We discuss the design principles for high current injection beam lines having a high degree of beam quality preservation. These principles are applied to designing a high current e-beam injection line delivering 10 MeV e-beams from the injector to an accelerator driving UV FELs, as proposed at CEBAF.

II. GENERAL DESIGN PRINCIPLES

A 200 MeV recirculating SRF e-beam accelerator is being designed at CEBAF for driving kW-level industrial UV FELs [1, 2]. This accelerator, like all the FEL drivers, should provide e-beams having the smallest possible emittances. This demands minimization of the emittance growth at all of the stages of beam creation, acceleration and transport. One of the most critical regions is the injection line which transports the e-beams from a CW high current 10 MeV injector [3] to the accelerator.

It is interesting to consider in general how a high current injection beam line should be designed to provide a high degree of beam quality preservation, while providing the required matching conditions into the accelerator. For a non-circulating accelerator, one can minimize the emittance growth in the bending system by minimizing the average beam size [5], and match the beam into the accelerator by adding quadrupoles after the last dipole of the beam bending line. However, this prescription does not apply to a circulating accelerator, since the quads added between the last dipole and the accelerator disturbs the lattice for high-pass accelerated beams [6]. Alternative prescriptions must be sought for the latter case.

III. EMITTANCE GROWTH

Beam emittance growth in bends has been studied in detail in [9]. The responsible mechanisms were attributed to the anomalous noninertial transverse space-charge force and the normal longitudinal space-charge force. Useful formulas have been given for estimating the emittance growth from these two mechanisms.

In some earlier particle simulations of high current beam transport through an injection line [4], we turned the transverse and axial space charge forces on and off alternatively, and identified the axial space charge force as the cause of the emittance growth. The underlying mechanism is that the momentum of each electron is modified by the axial space charge force, and the beam ends up with a residual dispersion or emittance growth.

We now derive the beam emittance growth in the bending plane defined by the transfer matrix

\[
R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ 0 & 0 & \delta \end{bmatrix}.
\]

where \( \delta = 1 \) is introduced to account for the momentum variation. Due to \( \sigma(B) = RA(B)R^T \), we have

\[
\sigma_{11}(B) = R_{11}^2\sigma_{11}(A) + 2R_{11}R_{12}\sigma_{12}(A) + 2R_{11}R_{13}\sigma_{13}(A) + R_{12}^2\sigma_{22}(A) + 2R_{12}R_{13}\sigma_{23}(A) + R_{13}^2\sigma_{33}(A),
\]

\[
\sigma_{12}(B) = R_{11}R_{21}\sigma_{11}(A) + (R_{11}R_{22} + R_{12}R_{21})\sigma_{12}(A) + (R_{11}R_{23} + R_{13}R_{21})\sigma_{13}(A) + R_{12}^2\sigma_{22}(A) + (R_{12}R_{23} + R_{13}R_{22})\sigma_{23}(A) + R_{13}^2\sigma_{33}(A),
\]

\[
\sigma_{22}(B) = R_{21}^2\sigma_{11}(A) + 2R_{21}R_{22}\sigma_{12}(A) + 2R_{21}R_{23}\sigma_{13}(A) + R_{22}^2\sigma_{22}(A) + 2R_{22}R_{23}\sigma_{23}(A) + R_{23}^2\sigma_{33}(A),
\]

\[
\sigma_{13}(B) = [R_{11}\sigma_{13}(A) + R_{12}\sigma_{23}(A) + R_{13}\sigma_{33}(A)] \delta,
\]

\[
\sigma_{23}(B) = [R_{21}\sigma_{13}(A) + R_{22}\sigma_{23}(A) + R_{23}\sigma_{33}(A)] \delta,
\]

\[
\sigma_{33}(B) = \sigma_{33}(A) \delta^2.
\]

