AMPLIFICATION OF EXTERNAL EM-WAVE BY NONLINEAR WAKE WAVES IN COLD PLASMA

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Abstract

The interaction of external monochromatic, linearly polarized, plane electromagnetic (EM) wave with the nonlinear one-dimensional wake-wave, generated by relativistic electron bunch, moving in cold plasma, is considered. At definite conditions on parameters of plasma and electron bunch, nonlinear plasma electron density wake wave has a pronounced spikes, where density value is nearing the wave breacking limit and plasma electron velocities, being directed along the bunch velocity reach, their maximum. Presented calculations exhibit, that external EM-wave, propagating through plasma at such a conditions normal to the bunch velocity direction and with polarization along the bunch velocity can be amplified inside the spikes. EM-wave, Thomson scattered on the spikes, is also amplified. Amplification factors are obtained for the both cases at different conditions for system plasma-electron bunch - external EM - wave parameters.

It is shown that amplification factor can be larger, especialy at the resonance condition, when frequency of external field is nearing to plasma frequency at spikes.

Presented results are obtained using perturbative approach, when dimensionless externnal EM-wave amplitude is used as a small parameter; the known exact nonlinear one-dimensional solution, obtained previously for cold plasma-relativistic electron bunch system, is taken as a zero order approximation.

Considered amplification process can serve as a physical ground for research and development of powerful klystron type amplifiers for future linear colliders.

1 INTRODUCTION

The problem of interaction of external electromagnetic wave (EM - wave) with electrons, moving in plasma, and plasma wake waves, generated by moving electrons or electron bunches, have been a subject of numerous investigations (see for example[1] and referencies therein).

The goal of the presented investigation is to find out the possibility and conditions, when EM wave inside the plasma with the frequency of external EM - wave and Thomson scattered EM - wave can be amplified, as compared to the external EM - wave. It is shown, that if the nonlinear plasma wake waves spikes, generated by relativistic rigid electron bunch, are taken into account as an active media, where interaction processes take place, the significant field amplification can be achived. Wake wave spikes are occure, when nonlinear waves in plasma are nearing wave breaking limit, described for free plasma waves in[2] - [4] and at some details considered in[5] - [6] for relativistic driving bunches and underdense and overdense plasmas. In[5] - [6], in particular, the connection between wave breaking limits on plasma electron density and velocities and bunch electron parameters are obtained.

Wave breaking has followed so called "blowout" regime, introduced in[7] for PWFA process. In the frame of laser - plasma based acceleration, wave breaking regime is used for generation high current relativistic electron beams[8]. In both cases[7] - [8] the unique features of the nonlinear plasma wake waves near wave breacking limit are used for particle acceleration purposes. Results of present work indicate the possible application of plasma wake wave breaking regime for amplification EM - wave, serving as a physical ground for research and development on powerful EM - wave generators and amplifiers for future colliders and accelerators with a high acceleration gradient.

2 BASIC EQUATIONS AND OUTLINE OF THE FORMALISM

The flat rigid electron bunch, moving in cold plasma with immobile ions along z - direction with velocity v_0 in lab system, is considered. Longitudinal length of the bunch is d, transverse directions of the bunch are taken infinite and problem in zero approximation (in absence of external EM - wave) is treated as one dimensional. This approach is valid for wide enough bunches, when bunch radius $r_0 \gg \frac{c}{\omega_p}$[9], where ω_p is plasma frequency $\omega_p^2 = \frac{4\pi e^2 n_0}{m}$, n_0 is plasma electron density at equilibrium.

This condition could be replaced at some sence more adjustable one, if external constant longitudinal magnetic field H_0 is applied to the system. Then one dimensionality conditions could be $r_0 \gg r_L, \Omega_L \gg \omega_p$, where r_L and Ω_L are subsequently the Larmour radius and frequency for plasma electrons.

