Electromagnetic Field Vector Components
Precise Measurements in Accelerating Structures

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Abstract
Precise method for resonator electric or magnetic vector components values and their space positions measurements, based on application of photosemiconductor plate with different con®guration lighted images, formed by projections forming and measuring optical system of amplitude modulated light radiation, is presented. The optical system for 433 MHz RFQ accelerating structure is realized by means of serial producted microalignment telescopes; the method allows to descriminate the @eld axis fluctuations on micron level and provides several percents and tenths of percents precision for accelerating ef®ciency and modulation period measurements respectively.

I. INTRODUCTION
An electromagnetic @eld distribution measurements in accelerator resonant structures are usually carried out by perturbation method, the data processing gives the @eld vector modulus as averaged volumetric value on the perturbation object. The measurements accuracy is limited in principle by the object carrying system distortions and unperturbed resonant frequency reading inadequacy. Operative frequency rise as well as applied @elds complication will cause inadmissible growth of these inaccuracies. Electro-optical-principle is proposed to exclude these errors and to realize different vector components measurements of electric or magnetic @elds distributions, the method and system development for RFQ structure is considered.

II. ELECTRO-OPTICAL PRINCIPLE
Perturbation object for the method is designed as a high resistivity photosemiconductor “at plate, that can be installed inside the resonator on a thin @lament as before. Light radiation of amplitude modulated source passes through a controller of the light beam spatial position and lights up desired con®guration region on the plate surface in required position. The resonant frequency difference between the readings in unlighting amplitude modulation half-cycle and in the next lighting one will determine the @eld in perturbed region. Electric @eld vector components measurements can be carried out by thin strip light con®gurations, oriented along the components; for the plate normal magnetic @eld component a closed-loop con®guration is suitable; average properties can be determined by a spot of accordant space. A normal to electric @eld lighting minimizes the dark and light readings difference (that vanish for in®nitesimal thickness of the normal), in quasistatic @eld it is equipotentials detection without quantity measurements.

Carrying system distortions are entirely excluded, because the perturbation is formed by exact straightforward light beam. The second mentioned error is excluded almost completely by the dark reading in any measurement point, because high-speed (acusto-optical, e.g.) devices allow to decrease amplitude modulation cycle up to the doubled transient duration. For all this, the precise measurement problem is reduced to implementation of correlated with the resonant mode light con®gurations.

III. RFQ MEASUREMENTS
The @eld symmetry axis can be detected by equal thin strips lighting on the round plate (II), @g.1. By (1) and (2) couple removing in OY direction the equal dark-light frequency differences for each strip can be achieved, i.e. $E_y$ components in (1) and (2) regions are equal, the axis coordinate $y_0$ is geometrical center of the couple; similar OX removing of (3),(4) gives $x_0$ coordinate. Vanes curvature is determined by the strips turning refer $O_1x$ and $O_2y$ until maximum (but a.m. equal) dark-light differences for $(1')$, $(2')$, $(3')$, $(4')$ will be obtained ± @g.1 presents symmetrical bend; for a single element distortion the geometrical centers will not form straight lines under OX displacing of (1),(2) couple, (3),(4) ± in OY direction. So, that kind positions research yields the @eld axis coordinates and symmetry distorting causes all information.

Obtainable precision analysis is conducted at known $l$ electric @eld in the bore with modulation period $l_m$, accelerating ef®ciency $\theta$, inner radius mean $r_0$:

$$
E_x = U x(\Lambda - 1/\sqrt{\Lambda})/\sqrt{\Lambda r}, \quad \Lambda = 2\theta \kappa \sin \kappa z I_1(\kappa r)/\kappa r,
$$

$$
E_y = U y(\Lambda + 1/\sqrt{\Lambda}), \quad r = \sqrt{x^2 + y^2},
$$

$$
E_z = (U/\pi)2\theta \kappa \cos \kappa z \cdot I_0(\kappa r); \quad \kappa = \pi/l_m.
$$

Spatial selectivity is determined by frequency deviations ratio of inter-unent and separated components, e.g. $l$ length, $a_x \times a_z$ cross-section strip (4) $E_x$ selectivity in (1) @eld according...
It is $\Pi_+ = (\delta f_{E_z} / \delta f_{E_{x_0}}) \leq 4 \cdot 48(a_*/2l)^3(\ln 2l/a_*)$ the cylinder circle perimeter is equal to square one. If the structure with $\theta \in [0.003; 0.5]$, $l_m \in [4; 20]$ mm, $r_0 = 3.6 \mu m$ and operating frequency $f_n = (k_n/2\pi \sqrt{\varepsilon_0 \mu_0}) = 433 MHz$ for $0.1 \times 0.1 \times 2.5 \mu m$ strip the averaged selectivity is $\Pi_+ < 2 \times 10^{-6}$, that is greatly less than instrumental resolution.

