The Incoherent Beam-Beam Effect in Linear-on-Ring Colliders

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Abstract

Linac-on-ring schemes for high-luminosity colliders are based upon collisions between an intense positron beam in a conventional storage ring and relatively low intensity bunches of electrons from a high-repetition rate superconducting linac. The electrons are highly disrupted as they pass through the positron bunch and the electron bunch envelope suffers a longitudinal pinch that varies in form according to the chosen beam parameters. This pinch must be incorporated into any beam-beam simulation program that is used to study the stability of the positron beam. The synchrobetatron resonances due to collision point oscillations are particularly sensitive to the form of the pinch and also to any longitudinal jitter on the electron bunches. The latter effect, peculiar to linac-on-ring colliders, causes a random-walk growth of the stored positron beam.

1. INTRODUCTION

Some twenty and more years ago, Csonka, Rees and Sessler put forward various schemes for increasing the center-of-mass energy in particle collisions [1, 2]. Of particular interest was the proposal to collide the SLAC linac beam with a stored beam of electrons or positrons, although the attainable luminosities were relatively low. With the advent of superconducting linacs, the potential advantages of linac-ring colliders are now being re-examined. Ambitious new proposals aim to achieve high luminosity by colliding relatively low-intensity bunches of electrons from a high-repetition rate superconducting linac against an intense bunch of positron beam in a conventional ring [3, 4].

In these schemes, each electron bunch is discarded after interacting with one of the stored positron bunches so that the positrons see a different electron bunch on every turn around the ring. A preliminary study of the machine parameters required to achieve a luminosity of $10^{34}$ cm$^{-2}$s$^{-1}$ in a linac-ring collider at the γ(4S) resonance has been published [5]. A typical set of main parameters is reproduced in Table 1. The ratio of positron to electron bunch intensities is in the region of $10^9$ with the result that the electron beam is highly disrupted. The disruption parameter is between 300 and 600, but is only 0.6 for the positrons. In the ideal case of a perfectly uniform train of electron bunches (each equally disrupted by the circulating positrons), one can calculate an instantaneous value for the linear tune-shift parameter, $\xi$, of the positron bunch. This is typically in the range 0.02 to 0.05. The limit on $\xi$ is directly related to the maximum attainable luminosity and one of the questions studied has been whether or not this limit is beyond that allowed in ring-on-ring colliders. A degree of optimism has been voiced in this respect for three reasons: a) we are only concerned with the effects of non-linear tune shift in one ring and so coupled-beam instabilities are absent; b) in an asymmetric collider the stored beam can be chosen to have the higher energy, i.e. less easily perturbed; and c) we have, in principle, some control over the beam-beam interaction by varying the matching parameters of the linac beam.

Table 1
Typical parameters for a linac-on-ring B-factory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron energy $E_{E-}$</td>
<td>3.1 GeV</td>
</tr>
<tr>
<td>Positron energy $E_{E+}$</td>
<td>9.0 GeV</td>
</tr>
<tr>
<td>Electron beam current $I_{E-}$</td>
<td>$\sim 1$ mA</td>
</tr>
<tr>
<td>Positron beam current $I_{E+}$</td>
<td>$\sim 1$ A</td>
</tr>
<tr>
<td>Positron beam emittance $\epsilon_0$</td>
<td>$1 \times 10^{-10}$ m</td>
</tr>
<tr>
<td>Beta function at I.P. $\beta^*$</td>
<td>$1 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Beam-Beam tune-shift parameter $\xi_0$</td>
<td>0.05</td>
</tr>
<tr>
<td>Transverse damping time $\tau_x$</td>
<td>$1 \times 10^{-2}$ s</td>
</tr>
<tr>
<td>Positron revolution frequency $\tau_{REV}$</td>
<td>$3 \times 10^5$ s$^{-1}$</td>
</tr>
<tr>
<td>Initial luminosity $\mathcal{L}_0$</td>
<td>$\sim 1 \times 10^{34}$ cm$^{-2}$s$^{-1}$</td>
</tr>
</tbody>
</table>

However, two factors conspire to reduce the allowed tune shift, and with it our degree of optimism. These are the longitudinal structure in the transverse pinch of the electron bunch, and the fast fluctuations of these pinched bunches that cause uncorrelated beam-beam kicks to the positrons and raise doubts regarding the stability of the positron beam. Beam interaction and tracking studies together with numerical simulations indicate a number of constraints that must be imposed on the linac beam quality.

