Results of Calculations on the Beam Deflection due to the 1 MHz Chopper for the Kaon Factory

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Abstract

Deflection of 1 GeV/c H− beam bunches to be eliminated by the 1 MHz chopper, for the proposed Kaon factory at TRIUMF, will be provided by an electric field between a set of deflector plates [1,2]. Deflection rise time is a function of beam transit time through the deflector plates and the rise time of the stored voltage pulse. This paper presents the results of time-domain mathematical simulations to assess the relationship between the above quantities: the results of these simulations allow an accurate determination of the required rise-time of the stored voltage pulse. The representation of the deflector plates is modified so that linear displacement of the beam, as well as angular deflection, may be assessed. Simulations have also been performed to assess the attenuating effect of the deflector plates upon both angular deflection and linear displacement of the H− beam caused by voltage ripple. A measured voltage pulse is simulated as driving the deflector plates, and beam deflection is predicted.

I. Introduction

Angular deflection rise [fall] time is a function of the beam transit time through the deflectors plates and the rise [fall] time of the stored voltage pulse [3,4]. The relationship between total angular deflection rise [fall] time \( t_{\Theta_r (r/f)}[a\%-b\%] \), between \( a\% \) and \( b\% \), the rise [fall] time of a trapezoidal driving voltage \( t_{\Theta_r (r/f)}[a\%-b\%] \), between \( a\% \) and \( b\% \), and the transit time of the beam through the deflector plates \( t_{\text{beam}} \) is assumed to be of the form:

\[
l_{\Theta_r (r/f)}[a\%-b\%] = l_{\Theta_r (r/f)}[a\%-b\%] + (M \times \tau_{\text{beam}})^x
\]

Where:
- \( x \) is a power function relating the variables;
- \( M \) is a multiplier for the beam transit time through the deflector plates: the value of \( M \) is related to the definition of the rise-time \( M = \frac{t_{\text{beam}}}{t_{\Theta_r (r/f)}} \).

The results of mathematical simulations are used to determine the dependence of the power function \( x \) upon the rise [fall] time of the driving voltage waveform and the physical length (ℓ) of the deflector plates [4].

A beam particle can exit the deflector plates with a trajectory parallel to the horizontal (\( \Theta_t = 0^\circ \)) but linearly displaced from the centre-line of the plates [4]. Thus in order to track beam particles through the deflector plates it is necessary to calculate both angular deflection and linear displacement of the particles. Hence the mathematical model of the deflector plates [5] has been modified such that linear displacement of beam particles is also predicted.

As a result of parasitic inductance and capacitance associated with the 1 MHz chopper there may be voltage ripple on the storage cable [6]. The deflector plates can act to attenuate the effect of the ripple [4,7]. Simulations have been performed to assess the attenuating effect of the deflector plates as a function of the frequency of the ripple and the physical length of the plates.

II. Calculation of Power Function \( x \)

If a voltage pulse with zero rise-time is applied to centered deflectors the total angular deflection (\( \Theta_{\text{tot}} \)) experienced by a particle exiting the plates will increase linearly from zero, at a time \( \tau_m \) after application of the ideal pulse, to its idealized flat-top value (\( \Theta_{\text{tot}} \)), at a time \( \tau_m + \tau_{\text{beam}} \) after application of the pulse [4,5] where:

\[
\tau_{\text{beam}} = \frac{\ell}{\beta_c}
\]

In the ideal case the total angular deflection increases from 0 % to 100 % of \( \Theta_{\text{tot}} \) in time \( \tau_{\text{beam}} \). However, if the limits of interest of angular deflection are 5 % and 95 % of \( \Theta_{\text{tot}} \), then, in the ideal case, the 90 % angular deflection excursion occurs in a time interval (\( \Delta t \)) given by:

\[
\Delta t = M \times \tau_{\text{beam}}
\]

where \( M = 0.9 \).

Previous mathematical simulations of the 1 MHz chopper assumed that the value of \( x \) is unity [2,3,6]; thus the permitted voltage rise [fall] time \( t_{\text{pu}}(r/f) \) was calculated from:

\[
t_{\text{pu}}(r/f)[a\%-b\%] = t_{\Theta_r (r/f)}[a\%-b\%] - M \times \tau_{\text{beam}}
\]

where \( t_{\Theta_r (r/f)}[a\%-b\%] = 30 \text{ ns} \) [8,9]. However the general form of equation 4 is:

\[
t_{\text{pu}}(r/f)[a\%-b\%] = \left( t_{\Theta_r (r/f)}[a\%-b\%] - (\tau_{\text{beam}} \times M)^x \right) \frac{1}{x}
\]