Then the bending-plane beam emittance at the exit is...
\[ \varepsilon_x^2(B) = \varepsilon_x^2(A) + \Delta \varepsilon_x^2(B), \]

where

\[ \Delta \varepsilon_x^2(B) = [\sigma_{11}(A) \sigma_{33}(A) - \sigma_{13}^2(A)] \varepsilon_1^2 + [\sigma_{22}(A) \sigma_{33}(A) - \sigma_{23}^2(A)] \varepsilon_2^2 + 2[\sigma_{11}(A) \sigma_{23}(A) - \sigma_{12}(A) \sigma_{13}(A)] a_1 + 2[\sigma_{12}(A) \sigma_{23}(A) - \sigma_{22}(A) \sigma_{13}(A)] a_2 - 2[\sigma_{13}(A) \sigma_{23}(A) - \sigma_{12}(A) \sigma_{33}(A)] a_1 a_2, \]

where \( a_1 = R_1 R_2 - R_1 R_2 \) and \( a_2 = R_1 R_2 R_2 - R_1 R_2 \).

We assume that the bending system is achromatic and symmetric between first half and the second half, and that the deflection in the second half is in the same sense as the first. Then the transfer matrices for the first half, second half and overall system are [10],

\[
\begin{bmatrix}
-2a_{12}a_{22} & \alpha_{12} & -\alpha_{13}a_{22} & 1 & 0 & 0 \\
0 & -1 & \alpha_{12} & \delta & 0 & 0 \\
0 & 0 & \delta^2 & 0 & \alpha_{12} & \alpha_{22} \end{bmatrix}
\]

where \( \Delta \eta = \alpha_{13}a_{22}(1 - \delta) \) and \( \Delta \eta' = \alpha_{13}a_{22}(1 - \delta) \) are the residual dispersions due to momentum variation. The total emittance growth (normalized) from one end to the other of the bending system is

\[
\Delta \varepsilon_{nx} = \eta_c \sqrt{\alpha_{11}^2(0) + \alpha_{22}^2(0)} \left( \sigma_{y\beta}(1) \Delta \langle \gamma \beta \rangle - \langle \sigma_{y\beta}(1) - \sigma_{y\beta}(0) \rangle \right),
\]

where \( \eta_c \) is the dispersion at the symmetry plane, \( \Delta \langle \gamma \beta \rangle / \langle \gamma \beta \rangle \) is the relative average momentum change, and \( \sigma_{y\beta}(1) - \sigma_{y\beta}(0) \) is the absolute change in the normalized rms momentum spread from the entry plane to the symmetry plane. This rms momentum change due to the axial space charge force has been given in [9], and substitution of it into Eq. (5) yields

\[
\Delta \varepsilon_{nx} = (3I_p^2 S / 8l_p I_A \lambda^2 \beta^2) (\eta_c / f_s) \sigma_x(0),
\]

where \( I_p = Q\beta_c / l_p \) is the peak current, \( l_p \) the total bunch length for a uniform distribution or equal to \((2\pi)^{1/2} \sigma_x \) for a Gaussian distribution, \( I_A = 17 \text{ kA} \), \( S \) the half length of the system, \( f_s = -1/\alpha_{23} \) is the focal length of the first half of the system, \( \sigma_x(0) \) is the initial rms beam size in the x-plane, and we assumed that the beam is nearly parallel at the entry plane, i.e., \( \alpha_{11}^2(0) \gg \alpha_{22}^2(0) \). This equation shows the parametric dependence of the emittance growth upon various beam parameters as well as the injection line parameters.

**IV. MAGNETIC BUNCHING**

Axial bunching occurs if the bending system is non-isochronous. The bunching process is shown in Fig. 1.

![Fig. 1 Bunching process through a magnetic buncher](image)

The bunch length after a magnetic buncher is

\[
\sigma_{55}(1) = \sigma_{55}(0) (1 - R_{56} f_{56})^2 + (R_{56} \varepsilon_c)^2 / \sigma_{55}(0),
\]

where \( \varepsilon_c = \sqrt{\sigma_{55}(0) \sigma_{66}(0) - \sigma_{56}^2(0)} \) is the longitudinal emittance, and \( f_{56} = -\sigma_{55}(0) / \sigma_{56}(0) \) measures the tilt of the longitudinal phase space distribution of the beam at the entrance of the magnetic buncher. The axial matching condition is \( f_{56} = R_{56} \) [11,12] under which the bunch length is minimum. The intermediate emittance growth at the symmetry plane is related to the beam momentum spread introduced for matching \( R_{56} \) with \( f_{56} \) in the form of \( \Delta \varepsilon_{nx} = (\eta_c / f_s) \sigma_{y\beta}(0) \sigma_x(0) \), which can hardly be removed completely at the end of the magnetic buncher. Therefore, a balance must be struck between matching and minimizing the beam momentum spread to ease the emittance cancellation through the achromaticity of the system.