2. BEAM-BEAM EFFECT

The electron bunch is highly disrupted by the intense positron bunch and the electrons leave the interaction region in a spray with an opening angle of a few milliradian. This in itself is not a problem, particularly if the electron beam has the lower energy, since then the detector will be on the electron upstream side of the interaction point. More problematic is the development of longitudinal nodes in the transverse pinch. So strong is the focusing that coherent waists are formed in the electron beam envelope as seen in the rest frame of the positron bunch. These will accentuate the synchrobetatron resonances due to longitudinal collision point oscillations of the positrons and thus degrade the positron beam.

The electron pinch has been extensively studied using weak-strong and strong-strong single-pass particle tracking simulations [5-7]. An example is presented in Fig. 1. More are to be found in the CBHEF study by Heifets et al. [6]. The coherence of the nodes is reduced by the non-linearity of the beam interaction and depends strongly on the longitudinal distribution of the positrons within the bunch - particularly the distribution in the tails (or more accurately the head, since it is in this region that the electron beam is "matched" to the strong core of the bunch). In addition, external matching of the electron beam parameters have been shown to reduce the coherence in certain cases [8].

The stability of the positron beam can only be studied by a multi-turn tracking program, and ideally this should employ the strong-strong beam-beam routine at each pass. However, to date this has not been attempted. We have resorted to a weak-strong program described in Ref. [9].
While developing the tracking program it was observed that the positron stability was severely affected by jitter in the longitudinal position (i.e. the timing) of the electron bunch, and to a lesser, but by no means negligible, extent by fluctuations in the intensity, transverse position and shape of the electron bunches [9]. These fluctuations give rise to uncorrelated beam-beam kicks that result in stochastic growth of the positron beam according to the Diffusion Model outlined by S. Kheifets and others [10-12]. Recently, Y. Baconnier [13] has shown that in the thin linear-lens approximation to the beam-beam interaction, the limits to the transverse jitter (dipole error) and the intensity fluctuations (quadrupole error) of the electron beam are given by the conditions:

\[
\frac{\delta I}{I} < \frac{1}{4\pi\varepsilon_0} \left( \frac{2}{T_d} \right)^{\frac{1}{2}} \tag{1}
\]

\[
\frac{\delta x}{\sigma_x} < \frac{1}{4\pi\varepsilon_0} \left( \frac{1}{T_d} \right)^{\frac{1}{2}} \tag{2}
\]

where \(T_d = f_{rev} \tau_x\).

It should be noted that the electron pinch, and its asymmetric shape for off-axis collisions may modify the latter condition, since the rigid electron beam model is an oversimplification. This detail is being studied.

A similar simple expression can be derived for the phase-error caused by longitudinal jitter. Consider a thin linear lens of strength \(1/f\), offset longitudinally with respect to the interaction point by an amount \(\delta z\). Then, ignoring variation of \(\beta^*\), if \(\bar{X}_0, \bar{X}_0'\) are the normalized transverse phase-space coordinates of a particle, referred to the interaction point, IP, then the displacement of the lens introduces the following perturbations in \(X\) and \(X'\):

\[
\Delta X = \frac{\delta z}{f} \bar{X}_0 + \frac{\delta z^2}{\beta^* f} \bar{X}_0', \quad \Delta X' = -\frac{\delta z}{f} \bar{X}_0'.
\]

Replacing \(1/f\) by \(4\pi\varepsilon_0/\beta^*\) and dropping the term in \((\delta z/\beta)^2\), gives:

\[
\Delta X = 4\pi\varepsilon_0 \frac{\delta z}{\beta^*} \bar{X}_0, \quad \Delta X' = -4\pi\varepsilon_0 \frac{\delta z}{\beta^*} \bar{X}_0'.
\]

After re-normalization the expressions are more complex, but the magnitude of the perturbation is little changed.