The cavity analysis in the form of coupled shortcircuited section radial waveguide sections, loaded by end capacitances, gives according to [App. (11)] the deviation

$$\left| \frac{\delta f_{E_x}}{f_n} \right| = M \left( c - 1 \right) \left( \frac{I}{r_0} \right)^2 ,$$  \hspace{1cm} (2)

$M \approx 5.5 \cdot 10^{-6}$. An error in desired equality of deviations at RF phase measurements is determined by minimal phase count discrete $dt_{m}$, and (2) result yields the strip displacement resolution $dx_0 \approx 2r_0 \pi MQ^{-1} dt_{m}$, $Q$ - quality factor. For $dt_{m} = 2 \cdot 10^{-5}, Q = 5 \cdot 10^3$ it is $dx_0 \approx 1.6 \mu m$, that cause the development of precise optical system for the images forming inside small aperture, lengthy ($L = 1445 mm$) bore without vanes lighting ± the plate excitation by reection phone is inadmissible. Autoreective image forming in convergent rays of telescopic objective can be affected by micro-alignment telescope only by placing the light source beyond the graticule on the eyepiece side.

The system @g.2 comprises (4) and (5) micro-alignment telescopes [2]. Collimated image former (1) contains transparency (2) with adjustable in independent square directions transparent region; projective telescope (4) forms the image along its datum axis with displacements possibility by optical micrometers.

![Image 2: Images forming and measuring system.](image-url)

Another telescope (5) is interconnected by pentaprisms with wedge (3) for the image dimensions measureings on the plate (1I). Alignment on the base datum targets (6)(7) sets optical axes of the former and both telescopes in coincidence; the resonator is @xed on the datum axis by (4) view nding on removable transparent targets (8);(9); and datum line of sight can be ascertainment always by (5) view nding on (7) target after (3) removing. Transparency (2) is equipped by rotary device with 300 count accuracy. The system allows to form rectangular images with variable 0.05 . . . 3 fm sides, total error of the image coordinates is $\pm (3 + 2D)$ km, $D$ - image distance, M, but the systematic error can be excluded, [2]. So, the strip position inaccuracy is 3.5 km and the @eld axis coordinates real error $\approx 4$ km. The structure @ne tuning properties $l_m$ are defned by the minimum dark-light deviation position detection under radial strip lighting refer to determined axis. According to [App.], this position corresponds to minimal integral value in (6) equation at the current
distribution (7) for (1) @eld. Direct variation yields the minimum deviation angle $\alpha$ between $Ox$ and the strip

$$\alpha = \frac{\arccos (G \theta \sin \kappa z)}{2} ; \quad G = \frac{2(kr_0)^2}{\pi} \int_0^l I_1(kx) F(x) dx ,$$  \hspace{1cm} (3)

$F(x) = k x - k l \sin k x \cdot \text{osc} k l$, and it is enough to measure $\alpha$ under the plate removing along the structure for $\alpha^{max}, \alpha^{min}$ values and $\alpha = \frac{\pi}{2}$ positions determination, because

$$\theta = (\cos 2\alpha^{min} - \cos 2\alpha^{max})/(2G)^{-1} .$$

Two counts phase method of $\alpha$ mesurement with rms error $(\delta \alpha/\varphi) = 2 \cdot 10^{-4}$ in (1) @eld gives inaccuracy $(\delta x_m/\alpha) \leq 1.7 \cdot 10^{-6}$, but the phase dependence is not symmetric refer $\alpha$ (except $\alpha = \pi/4$), and systematic inaccuracy $(\delta x_m/\alpha) \leq 4.4 \cdot 10^{-5}$. Spatial selectivity is ample as before, sensitivity decrease is $2.5 \cdot 10^{-5}$ only. The angle reading rms error of the optical system is $1.4 \cdot 10^{-4}$, that determines real precision: for longitudinal coordinate count error $\approx 5 \cdot 10^{-5}$ the modulation period inaccuracy is $(\delta t_m/l_m) \leq 7.1 \cdot 10^{-5}$, and (3) result gives accelerating efciency with $(\delta \theta/\theta) \leq 1.3 \cdot 10^{-5}$ error.

