Effect of Power Supply Ripple on Emittance Growth in the Collider.

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Abstract

Power supply ripple at frequencies of 720 Hz and its harmonics is expected to affect the motion of particles in the collider. These ripple frequencies are nearly resonant with the betatron frequencies. To estimate the tolerable ripple levels, we have tracked particles through the complete nonlinear lattice for $10^4$ turns with ripple fed from 10 different power stations and including up to 7 different ripple frequencies. We present an estimate that relative ripple amplitudes must be below the $10^{-8}$ level for there to be no significant impact on the emittance over the short term.

I. INTRODUCTION

Experience at the CERN SpS and other colliders has shown that power supply ripple affects the long term dynamic aperture through modulation of the tune. A problem specific to the SSC is that betatron frequencies may be nearly resonant with ripple frequencies, due to the relatively low orbital frequency. This can lead to emittance growth through driven betatron oscillations. Since the working tune may not be known precisely until commissioning, it is important to set tolerance levels for ripple amplitudes at or near betatron frequencies. In this paper we present a preliminary estimate by tracking particles for $10^4$ turns through the complete nonlinear lattice with a reasonably realistic ripple distribution.

II. DESCRIPTION OF POWER SUPPLY RIPPLE

The 87 km long collider is powered by 10 power supply stations. One station will feed approximately 480 dipoles and 90 quadrupoles. Each string of magnets is connected to a power supply through a low pass filter. Voltage ripple in the power supply will lead to a ripple in the magnetic field in the dipoles and quads. The string of magnets can be modelled as a damped transmission line which attenuates the ripple as it propagates. The ripple field behaves as an exponentially decaying sinusoid, i.e.

$$ B_{\text{rip}}(s,t) = \sum_i \Delta B^i(\omega_i) \cos(\omega_i t - k_i s) e^{-\alpha_i|m|} \quad (1) $$

The attenuation length $1/\alpha$ is a function of the ripple frequency, decreasing for high frequencies. Assuming that pulse SCR power supplies are used as planned, the largest ripple field will be at 720 Hz. The attenuation length at 720 Hz is about 13 dipoles.

Ripple in the beam tube will be reduced by a combination of factors, i) appropriate choice of the low pass filter, ii) eddy current losses in the shielding of the beam tube and in the proposed copper liner and iii) by a damping resistor placed in parallel with the magnets. According to present estimates [1], the magnetic field ripple at 720 Hz can be reduced to 1 part in $10^7$ at 20 TeV and smaller for other ripple frequencies.

III. EFFECTS OF RIPPLE

The modulating dipole field changes the closed orbit and makes it time dependent while the ripple in the quads causes a modulation in the tune. More importantly, the changes in the fields can also cause a growth in emittance and the tune modulation can reduce the long term dynamic aperture.

Analytic calculations of both the tune shift and the orbit shift show that they oscillate with the ripple frequency but with very small amplitudes. For example, a ripple field at 720 Hz with relative amplitude $10^{-7}$ leads to a maximum orbit shift of 1 micron and a tune shift of less than $5 \times 10^{-7}$. We conclude that a distribution of ripple frequencies with maximum relative amplitude of $10^{-10}$ will have negligible impact on the closed orbit and tune.

The primary concern with ripple is emittance growth due to resonantly driven betatron oscillations arising either from dipole kicks at the betatron frequency or quadrupole kicks at twice the betatron frequency. One choice of working tunes for the collider has been (123.765, 122.791) which correspond to betatron frequencies of (808 Hz, 719 Hz) in the horizontal and vertical planes respectively. In this case, the core of the beam will have its dipole mode driven resonantly in the vertical plane by the ripple at 720 Hz and the higher order modes by the superharmonics of 720 Hz. Substantial emittance growth is expected in this case. Even though this is easily avoided by shifting the tune, some particles in the beam might still be resonantly driven if there is sufficient tune spread. In the collision mode, the major sources of tune spread are i) the beam-beam interaction, ii) multipole fields in the magnets and iii) uncorrected chromaticity. The total tune spread might be as large as 0.02 leading to a frequency spread of about 69 Hz.

In order to provide the tightest tolerance on allowable ripple levels, we study the emittance growth when the rip-
ple frequency is resonant with the betatron frequency. If the filters and damping mechanisms can be designed to meet this tolerance, then power supply ripple will not be a significant factor in the choice of tune.