So, following the arguments in Ref. [13], the rate of emittance blow-up is approximately given by:

\[
\dot{\varepsilon} = f_{rev} \varepsilon \left( 4\pi\varepsilon_0 \right)^{\frac{3}{2}} \left( \frac{\delta z}{\beta^*} \right)^2
\]

which sets the condition that:

\[
\frac{\delta z}{\beta^*} \leq 0.017
\]

In each transverse plane the growth rates from the different sources will add in quadrature and for round beams the luminosity will decrease inversely as the product \(\sigma_x \sigma_y\), until equilibrium with the radiation damping is established. Using the values given Table 1, the numerical limits for simultaneous fluctuations given by (1), (2) and (3) for our typical machine are:

\[
\frac{\delta I}{I} \leq 0.024, \quad \frac{\delta x}{\sigma_x} \leq 0.017, \quad \frac{\delta z}{\beta^*} \leq 0.017
\]

Fluctuations at this level would reduce the luminosity by about a factor two for round beams.

Transverse position fluctuations cause a simple random walk of the beam center in phase space and this leads to a linear rate of emittance growth. Intensity or timing jitter perturb the particles in proportion to their amplitudes and, in the linear approximation, the emittance grows exponentially. In the collider with non-linear beam-beam forces, synchrotron motion and the electron pinch, the initial growth rates will not far from the values given above. At large amplitudes the rates will decrease, but then the perturbation from the nonlinearities, the synchrobetatron effects and the pinch will be stronger. In addition, we can expect some general increase in beam resonance due to the large bandwidth of a typical noise spectrum. This point has also been discussed in Ref. [13].

The need for complete beam-beam simulation and tracking is evident.
amplitude gives the rate of beam emittance growth, the individual particle excursions are more relevant to studies of loss to aperture limits (hard or soft).

Fig. 3 Fluctuations in intensity timing and position - effect on groups of 100 positrons over 10^6 turns

In all of these simulations the radiation damping is turned off to allow the effects of fluctuations to show up. In the last example, the damping would contain the transverse emittance to within a factor 2 of the starting condition. The tolerances on beam jitter are extremely tight.

5. CONCLUSIONS

The simulations presented above are a small fraction of those computed, which themselves represent only a part of the study required to optimize the linac-on-ring parameters taking into account realistic models to linac fluctuations. Nevertheless, the following conclusions can be drawn:

- Although the non-linear beam force and synchrotron motion are essential to a full description of the beam diffusion, including the stochastic excitation of resonances, the initial emittance growth rates are fairly well described by the simple linear theory and the expressions (1) to (3) are good design guidelines.
- Weak-strong simulations give a good insight into the magnitude of the problem of fluctuations, but a strong strong model is needed to compute the resulting (lowered) luminosity.
- Strong-strong computation is needed to normalize the magnitude of the transverse position jitter (incoming electron beam) to the effective offset after the pinch. This is under way at CERN and at CEBAF.
- Timing jitter, giving an exponential beam growth that could be worsened by the pinch, is probably the most critical source of positron beam instability. A mitigating factor is that in the simulations the jitter is expressed as a longitudinal position offset of the electron bunch relative to the interaction point. Due to the relative motion of the two beams the tolerance on the absolute timing of the electron bunch is relaxed by a factor 2.

- The required jitter tolerances (cf. Fig. 4) will be small and maybe difficult to measure with the desired precision. Experience with the SLAC and, more appropriately, the CEBAF linacs will help in clarifying this point. The frequency spectra of any jitter must also be specified and measured. Some simulations involving empirical noise spectra have been given in Ref. [9].
- The positron ring must have strong damping (T_0 << 10^6).

ACKNOWLEDGEMENTS

The comments, advice and information given by J. Boyce and R. Rossmanith, CERN, P. Grosse-Wiesmann, CERN, and J.T. Seeman, SLAC, and the typesetting work of Anne Molat are gratefully acknowledged.

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