Asymmetric positioning of $R_d$ radius, $T$ thickness, $\varepsilon$ dielectric constant plate will distort investigating @eld (on @g.1:F is fastening point, $\sqrt{2}/2$ ± sag to 3 quadrant). But only transverse @eld components inside the plate are used, $l_m$ accuracy under $\alpha = \pi/4$ position determination in 1 and 3 quadrants is not changed, because $\delta x_m$ arise even to two orders due to sensitivity lowering will not exceed determinative value. The $\theta$ error electrostatic estimating for $R_d = l, \varepsilon = 10, T = 0.1 \mu m$, $b = 0.05 mm$ gives $(\delta \theta/\theta) \leq 6 \cdot 10^{-3}$, the efciency error grow up to $1.5 \cdot 10^{-2}$.

Photomolecular characteristics analysis by [App.] method yields optimized ratio of the active light-dark conductivity: for $\varepsilon \leq 10$ the ratio is 125...130, and CdS or CdSe materials suit perfectly well.

IV. CONCLUSION

Electro-optical principle and its application for RfQ structures precise measurements method have been developed. Designed optical system provides several micrometers accuracy not only for the @eld axis coordinates, but for geometrical shapes of the accelerating bore forming elements also ± autoreection method is efected by contact bore targets or master gauges fastening on the same @ament. Developed balance technique (that excludes a @eld quantity measurements) for the structure @ne tuning could be used for precise measurements in other types cavity resonators.

APPENDIX. Thin cylinder formfactor for RF nonhomogeneous fields

Use of retarded potential $\tilde{A}$ @eld operator singularity for axially symmetric $l$ length $\alpha$ radius longitudinal current $\tilde{I}(x)$ yields on its circumference in $\rho, \phi, x$ local coordinates

$$\tilde{A} = N(\rho) \cdot \tilde{I}(x) \bigg|_{\rho = \alpha},$$  \hspace{1cm} (4)

where $N(\rho) = (1/2\pi)[\ln(2l/kp\rho)] + C_i(kl) - (\sin kl/kl)$, $\gamma = 0, 577...$ - Euler’s constant, $C_i(kl) - \text{integral cosine}$. In two-component case (4) result inaccuracy will not exceed $(a/2)^2$
even for equal longitudinal and transverse fields components, that follows, e.g., from ellipsoid depolarization tensor principal values \([3]\); however, the practically used disposing along the supposed \(\theta\)eld vector will sum up decrease of inaccuracy to \(3 \ldots 4\) orders. Boundary conditions in external \(E_x^s\) \(\theta\)eld lead to
\[
\frac{\partial I}{\partial x} = -\frac{i \omega \varepsilon_0 \varphi(x)}{N(a)}; \quad \frac{\partial \varphi}{\partial x} = E_x = I(x)(i \omega \mu_0 N(a) + z_0), \tag{5}
\]
where \(z_0\) - line active resistance of the cylinder material, \(\varphi\) - scalar potential of (4) \(\theta\)eld. Now a resonator-\(m\)ode \(\{E_x, H_y, I\}\) with \(\omega_r\) resonant frequency, \(Q_r\) quality factor is excited by some source \(S\) together with the cylinder \(j\) current density, and magnetic \(h_z\), electric \(e_v\) \(\theta\)elds amplitudes equations are
\[
e_v = i \omega h_v/\omega_v; \quad \text{where } W_v = \varepsilon_0 \int_{\text{vres}} \left| E_v \right|^2 dv; \quad v_{\text{res}}, v\) - resonator and cylinder volumes. Thus, for any \(E_{\nu\nu}(x)\) function the \(I(x)\) distribution is defined at \(E_x^s\) = \(e_v E_{\nu\nu}(x)\) substituting in (5) equations:
\[
\frac{\partial^2 I(x)}{\partial x^2} + k_l^2 \left(1 - \frac{i \omega \varepsilon_0 z_0}{N(a) \omega_v} \right) I(x) = \frac{\omega^2 \varepsilon_0}{N(a) \omega_v} E_{\nu\nu}(x) h_v, \tag{7}
\]
boundary values are \(I(0) = I(l) = 0\) for distant from the cavity walls cylinder. So defined \(I(x)\) determines integral value in equation (6), forming the amplitude equation for only excitation of the resonator with new resonant frequency \(\omega_n\) and new quality factor \(Q_n\), i.e. the formfactor is defined. Eigenfunctions mean substantiate that the summary \(\theta\)eld of all other modes with gradient summand is described by cylinder own \(\theta\)eld (4), and only condition for exact measurements is excluding of other modes excitation by \(S\) source in the cylinder region.