The considered cold plasma - relativistic electron bunch system is interacting with the external monochromatic, linearly polarized electromagnetic (EM) wave, propagating through plasma in x - direction and with electric vector directed along z - axis (p -polarization). The external EM-wave inside the plasma is described by:

$$\mathcal{E}_z = \mathcal{E}_{0z} e^{-i\omega_0 t + ik_0 x},$$
$$H_y = -\sqrt{\epsilon}\mathcal{E}_{0z} e^{-i\omega_0 t + ik_0 x},$$
$$k_0 = \sqrt{\epsilon}\frac{\omega_0}{c}, \qquad (1)$$

where ϵ is a dielectric constant for plasma:

$$\epsilon \equiv \epsilon' + i\epsilon'' = 1 - \frac{\varpi_p^2}{\omega_0} + i\frac{\varpi_p^2 \nu_{eff}}{\omega_0^3}; \qquad (2)$$

$$\sqrt{\epsilon} \equiv n + i\eta, n = \frac{1}{\sqrt{2}}(\epsilon' + (\epsilon'^2 + \epsilon''^2)^{1/2})^{1/2},$$

$$\eta = \frac{1}{\sqrt{2}}(-\epsilon' + (\epsilon'^2 + \epsilon''^2)^{1/2})^{1/2};$$

$\varpi_p^2 = \frac{4\pi e^2 n^{(0)}(z)}{m}$, ν_{eff} - effective colissions frequency of plasma electron, $n^{(0)}(z)$ - plasma electron density in wake waves in the absence of external EM-wave, $\nu_{eff} \ll \varpi_p < \omega_0$. It is assumed that plasma column has a thickness along the EM- wave propagation direction (0x-axis) equal d_x, which is smaller than coressponding skin depth in plasma.

The considered problem consists in finding out the component of EM-field with frequency ω_0 inside and outside the plasma, induced by the field (1) after interaction with plasma electrons, perturbed by rigid relativistic electron bunch, moving in plasma. The back influence of plasma wake wave on the bunch, considered in[10], as well as interaction of the bunch electrons with external EM-wave (1) are disregarded, according to the assumption that bunch is relativistic.

The considered system cold plasma-rigid relativistic electron bunch- external EM-wave is described by the hydrodinamic equation for the motion of plasma electrons in external $\mathcal{E}, H)$ and internal $(\vec{E}, \vec{\mathrm{H}})$ electromagnetic fields and Maxwell equations for the EM-fields generated by bunch and plasma electrons.

The dimensionless amplitude of the external EM-wave (1) $a = e\mathcal{E}_0/mc\omega_0$ assumed to be small $a \ll 1$ and external EM- wave considered as perturbation.

In zero order approximation $(a = 0)$ the system plasma-electron bunch in steady state regime described by known nonlinear, exact, one dimensional solution, obtained in[5],[6]. In particular the plasma electron density $n^{(0)}(\tilde{z})$ in dimensionless variables is given[6] by:

$$n'^{(0)}_{(\tilde{z})} \equiv \frac{n^{(0)}(\tilde{z})}{n_0} = \frac{\beta_0}{\beta_0 - \beta_z^{(0)}(\tilde{z})} = = \frac{\beta_0(1+\rho_z^{(0)2})^{1/2}}{\beta_0(1+\rho_z^{(0)2})^{1/2} - \rho_z^{(0)}(\tilde{z})}, \tag{3}$$

where $\beta_0 = v_0/c, \beta_z^{(0)} = \frac{v_e}{c} = \rho_z^0/(1+\rho_z^{(0)2})^{1/2}, \rho_z^{(0)} = \frac{P_{ze}}{mc}$. P_{ze} - is the plasma electron momentum in z-direction, $\tilde{z} = z - \beta_0 t$. Inside the electron bunch $\rho_z^{(0)}(\tilde{z})$ is always negative; behind the bunch in wake waves $\beta_z^{(0)}(\tilde{z})$ is periodic and could be positive. At some places, $\tilde{z} \rightarrow \tilde{z}_m$ of wake wave $0 < \beta_z^{(0)}(\tilde{z}) \rightarrow \beta_{max} < \beta_0$ tends to it's maximum value $\beta_{max} \equiv \beta_m$ and $n^{(0)}(\tilde{z} \rightarrow \tilde{z}_m)$ could be large enough, nearing wave breacking limit. The values of β_m are fixed by plasma and electron bunch parameters $n_0, n_b, d, \gamma_0 = (1-\beta_0^2)^{-1/2}$ and could be estimated, using analytic results of work[6].