IV. LINEAR LATTICE

Preliminary studies were done with a linear lattice to study the relative effects of dipoles and quads at a ripple frequency of 720 Hz. For this simplified linear lattice the betatron frequency in both planes was 743 Hz. A beam of 1000 particles with a gaussian distribution in all six dimensions was tracked through the lattice. The natural chromaticity was not corrected resulting in a large frequency spread of about 90 Hz. With relative ripple amplitudes of up to $10^{-4}$ in only the quads, there was no emittance growth over a period of 1000 turns. However there was substantial emittance growth over the same period with ripple in only the dipoles. This is expected since the quads do not cause a resonant growth at the betatron frequency but they do cause a parametric resonance at twice the frequency. The ripple distribution shows that ripple frequencies above 1 kHz are attenuated by an order of magnitude compared to the amplitude at 720 Hz. Hence we expect that dipole ripple will dominate the effects on the beam emittance.

V. NONLINEAR LATTICE

The cycle time intended for the collider at collision energy of 20 Tev is 24 hours. A variety of processes such as beam-gas scattering, intra-beam scattering, power supply ripple, and noise can lead to substantial emittance growth over this interval. Since it is not practical to track particles for this length of time through the complete non-linear lattice, simulations alone cannot provide accurate tolerances for all these effects. However, simulations can be used as a guide to understand the effects over shorter time periods. We have this purpose in mind.

The lattice used for tracking has as complete a description of all magnets as presently available. It includes the field errors in all the dipoles and quadrupoles in the arcs and straight sections, particularly those in the Interaction Regions where the $\beta$ function is very large. The linear chromaticity and coupling has also been corrected with an appropriate set of sextupoles and skew quadrupoles. The tracking routine is a modified version of "Ztrack" which uses the tracking algorithm of Teapot [2]. Ripple, with the spatial and temporal distribution given in Equation (1), was propagated from ten feedpoints. 100 particles with a gaussian distribution in all six dimensions were tracked through the lattice. No beam-beam effects were included in the simulation so the tune spread resulted from the non-linearity and residual chromaticity. Since the study with the linear lattice had shown that dipoles cause the dominant emittance growth at the ripple frequencies of interest, ripple in the quads was not included in the results reported here.

We report first the results of a study with the ripple not resonant with the betatron frequencies in a distribution of 100 particles. Two lattices were used, one with the nominal betatron frequencies (749Hz, 771Hz), the other with (981Hz, 909Hz). The frequency spread in either case was less than $15$Hz. Two runs were done at a ripple frequency of 720 Hz, one at a relative amplitude of $10^{-3}$, the other at $10^{-4}$. In both cases there was no evidence of emittance growth over $10^4$ turns.

The resonant case has been studied in more detail. The lattice was tuned to betatron frequencies (808Hz, 719Hz). Frequencies with the 7 largest amplitudes were chosen from the ripple distribution.

First, particles were tracked with only one of these frequencies at a time to determine the relative effects of each on the emittance. The rms normalized emittance for these particles was specified to be the nominal beam emittance of 1mm-mrad. The relative amplitude of ripple in each case was the same at $10^{-6}$. The average emittance growth at the end of $10^4$ turns due to each of these frequencies is shown in Table 1.

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$k$ (m$^2$)</th>
<th>$\Delta \epsilon_B / \epsilon_B$ (%)</th>
<th></th>
<th>$\Delta \epsilon_Y / \epsilon_Y$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.</td>
<td>0.002</td>
<td>925</td>
<td>0.45</td>
<td>-0.04</td>
</tr>
<tr>
<td>120.</td>
<td>0.002</td>
<td>561</td>
<td>0.68</td>
<td>0.17</td>
</tr>
<tr>
<td>240.</td>
<td>0.003</td>
<td>364</td>
<td>1.56</td>
<td>0.27</td>
</tr>
<tr>
<td>720.</td>
<td>0.005</td>
<td>175</td>
<td>58.92</td>
<td>221.57</td>
</tr>
<tr>
<td>1440.</td>
<td>0.007</td>
<td>136</td>
<td>0.00</td>
<td>2.21</td>
</tr>
<tr>
<td>2160.</td>
<td>0.009</td>
<td>105</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>2880.</td>
<td>0.010</td>
<td>91</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