**V. APPLICATION**

The original injection line design for the proposed CEBAF UV FEL injector/accelerator is shown in Fig. 2.

![Fig. 2 The original 180°-bend injection line \((R_{56} = 0)\).](image)

![Fig. 3 A staircase injection line \((R_{56} = -0.282 \text{ m})\).](image)
beam emittance growth and to decouple the high-pass accelerated beams with the injected beams from that 180 degree bend. Fig. 3 is a staircase-line consisting of 7 quads and 4 parallel dipoles, and Fig. 4 is a slide-line consisting of 4 quads and 3 sector dipoles.

![Image of a slide injection line](R56 = -0.138 m).

Fig. 4 A slide injection line (R56 = -0.138 m).

Table 1 shows the performance comparisons among these three designs. The numbers in the table are the beam parameters at the end of each injection beam line.

<table>
<thead>
<tr>
<th>Beam parameters</th>
<th>180°-bend</th>
<th>staircase</th>
<th>slide</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{\text{rms}} (\pi \text{ mm mrad}) )</td>
<td>8</td>
<td>6.8</td>
<td>4.6</td>
</tr>
<tr>
<td>( \varepsilon_{\phi\text{rms}} (\pi \text{ keV-deg}) )</td>
<td>27</td>
<td>19</td>
<td>15</td>
</tr>
<tr>
<td>( 4\sigma_t ) (ps)</td>
<td>11</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

The simulations started from the cathode with a 135 pC charge/bunch, and continued to the cryounit exit, with the intermediate results saved. Then we restarted the simulation by incorporating each injection line with the preceding part. The restarted beam parameters are: transverse normalized emittance \( \varepsilon_{\text{rms}} = 4.2 \pi \text{ mm mrad} \), longitudinal emittance \( \varepsilon_{\phi\text{rms}} = 11.6 \pi \text{ keV-deg} \), and bunch length \( 4\sigma_t = 6.7 \) ps.

Fig. 5 shows the beam envelope evolution along these lines. It is seen that the beam’s betatron-oscillation becomes more and more regular, and the number of crossovers (very small beam waists) becomes smaller and smaller, from the 180°-bend-line to the slide-line. Due to its simplicity and better performance, the slide-design is chosen as the nominal injection beam line for the CEBAF FEL injector.

As is seen from Fig. 5 (c), the 4 quads before the three sector dipoles in the slide-line constitute a telescopic lens which can variably magnify the beam’s \( \beta \)-functions from the cryounit exit to the entrance of the first dipole. The 3-magnet bending system preserves the beam’s parallelism in both \( x \)- and \( y \)-plane [13, 14] while transporting the beam to the main accelerator. Betatron-function magnification and beam deflection are thus decoupled, which will greatly ease the operation.

Finally, we note that this 3-magnet bending system is non-isochronous with \( R_{56} = \rho (3\alpha - 4\sin \alpha) \), where \( \rho \) and \( \alpha \) are the bending radius and angle of each dipole. The achromaticity condition is \( L/\rho = \cot \alpha - \tan (\alpha/2) \), where \( L \) is the edge-to-edge distance between the two adjacent dipoles. The dispersion at the symmetry plane is \( \eta_s = \rho (2\cos (\alpha/2) - 1) \), and the focal length is \( f_x = \rho / 2\sin (\alpha/2) \). For small \( \alpha \)'s, we have \( \eta_s / f_x \approx \alpha \), which reduces Eq. (5) to Eq. (11) of [9].

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**VI. REFERENCES**

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