte Carlo, its center-of-mass momentum is balanced by a single pion which is then forced to undergo a deep inelastic interaction in the target nucleus. The latter approximates particle production associated with deep inelastic neutrino events.

3 NEUTRINOS ON PHANTOM

A neutrino induced dose delivered to a person—represented in this study by a 30 cm thick TEP—depends strongly on whether any material is present upstream of the TEP and on the composition and location of that material. The minimal dose results from cascades developed by particles produced in $\nu$-interactions within the TEP itself. Any material immediately upstream the TEP would only amplify the maximum dose which in all cases occurs at the TEP exit plane. Dose reduction due to removal of the $\nu$ by interactions or scattering is completely negligible—even after hundreds of kilometers of soil. Fig. 1 shows maximum dose in a TEP for a
νµ broad beam as a function of energy for for a bare TEP suspended in vacuum and for the equilibrium case in comparison with [5]. Instead providing shielding, the presence of soil upstream enhances the dose by a factor of \(\sim 1000\) in the TeV region compared to the bare TEP.

Calculations show that in a bare TEP, dose for \(e^\pm\) is very close to that for \(ν\), and is about a factor of two lower for the \(τ^\pm\) beams. Equilibrium dose is practically achieved after some 5 m of soil at all energies of interest here. Maximum dose in a TEP downstream of a thick wall grows with \(Z\): in \(pSv\) per incident neutrino it is 0.12 for water or tissue, 0.19 for soil, 0.25 for steel and 0.48 after a lead wall.

4 STRAIGHT MUON BEAM

Using a 2 TeV \(μ^+\) decay neutrino beam as a source term—a typical situation downstream of a straight section or spent muon beam—a series of calculations is performed on dose in a TEP for various shielding configurations. Fig. 2 shows that the dose remains roughly constant through the soil shielding, then drops quickly in the air downstream. Any object thick enough (e.g., a 3 m concrete wall) restores the dose back to its original level in soil after which dose decays again in air or vacuum. For a TEP embedded into soil ('equilibrium case'), the dose is mainly due \(e^\pm\), with photons and charged hadrons contributing noticeably. In the air downstream, the dose is first determined by \(e^\pm\), then by photons and—after several hundred meters—by muons.

5 RADIATION AROUND RING

The magnet and beam parameters [2] for both low- and high-energy muon colliders, assumed embedded into Fermilab type soil, are implemented into MARS [3]. For a strictly planar orbit \(ν\)-spreading is exclusively due to the transverse momentum acquired at decay. For 2 TeV muons, the fraction of \(ν\)-energy—or dose—contained within 10 \(µrad\) spreads only over 1 cm after traversing 1 km and dose decreases rather slowly with the radial distance in the orbit plane (Figs. 3-4). The DOE off-site annual dose limit of 100 mrem (=1 mSv) and the Fermilab recommended limit of 10 mrem are reached at radial distances shown in Table 1. Assuming a spherical earth, this radial distance tells us how deep below ground the collider must be placed by equating dose limit(s) with surface dose—apart from legal considerations pertaining to dose delivered deep underground. Note that several meters of soil everywhere around the tunnel are needed in all cases to protect against hadrons and muons.
6 MITIGATION

Since the \( \nu \)-beam is highly collimated and directional—the intrinsic divergence is only 50\( \mu \)rad from 2 TeV muon decay—it was proposed to vary the direction in which the secondary \( \nu \)-beam is produced [3, 8]. The beam is already spread in a horizontal disc by the collider dipoles. A vertical wave can be introduced to distribute the radiation over a larger area with lower average dose. This vertical wave should vary in strength and phase over time so as to best dilute the dose. MARS calculations indicate that such a floating vertical wave installed in the arcs can reduce the \( \nu \)-flux by more than an order of magnitude (Fig. 5). The \( \sim 8 \) m arc dipoles can be rolled by 20 mrad to achieve the desired 200 \( \mu \)rad kick (B\( \sim 0.2 \) T in Fig. 5). To avoid the complication of skewed quadrupoles, net rolls or horizontal magnetic fields are canceled before entering quadrupoles. That is, the first dipole in a set of three is rolled 10 mrad horizontally, the next double that in the opposite direction, and the last by the same amount in the original direction to almost exactly cancel coupling, vertical dispersion, and amplitude effects. Reverse rolls and other changes can be executed from time to time to reduce dose levels in all directions.

Table 1: Radial distance, \( R \), from the ring center with center-of-mass energy, \( \sqrt{s} \), and depth, \( d \), needed to reduce neutrino-induced dose at surface to DOE (100 mrem) and Fermilab (10 mrem) annual off-site limits at \( N_D \) decays/yr.

<table>
<thead>
<tr>
<th>( \sqrt{s} ) (TeV) ( N_D \times 10^{21} )</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mrem</td>
<td>( R ) (km)</td>
<td>0.4</td>
<td>1.1</td>
<td>6.5</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>( d ) (m)</td>
<td>1</td>
<td>3.3</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>10 mrem</td>
<td>( R ) (km)</td>
<td>1.2</td>
<td>3.2</td>
<td>21</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>( d ) (m)</td>
<td>1</td>
<td>34</td>
<td>107</td>
<td>254</td>
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</table>

7 CONCLUSIONS

Neutrino induced radiation is one of the main challenges to the design and civil engineering aspects of a high-energy muon collider. The newly updated MARS provides a valuable tool to calculate the extent of the problem and address proposed mitigations. Preliminary results presented here show how dose depends strongly on muon collider energy and on the geometry between source and TEP. Beam wobbling holds promise to significantly alleviate the problem.

8 REFERENCES