Equations for first order approximation ($\sim a$) constitute a set of quasilinear equations, with coeficients known from zero order approximation and depending only on $\tilde{z}$. This last feature of the set permit to search a solution in the form

$$\rho_z^{(1)} = \rho_{z0}^{(1)}(\tilde{z})e^{-i\omega_0 t + ik_0 x}, etc., \tag{4}$$

which reduces the set of quasilinear differential equation in partial derivatives to the set of quasilinear ordinary differential equations. It is still complicated enough and in order to find an analytic solution in what follows the region of $\tilde{z}$ near to $\tilde{z}_m$ is considered.

In this region, approximately[6]

$$\beta_z^{(0)} \approx \beta_m + \frac{1}{2}\left(\frac{d^2\beta_z^{(0)}}{d\tilde{z}^2}\right)_{z_m}(\tilde{z} - \tilde{z_m})^2 = = \beta_m - \frac{n_m(n_m-1)}{2\beta_0(1+\rho_m^2)^{3/2}}(\tilde{z} - \tilde{z}_m) \tag{5}$$

$$|\tilde{z} - \tilde{z}_m| \ll \left(\frac{2\beta_0\rho_m^3}{n_m^2}\right)^{1/2} \sim \frac{\gamma_0^{3/2}}{n_m}, \tag{6}$$

where n_m, ρ_m are the plasma electron density and momenta at wake wave spike region (6) at $\tilde{z} = \tilde{z}_m$. Supposing also that quantities $1-\beta_0^2 = \frac{1}{\gamma_0^2}, 1-\beta_m^2 = \frac{1}{\gamma_m^2}$ are small enough it is possible essentialy simplify the obtained set of equations and then reduce it to one first order equation for $\rho_{z0}^{(1)}$, particular solution of which in the region (6) is not difficult to obtain.

3 ESTIMATES FOR AMPLIFICATION FACTORS

From approximate expression for $\rho_{z0}^{(1)}$, obtained by above mentioned way, it is possible to estimate the amplitudes of internal electric field $E_{x,z,0}^{(1)}$

$$E_{z0}^{(1)} \approx -2\mathcal{E}_{z0}\left(1 - \frac{n_m\omega_p}{\rho_m\omega_0}\right),$$

$$E_{x0}^{(1)} \approx -\frac{\mathcal{E}_{z0}}{2\sqrt{\epsilon}}\frac{\omega_p}{\omega_0}\times \times\left[2n_m\left(\frac{\omega_p}{\omega_0}\right)\left(1 - \frac{n_m\omega_p}{\rho_m\omega_0}\right) - -\frac{n_m\omega_p}{\rho_m\omega_0}\left(n_m\frac{\omega_p^2}{\omega_0^2} + 2\frac{n_m\omega_p}{\rho_m\omega_0} - 1\right)\right]; \tag{7}$$

When $\frac{n_\omega\omega_p}{\rho_m\omega_0} \gg 1, \frac{\omega_p}{\omega_0} \ll 1$ i.e.

$$\beta_0 - \beta_m \ll \frac{\omega_p}{\omega_0}\gamma_0^{-1}, \frac{\omega_0^2}{\omega_p^2} > n_m \gg \frac{\omega_0}{\omega_p}\gamma_0, \gamma_0 \ll \frac{\omega_0}{\omega_p} \tag{8}$$

the field amplification factors

$$K_{x,z} = |E_{x,z0}^{(1)}/\mathcal{E}_{z0}|, K_z \approx 2\frac{n_m\omega_p}{\rho_m\omega_0} \gg 1,$$

$$K_x = \frac{1}{\sqrt{\epsilon}}\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right)n_m\left(\frac{\omega_p}{\omega_0}\right)^2 < \frac{1}{\sqrt{\epsilon}}\times \times\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right), K_x \gg 1; \tag{9}$$