Time domain PSpice simulations have been carried out to evaluate \( x \). An 80 section representation of the deflector plates [4,5] was utilized to assess the dependence of the power function \( x \) [see equation 1] upon the beam transit-time multiplier \( M \) for 5 different conditions:
- \( \ell = 4 \text{ m} \) (\( \tau_{\text{beam}} = 18.06 \text{ ns} \), \( t_{\Theta_r (r/f)}[0\%-100\%] = 20 \text{ ns} \);
- \( \ell = 3.78 \text{ m} \) (\( \tau_{\text{beam}} = 17.1 \text{ ns} \), \( t_{\Theta_r (r/f)}[0\%-100\%] = 20 \text{ ns} \);
- \( \ell = 2 \text{ m} \) (\( \tau_{\text{beam}} = 9.03 \text{ ns} \), \( t_{\Theta_r (r/f)}[0\%-100\%] = 20 \text{ ns} \);
- \( \ell = 4 \text{ m} \) (\( \tau_{\text{beam}} = 18.06 \text{ ns} \), \( t_{\Theta_r (r/f)}[0\%-100\%] = 36 \text{ ns} \);
- \( \ell = 3.78 \text{ m} \) (\( \tau_{\text{beam}} = 17.1 \text{ ns} \), \( t_{\Theta_r (r/f)}[0\%-100\%] = 6.67 \text{ ns} \).

The dependence of \( x \) upon \( M \) is shown in fig. 1 for each of the last 4 simulations: the value of \( x \) for
\( t_{\text{ns}(5\%-100\%)} = 20 \text{ ns} \), is almost identical for the cases where \( \ell = 3.78 \text{ m} \) and \( \ell = 4 \text{ m} \). The value of the power function increases with reducing beam transit-time multiplier. If the total angular deflection rise [fall] time is defined as 5 \% to 95 \% \((M=0.9)\) then the power function lies in the range 1.6 to 1.7 (fig. 1). For \( \ell = 1.6 \) \((M=0.9)\), and 4 \text{ m} \) deflector plates \((t_{\text{beam}}=18.06 \text{ ns}) \Rightarrow t_{\text{ns}(5\%-95\%)}=32.6 \text{ ns} \) (c.f. \( t_{\text{ns}(5\%-95\%)}=22.7 \text{ ns} \) if \( \ell \) is assumed to be 1.0).

\[ \begin{align*}
\text{Multiplier (M)} & \quad 0.80 & 0.85 & 0.90 & 0.95 & 1.00 \\
\text{Power Function (}x\text{)} & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
\end{align*} \]

Figure 1: Power function \((x)\) versus beam transit time multiplier \((M)\)

III. Modified Representation of Plates

A. Improved Precision

Equations given elsewhere \([10,11]\) for the electric and magnetic components of angular deflection sum the incremental deflections \((\Delta\Theta_{\text{en}(\ell n)}\) and \(\Delta\Theta_{\text{en}(\text{mn})},\) respectively), at \(N+1\) circuit nodes in the mathematical model of the deflector plates, while giving each of the incremental deflections an equal weighting function. However \(\Delta\Theta_{\text{en}(\ell n)}\) and \(\Delta\Theta_{\text{en}(\text{mn})}\) represent the incremental deflections, over a length \(\Delta\ell\) of the deflector plates, assuming that instantaneous node voltage \((V_{\text{n}(\ell n)})\) and instantaneous branch current \((I_{\text{b}(\ell n)})\) are constant over length \(\Delta\ell\) (version 1.0 of deflector plate mathematical model). But \(\Delta\Theta_{\text{en}(\ell n)}\) and \(\Delta\Theta_{\text{en}(\text{mn})}\) at the ends of the plates should ideally be calculated assuming that \(V_{\text{n}(\ell n)}\) and \(I_{\text{b}(\ell n)}\) are constant over a length \(\ell_2\); version 1.1 of the mathematical model of the deflector plates simulates the ends of the plates accordingly. Details of the revised mathematical model can be found elsewhere \([4]\).

