Abstract

Medium energy (1 to 30 GeV) accelerators are often confronted with transition crossing during acceleration. A lattice without transition is presented, which is a design for the Fermilab Main Injector. The main properties of this lattice are that the γ is an imaginary number, the maxima of the dispersion function are small, and two long-straight section with zero dispersion.

I. Introduction

Most medium energy proton accelerators exhibit transition between injection and the end of the acceleration cycles. There are many unfavorable effects which can occur during and after transition[1]. For example, the momentum spread of a bunch around transition can become so large that it exceeds the machine momentum aperture and beam loss occurs[2]. There is little or no Landau damping against microwave instability near transition. As a result, the bunch area grows due to internal space-charge force as well as external beam-pipe wake forces. Particles with different momenta cross transition at different times leading to longitudinal distortions as well. The transition gamma for a particle with momentum p is defined as 1/γ² = (dC/dp)/(dp/p), where C is the total path length of the particle around the accelerator. One way to avoid transition is to make 1/γ² less than zero; or dC/dp/p = ∑ Dᵢ δᵢ ≤ 0, where Dᵢ is the dispersion at the same location[3]. In Sect. II, a design method of such a lattice is reviewed. In Sect. III, a design for the Fermilab Main Injector is presented. We call this design an “advanced” γ lattice because it has a better compactness factor with respect to the one previously presented[4].

II. Review of the Design Method

The horizontal dispersion function is presented in a normalized Floquet’s coordinate system as ξ = D'√β = A sin φ and ξ = D'√β + α A cos φ, where D and D' are the dispersion and the slope of the dispersion function, respectively, while β, α, and φ are the Twiss parameters[6]. To provide an average negative value of the horizontal dispersion through the dipoles, most dipoles should be placed in a lattice with negative dispersion (within the third and fourth quadrants of the x-ξ space.) The basic block of the Imaginary γ lattice is presented in the normalized dispersion space in Fig. 1.
III. Lattice Properties

The design of the imaginary $\gamma_1$ lattice in this example represents a possible solution for the 150 GeV future Main Injector in Fermilab. The transition gamma is an imaginary number, $\gamma_1 = i23.77$. The lattice has to follow many geometrical constraints due to the limited site space. It also has to follow the shape of the tunnel of the existing FODO-cell-based design. There are 302 dipoles available; 8 straight sections at specific locations (two of which need to have zero dispersion), and the total length has to be exactly 3319.4186 meters.

The whole ring consists of three types of blocks. The basic block "BLR" (Fig. 2) consists of two 60° FODO cells containing 12 dipoles and a low-beta insertion containing 3 dipoles. The gradient of every quadrupole in the ring is $220 \text{ kG/m}$. The horizontal and vertical betatron functions in the FODO cell are 77.42 m and 76.78 m, respectively. The quad lengths are 1.0325 m while the dipole length is defined as 6.096 m by the prototype magnet already built. The maximum values of the horizontal and vertical betatron functions within the block (BLS) is very similar, just with the 3rd, 4th, 7th, 8th, 9th, 12th, and 13th dipoles removed.

The zero-dispersion straight section is defined within the block (BLN) with zero-dispersion straight of the imaginary $\pi$ lattice. The long-straight section block (BLS) is designed to be used for extraction and injection purposes. There are eight dipoles available; 8 straight sections at specific locations (two of which need to have zero dispersion), and the total length has to be exactly 3319.4186 meters.

The natural chromaticities of this imaginary $\gamma_1$ example are $\xi_x = -26.02$ and $\xi_y = -23.95$, of the same magnitude as $|\gamma_1|$. The horizontal and vertical tunes are 19.714 and 14.188, respectively. The chromaticity sextupoles $S_F$ are located at the FODO horizontal focusing quads where the horizontal dispersion has negative values of $-2.6$ m and $\beta_x = 77.0$ m, while the $S_D$ are at the defocusing quads where the dispersion is $-2.2$ m and $\beta_y = 75.9$ m. The integrated strength of the sextupoles, at a momentum of 150 GeV/c, to compensate for the natural chromaticities are $k_{S_F} = -0.056 \text{ m}^{-2}$ and $k_{S_D} = +0.126 \text{ m}^{-2}$, respectively. These strengths correspond to pole-tip fields of 1.74 and 3.49 kG, respectively, at 150 GeV for sextupoles of length 20 cm and aperture 5 cm in radius. There are only 44 $S_F$’s and 22 $S_D$’s, so they are rather strong and introduce quite large nonlinearities, which must be counteracted by the introduction of a family of harmonic sextupoles in order to achieve a large dynamical aperture. The latter will be studied in a separate paper [5].

The betatron functions dependence on momentum of the chromaticity compensated lattice were examined with two computer programs, SYNCH and TEVLAT. The momentum offsets were introduced in small steps within a range of $\Delta p/p = \pm 2\%$ although the estimated momentum spread in the future Main
Injector should be less than ±0.2%. With the off momentum closed orbit, the maxima of the \( \sqrt{\beta_x} \) and \( \sqrt{\beta_y} \) (which define the beam size) as plotted in Fig. 4 show very small dependence on momentum. At the maximum momentum offsets of \( \Delta p/p = ±2\% \), \( \sqrt{\beta_x} \) changes by ±0.12% while \( \sqrt{\beta_y} \) changes by +1.6/−0.9%.

![Fig. 4. A dependence of the maxima of the betatron functions on momentum (\( \sqrt{\beta_x} \) and \( \sqrt{\beta_y} \)).](image)

The momentum acceptance of this imaginary \( \gamma_1 \) lattice is very good. The dispersion function propagates through the ring equally between the positive and negative values. The transverse motion of the off momentum particles is especially advantageous, because particles with the higher/lower momenta travel through the dipole magnetic field inside/outside of the central curvature. A displacement of the off momentum particles after the dipole \( \Delta x = D\Delta p/p \) in the imaginary \( \gamma_1 \) lattice has opposite sign than in the regular FODO cell lattice. With this lattice there is no need for the dipoles with an opposite bend angle[7] to cancel out the longer/shorter path length of the higher/lower momenta particles to avoid transition. In the imaginary \( \gamma_1 \) lattice the horizontal offsets and their slopes at the dipoles were designed to provide this cancelation.

The values of the dispersion function with this method are much lower than any previous transitionless design. The betatron functions in all imaginary \( \gamma_1 \) lattices designed by this method were within the same range as the corresponding FODO cell lattices or the other imaginary \( \gamma_1 \) lattices.

V. Conclusion

We have presented a very compact imaginary-\( \gamma_1 \) lattice for the Fermilab Main Injector. The whole lattice contains only 220 quadrupoles, only ≈ 10% more than the usual FODO design. It is important to note that the desired value of the \( \gamma_1 \) can be selected easily during the design procedure because the value depends on the average value of the dispersion function through the dipoles. To avoid head-tail instability, one usually need to operate at slight negative (positive) chromaticities below (above) transition. An imaginary-\( \gamma_1 \) lattice is always below transition so that no positive-chromaticity operation is necessary. As a result, the correction sextupoles need not be as strong as those in the FODO design.

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References


[5] K.Y. Ng, D. Trbojevic, and S.Y. Lee, "Examination of the Stability of the Advanced Imaginary \( \gamma_1 \) Lattice, in these proceedings.