The plasma current density, generated by external EM-wave in considered system, in the first approximation is given by

$$j_{x0}^{(1)} = n^{(0)}\beta_{x0}^{(1)}, j_{z0}^{(1)} = n^{(0)}\beta_{z0}^{(1)} + n_0^{(1)}\beta_z^{(0)} \tag{10}$$

and it can be shown that at the condition (8)

$$j_{x0}^{(1)} \approx \frac{2i}{\sqrt{\epsilon}}\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right)^2 \mathcal{E}_{z0}, j_{z0}^{(1)} \approx -2i\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right)^2 \mathcal{E}_{z0} \tag{11}$$

and corresponding amplification factors for the intensity of EM wave Thomson scattered from the plasma wake wave spikes are:

$$K_{radx,z} = \frac{1}{W_0}\frac{dW_{x,z}}{d\omega} \sim \frac{|\mathrm{j}_{0,x,z}^{(1)}|^2}{|\mathcal{E}_{z0}|^2}, \tag{12}$$

where $\frac{dW_{x,z}}{d\Omega}$ is radiated energy flux in unit solid angle per second from unit volume of plasma wake wave spike, andW_0 is the incident energy flux of the external EM-wave on unit area of plasma wake wave spike cross section, normal to direction of EM - wave propagation, per second. From (11) and (12) it follows that amplification factors can be large enough at the conditions (8).

$$K_{radx} \sim \frac{4}{|\epsilon|}\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right)^4 \gg 1 \tag{13}$$

$$K_{radz} \sim 4\left(\frac{n_m\omega_p}{\rho_m\omega_0}\right)^4 \gg 1$$

Further increase of amplification factors K_x and K_{radx} (9, 11, 12) can be realized at resonnance condition $\omega_0 \to \varpi_p$ when $\epsilon' \to 0$. Then (see (2))

$$(\sqrt{\epsilon})^{-1} \to \left(\frac{\omega_0}{\nu_{eff}}\right)^{1/2}, \omega_0 \gg \nu_{eff} \tag{14}$$

$$|\epsilon|^{-1} \to \left(\frac{\omega_0}{\nu_{eff}}\right) \gg 1$$

and corresponding increase of K_x and K_{radx} from (9, 12) can be easily noticed. Resonance increase due to factors (14) for K_x, K_{radx} can take place independently from conditions (8).

4 CONCLUSION

The obtained analytical results demonstrate that at certain conditions on cold plasma - relativistic electron driving bunch - external EM wave parameters (8), essentially large amplifications are existed of electric field inside the plasma (9,14) as well as of intensity of Thomson scattered on plasma spikes EM-wave (13,14). Presented results could be used in research and development of powerful klystron type amplifiers and generators of high frequency EM - waves for future linear colliders.

The presented estimates have at some extent qualitative character and must be complemented by more quantative calculations, presumably by computer simulations. May be, more elaborate perturbative approach, for example, based on multiple scales method, could provide the possibility to go further in directions, outlined in the present work.

In order to find out the optimal geometry for experimental deffection of the predicted EM - wave amplification, it is necessary to consider also different directions of propagation and different polarisations of external EM - wave.

5 REFERENCES

[1] E.A. Akopian, G.G. Matevosian in Proc. of Workshop "New Developments in Particle Acceleration Techniques", v.2, p. 476, CERN 87-11, ECFA 87/110, 1987.

[2] A.I. Akhiezer, P.V. Polovin JETP **3**, 696, (1956).

[3] J.M. Dawson Phys. Rev. 113, 383 (1959).

[4] R.C. Davidson "Methods in Nonlinear Plasma Theory", ch. 3, Acad. Press, NY and London, 1972.

[5] A.Ts. Amatuni, M.R. Magomedov, E.V. Sekhpossian, S.S. Elbakian Physica Plasmi **5**, 85, (1979).

[6] A.Ts. Amatuni, S.S. Elbakian, E.V. Sekhpossian, R.O. Abramian Part. Acc. **41**, 153, (1993).

[7] J.B. Rozenzweig, B. Breizman, T. Katzouleas, J.J. Su Phys. Rev. A. **47**, R6189, (1991).

[8] S. Bulanov, F. Pegoraro, J. Sakai NIM Sec. A, A**410**, 477, (1998).

[9] W.B. Mori, T. Katzouleas Proc. EPAC-90, Nice, France, June 1990, v. I, p. 603.

[10] A.Ts. Amatuni Proc. PAC'97, p. 693, Vancouver, Canada, May, 1997.