A drawback of the modified mathematical model of the deflector plates is the increased CPU time required; the ceiling compute step which PSpice uses is proportional to the delay of the shortest transmission line represented. The shortest delay is reduced by a factor of \(\frac{3}{25}\); thus the CPU time and segment size required all increase significantly \([4]\).

\[ \begin{align*}
\text{Deflection (mmrad) / Displacement (p.u.)} & \quad 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\text{Frequency (MHz)} & 0 & 50 & 100 & 150 & 200 \\
\text{Linear Displacement (normalized to } \frac{\ell}{2} \text{, and total angular deflection upon frequency and plate length)} \\
\end{align*} \]

Figure 2: Dependence of linear displacement, normalized to \(\frac{\ell}{2}\), and total angular deflection upon frequency and plate length

B. Linear Displacement of Beam

The idealized linear displacement \((dy_{\ell(x)})\) of a beam particle, at the exit to the deflector plates, is given by \([4]\):

\[ dy_{\ell(x)} = \Theta_{\text{en}} \times \frac{\ell}{2} \]  

However a beam particle which exits the deflector plates with a trajectory parallel to the horizontal \((\Theta_i = 0^\circ)\) may be displaced from the centre line of the deflector plates \([4]\). Thus in order to determine the linear displacement of a particle at the exit of the deflector plates \((dy_{\ell(x)})\), and a distance \(\ell_d\), down-stream of the exit, the PSpice equivalent circuit of the deflector plates has been modified; details of the revised model are given elsewhere \([4]\).

The modified representation of the plates is used for the remainder of the simulations reported in this paper.

IV. Effect of Deflector Plates Upon Ripple

As a result of parasitic inductance and capacitance associated with the 1 MHz chopper there may be voltage ripple on the storage cable \([6]\); the deflector plates can act to attenuate the effects of the ripple. A frequency domain analysis of a 40 section representation of the deflector plates has been performed using version 4.6sp5 of PSpice \([12]\); the results of these simulations are shown in fig. 2. For the 1 MHz chopper any significant voltage ripple is likely to be in the frequency band up to 50 MHz \([6]\). Thus, in general, the longer the deflector plates the less is the effect of a given frequency voltage ripple upon the angular deflection and normalized linear displacement of the beam (fig. 2). For a given product of deflector plate length and frequency (e.g. 4 \text{ m} plates and 25 MHz ripple, or 2 \text{ m} plates and 50 MHz ripple) the angular deflection of the beam is a constant; similarly the normalized linear displacement of the beam, at the exit of the plates, is also
Figure 3: Measured voltage waveform and predicted total angular deflection

A constant. Thus 20 MHz voltage ripple applied to 5 m deflector plates, or the same magnitude of 33.3 MHz ripple applied to 3 m plates have the same effect upon the beam.

Version 4.05p of Probe [12] was utilized to output tables of frequency versus magnitude (in dB) and phase (in degrees) for total angular deflection and linear displacement, at the exit of the plates, for a 40 section representation of 4m plates. These tables are used as data for the frequency response extension of the controlled sources, of the Analog Behavioral Model option, for PSpice simulations: the frequency response tables are the total angular deflection and linear displacement transfer functions for the deflector plates.

A time domain simulation was then performed using the frequency response tables (1 MHz - 2000 MHz) for the deflector plates, and a piece-wise linear (PWL) approximation of a representative driving voltage was simulated [7]. The predicted angular deflection and linear displacement are virtually identical to those predicted when a discrete element representation of the plates is used: however the CPU time for the transient analysis is reduced to 6% of that required for the discrete component representation [7].

Fig. 3 shows a plot of measured voltage for the prototype 1 MHz chopper [9]: the digitized waveform was stored on a p.c. and subsequently used as time-voltage corner points for a PWL independent voltage source in a PSpice simulation. The PWL source is used as input to the frequency response table representations of 4 m deflector plates. The predicted angular deflection is also shown in fig. 3. The rise-time of the measured voltage shown in fig. 3 is 17.7 ns (5% → 95%): the predicted rise-time for the angular deflection (21.7 ns) is less than that which would be calculated using equation 5, with $x=1.6$. The deflector plates reduce the effect of the pre-pulse and ‘flat-top’ ripple upon the angular deflection (fig. 3).

V. CONCLUSION

Two quantities are used to relate beam transit time through the deflector plates (and hence the physical length of the plates), the rise [fall] time of a trapezoidal driving voltage and the rise-time of the total angular deflection: a power function and a multiplier for the beam transit time. For angular deflection rise-time defined between 5% and 95% the power function $x$ has a value of approximately $x=1.6$ for a trapezoidal driving voltage: however $x=1.6$ is a conservative ‘rule-of-thumb’ for determining required rise [fall] time for stored voltage pulses.

A frequency domain analysis of the mathematical model of the deflector plates shows that, for a given product of frequency and plate length, the effect of voltage ripple upon the beam is a constant. In addition, for voltage ripple in the frequency band up to 50 MHz, the longer the plates the less is the effect of a given frequency ripple upon the beam.

VI. REFERENCES