LONGITUDINAL EMITTANCE IN HIGH-CURRENT LINEAR ION ACCELERATORS*

T. P. Wangler, T. S. Bhatia, G. H. Neuschafer, and M. Pabst†

ABSTRACT

The control of longitudinal emittance in an ion linear accelerator is important for minimizing both chromatic aberrations and beam halo. The root-mean-square (rms) longitudinal emittance growth can result from either the nonlinear rf focusing fields or the nonlinear space-charge fields. We will present conclusions based on numerical beam-dynamics studies for both the radio-frequency quadrupole (RFQ), and the drift-tube linac (DTL). We will discuss the scaling of longitudinal emittance produced during the adiabatic bunching in an RFQ and will show the benefits of ramped DTL accelerating field designs to maintain high longitudinal focusing strength with increasing particle energy.

INTRODUCTION

The control of rms longitudinal emittance growth in linear ion accelerators can be important for two reasons. First, longitudinal emittance growth produces a beam halo in longitudinal phase space that, because of coupling from longitudinal to transverse motion, also may result in a beam halo in transverse phase space. This can cause undesirable beam spill that may be a serious limitation for linacs with high average current. Second, for some applications, the output beam-optics requirements put limits on longitudinal emittance to minimize chromatic aberration effects. Designers of ion linacs need to know how to limit the growth of rms longitudinal emittance. However, few measurements of longitudinal emittance have appeared in the published literature, and in spite of progress in the understanding of emittance growth effects, there is insufficient guidance from analytic theory.

Nevertheless, we can use the numerical simulation codes PARMTEQ² and PARMILA³ to study emittance growth phenomena that are induced by both external and space-charge fields, under a variety of conditions. These codes make use of a particle-in-cell (PIC) approach and have been subjected to experimental tests with a good record of consistency between theory and experiment. Therefore, we can use the codes as a tool to allow a study of different emittance growth mechanisms and to determine approximate scaling laws that will aid the linac designer.

A typical configuration of a high-current, high-energy proton linac is shown in Fig. 1. The dc injector supplies a (50- to 100-keV) beam to the radio-frequency quadrupole (RFQ)⁴,⁵ linac. The RFQ adiabatically bunches the dc beam and accelerates it to an energy of about 2 MeV. In this energy range, the conventional drift-tube linac (DTL) can begin to provide strong transverse focusing and more efficient acceleration. Above about 100 MeV, a class of accelerating structures called coupled-cavity linacs (CCL) become more efficient than the DTL.

The RFQ forms the bunches from an initial dc beam and determines the longitudinal emittance through the bunching process.⁶ The bunching is done over many cells (adiabatically) as the accelerating field and synchronous phase are varied. Both nonlinear rf fields and nonlinear space-charge forces act on the beam.

We have studied the RFQ bunching using the PARMTEQ numerical simulation code with 2500 macroparticles per run in an initial 4-D waterbag distribution for the transverse coordinates and momenta (uniform filling within a 4-D hyperellipsoid). The initial transverse ellipse parameters were chosen to produce an rms match into the RFQ radial matching section, and the particles in longitudinal space were monoenergetic and spaced uniformly over 360° of phase. An earlier study has shown that the beam characteristics after bunching in the RFQ are very insensitive to the initial beam distribution. In Figs. 2a through 2c, we show how longitudinal phase-space plots at the end of the bunching section (0.244 MeV) for three different beam currents for a 200-MHz RFQ with an injection energy of 0.05 MeV and a current limit of 200 mA. These plots show a decreasing emittance as beam current is increased, in contrast to the space-charge-induced growth of emittance that is a characteristic result for transverse phase space in accelerators and beam transport systems. In Figs. 3 and 4, we show the rms longitudinal and transverse emittances versus cell number in the RFQ bunching section for the same three beam currents. The correlation between transverse emittance growth and longitudinal emittance decrease as the current increases is evident in the figures and corresponds to energy transfer from the longitudinal to the transverse planes.

Fig. 1. Block diagram of an ion-linac configuration for high beam-current applications.

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Fig. 2. Longitudinal phase-space plots at end of RFQ bunching section.

Fig. 3. The rms normalized longitudinal emittance vs RFQ cell number.

Fig. 4. The rms normalized transverse emittance vs RFQ cell number.