As expected, the ripple at 720 Hz causes the largest emittance growth by orders of magnitude. In the next study we included all 7 frequencies with the ripple at 720 Hz having the largest amplitude and the amplitudes of the others scaled appropriately from the ripple distribution. The emittance was also increased to 5mm-mrad so that the tune spread would be larger due to the nonlinearity. Two tracking runs were done, one with relative amplitude ($\Delta B_{max}/B$) of the 720Hz ripple set to $10^{-6}$ and the other with $\Delta B_{max}/B$ equal to $10^{-7}$. Figures 1 and 2 show the growth in emittance in the two planes at these amplitudes. With fewer particles resonant at 720 Hz for these cases, the average vertical emittance growth has dropped to 63% at $\Delta B_{max}/B_0 = 10^{-6}$ compared to the 222% growth (shown in Table 1) when the beam emittance was five times smaller. Figure 2 also shows that the emittance after an initial linear increase settles down to an oscillatory behaviour about a larger equilibrium value. This is exactly the behaviour shown by a harmonic oscillator driven by a sinusoidal force whose frequency changes in time and which passes through resonance at some instant. In our case, the initially resonant particles get driven to larger amplitudes where they get detuned from resonance. In the
absence of other mechanisms driving particles which are off-resonance into resonance with the ripple frequency, no systematic growth of emittance will occur after this initial increase due to the ripple. Decreasing the ripple amplitudes by one order of magnitude to $\Delta B_{\text{max}}/B_0 = 10^{-7}$ reduces the average growth in vertical emittance to 0.8%.

The vertical emittance growth seen here is due to the residual coupling in the lattice since the ripple in the dipoles kicks the beam horizontally. We expect greater sensitivity to ripple frequencies resonant with the horizontal betatron frequency. A single ripple field with frequency 808 Hz at $\Delta B/B_0 = 10^{-7}$ with the same transverse beam size as above gives rise to an average growth of 61% in the horizontal emittance and 1% in the vertical emittance over $10^4$ turns. At $\Delta B/B_0 = 10^{-8}$, the horizontal emittance growth drops to 0.01%.

VI. NOISE TOLERANCES

The required tolerances on noise levels can be estimated using the expressions given, for example, in [3]. The growth in the normalized emittance due to noise in dipoles is given by

$$\frac{d\epsilon_N}{dt} = \pi(\beta\gamma)f_0\beta_4\langle(\delta\theta)^2\rangle$$

where $f_0$ is the revolution frequency, $\beta_4$ is the beta function at the dipole, and $\langle(\delta\theta)^2\rangle$ is the average of the square of the angular kick. We assume that the noise also attenuates with distance from each power supply, that the 10 power stations are uncorrelated and set

$$\langle(\delta\theta)^2\rangle \approx \sqrt{10}N_d(\Delta B/B)_\text{noise}$$

where $N_d \approx 30$ is the number of dipoles next to each station with significant noise and $\theta$ is the bend angle per dipole. Requiring that the emittance growth not exceed 10% over the cycle time of 1 day leads to $(\Delta B/B)_\text{noise} < 1 \times 10^{-11}$.

Noise in quadrupoles leads to an emittance growth given by

$$\frac{d\epsilon_N}{dt} = \frac{1}{2} f_0\beta_q^2(\Delta K)^2\epsilon_N$$

where $\beta_q$ is the beta function at a quadrupole and $K$ is the inverse focal length of the quadrupole. We assume that noise from each station will propagate with sufficient amplitude into 4 focusing quadrupoles. Requiring that emittance growth not exceed 10% leads to a more relaxed tolerance of $(\Delta B'/B')_\text{noise} < 4 \times 10^{-7}$, where $B'$ is the gradient field.

VII. CONCLUSIONS

Taking into account only the ripple in the dipole fields we have found that the relative amplitude of the ripple must be at or below the $10^{-8}$ level for there to be no emittance growth over $10^4$ turns, if the main ripple field is resonant with the betatron frequency. Another possible source of significant emittance growth is ripple in the Interaction Region quadrupoles, a subject now under study. Present power supply designs call for the ripple fields to be damped to the $10^{-10}$ level. With the proper choice of tune, this may be sufficient to not cause any significant emittance growth over the time scale of collider operation.

VIII. REFERENCES