Simplest homogeneous \(\theta\)eld analysis fork \(l \leq 0.1\) gives
\[
Q_n \approx Q_v \frac{\omega_v}{\omega_n} \left(1 + \frac{\omega^2 - \omega_n^2}{\omega_v^2 - \omega_n^2} Q_v \sqrt{\frac{2 \varepsilon_0 N(a)}{\omega_v \omega_n}} \frac{k_n l}{(k_n l)^2 + 10} \right)^{-1};
\]
quality factor decrease due to finite conductivity of a thin \((N(a) > 1)\) cylinder even with \(z_0 l = 10\) Ohm will be in 4 order only. Therefore, \(z_0 = 0\) value can be used indeed, and for any \(k l\) the method yields (in conventional Slater's form writing):
\[
\frac{\omega^2 - \omega_v^2}{\omega_n^2} = \left[ \frac{E_{\nu\nu}(x_0)}{W_v} \right]^2 \varepsilon_v v K, \tag{8}
\]
where \(x_0\) - coordinate of the \(\theta\)eld reading, \(K = K_0\) - formfactor:
\[
K_0 = \frac{\left(\frac{l}{a}\right)^2 2 \tan \frac{k_n l}{2} - k_n l}{\pi N(a) (k_n l)^3}.
\tag{9}
\]
Product \(v K_0\) for \((a/l) \ll 1, k \to 0\) coincides with the result of electrostatic analysis [3]. However, if the cylinder approaches the cavity walls (e.g. \(x = 0\) endpoint is near the wall), \(I(0) = 0\) value must be interchanged by the \(\varphi(0) = -I(0)/i \omega C\), or \(I(0) = \beta(\partial I/\partial x)|_n, C\) = the cylinder end-wall and the cavity wall capacitance, \(\beta = C_r N(a)/\varepsilon_0\):
\[
K_w = K_0 \left(1 + \beta k_n \frac{1 - 2 \tan \frac{\tau}{2} \cot \tau}{2 \tan \frac{\tau}{2} - (1 + \beta k_n \cot \tau)}\right), \tag{10}
\]
where \(\tau = k_n l\). The end contiguity \((\beta \to \infty)\) yields \(K_w \approx 4 K_0\) for \(k l \leq 0.1\) and than \(K_w\) decreases up to \(K_0\) for \(\beta \to 0\) with the cylinder moving off, (10) general relation corresponds to electrostatic model [4].

Practically interesting results of nonhomogeneous \(\theta\)eld analysis are obtained at polinomial representation, e.g. for \(E_{\nu\nu} = A_x^2 x^2 + A_x x + A_0\) the method gives
\[
K_p = K_0 \left(\frac{\eta}{4(\eta^2 + 1)} - \frac{A_1 l}{A_0} \eta - \frac{A_0 l^2}{4 A_0 (1 + \xi)}\right), \tag{11}
\]
where \(\eta = 1 + (4 s_1 - 1) \left(\frac{\xi}{1 + \xi}\right)^2; \quad \xi = \frac{A_1 l}{A_0} l, \quad \eta = \frac{A_0 l^2}{4 A_0 (1 + \xi)} - \text{normalised variability},\)
\[
s_1 = \frac{2 \tau^2 + \tau \cot \tau - 1}{\tau \cot \tau - 1}, \quad s_2 = \frac{2 \tau^2 + \tau \cot \tau - 1}{2 \tau \cot \tau - 1}, \quad s_3 = \frac{1}{2} - \frac{2 \tau}{\tau^2 + \tau \cot \tau}, \quad s_4 = 1 - \frac{4 \tau^2 - \tau \cot \tau}{\tau^2 + \tau \cot \tau},
\]
and \(K = K_p, x_0 = \frac{l}{2}\) in form (8).

References