In an earlier work,\textsuperscript{11} the scaling of transverse rms emittance growth was reported. In this paper we present the results of a systematic study of RFQ designs with different beam currents, current limits, injection energies, and rf frequencies to determine a scaling law for rms longitudinal emittance after bunching. We find that the rms longitudinal emittance $e_z$ at the end of the bunching section can be approximately expressed as a product

$$e_z = e_z \eta,$$

where $e_z$ is equal to the product of the horizontal and vertical semiaxes of the zero-current separatrix area and is given in energy-time phase space by

$$e_z = \frac{3}{2 \sqrt{2 \pi}} \sqrt{qAVW^2(\phi \cos \phi - \sin \phi)}$$

where $q$ is the charge per particle, $V$ is the intervane voltage, $\lambda$ is the rf wavelength, $c$ is the speed of light, and $A$, $\phi$, and $W$ are the acceleration efficiency, synchronous phase in radians, and energy at the end of the bunching section. The factor $\eta$ depends on the ratio $I/I_0$, where $I$ is the captured beam current at the end of the bunching section, and $I_0$ is the longitudinal current limit, which is calculated from the uniform 3-D ellipsoid model.\textsuperscript{12} In Fig. 5 we show $\eta = e_z/e_z$ as a function of $I/I_0$ for two different RFQ linacs. The decreasing value of $\eta$ as $I/I_0$ increases is the result of increasing energy transfer to transverse motion. This curve may depend somewhat on the details of the design procedure.

**LONGITUDINAL RMS EMITTANCE GROWTH IN THE DTL**

We have generated a 200-MHz DTL for acceleration of a proton beam from 2 to 50 MeV to study longitudinal rms emittance growth for low-emittance bunched beams. Numerical simulation studies have been made with PARMILA with 2000 macroparticles per run. Two different initial, random particle distributions have been generated, both of which were rms-matched to the DTL in all three
planes using the program TRACE. The first distribution was Gaussian in 6-D space and truncated at four standard deviations. The second distribution was uniform within a 3-D ellipsoid in real space; it was slightly modified from a 3-D uniform ellipsoid in momentum space to produce elliptical boundaries in the 2-D phase-space projections. Both the axial accelerating field and the synchronous phase were chosen to be constant along the linac at $E_T = 1.6$ MV/m and $\phi = -40^\circ$. The zero-current phase advances per 2pph period in the transverse planes were held constant throughout the linac at $\sigma_0 = 60^\circ$ and for the longitudinal motion $\sigma_0 = 46^\circ$ at 2 MeV, decreasing to $13^\circ$ at 50 MeV. The phase advances were significantly depressed by space charge. The initial $u_0/o_0$ ratios were 0.19 and 0.17 for the transverse and longitudinal planes, respectively, and the beams were initially equipartitioned so that the ratio $r = 1$.

Figure 6a shows the rms longitudinal emittance versus cell number for the initial Gaussian beam at $I = 0$ and 100 mA. For $I = 0$, no longitudinal rms emittance growth is observed, which implies that the nonlinear external field has a negligible effect on the emittance. At 100 mA, a very rapid emittance growth of about a factor of 2 is observed in the first cell, which is followed by beam oscillations and a slow growth throughout the remainder of the DTL. The corresponding transverse emittance plots (Fig. 6b) show the presence of the rapid initial growth, but not the slow growth component. The beam becomes nonequipartitioned; at 50 MeV the ratio $r = 3.3$. We attribute the rapid growth to charge redistribution of the beam as the beam matches itself internally to the accelerator, and field energy is transformed to particle kinetic energy. This conclusion is supported by the following facts: (1) the observed growth rate occurs within a beam plasma period, (2) the magnitude of the observed growth in all three planes is nearly equal to that expected for a Gaussian spherical bunch, and (3) the rapid growth is not observed for the initial uniform distribution (see Fig. 7a), which has no available field energy for the rapid emittance growth. Figure 7a shows, however, that the slow growth is still observed for an initial uniform beam.

We interpret the slow growth as caused by a slow charge redistribution as the bunch length expands during acceleration. Also, we cannot exclude a contribution resulting from energy transfer from the transverse motion as the beam becomes more nonequipartitioned during the expansion. The expansion occurs because of a decrease in the longitudinal focusing force as the beam velocity increases. This effect can be seen in the longitudinal oscillation frequency $\omega_0^2 = 2nq E_T \sin(-\phi)/\gamma^4m$. When $E_T$ and $\phi$ are constant, $\omega_0$ and the external focusing force...
We believe the explanation for the slow growth is that the initial uniform beam does not generally remain uniform for electric breakdown and structure cooling. The transverse emittance for constant $\omega_{\text{sep}}$ (not shown) shows no significant growth and the resulting beam remains equipartitioned.

The conventional linac design generally keeps $E_0 T$ constant throughout most of the linac. An approximate empirical formula for the slow emittance growth rate in the constant $E_0 T$ linacs of this PARMILA study is,

$$c_{\text{r}} = \sqrt{1 + c_{\text{c}} N_{\text{ion}} / c_{\text{r}}}$$

where $c_{\text{c}} = 0.00027 \text{MV}^{-1}$, $N_{\text{ion}}$ is the cell number, and $c_{\text{r}}$ is the initial value of a space-charge parameter, given in terms of the initial rms radius $a$, bunch length $b$, and rms longitudinal emittance $\epsilon_{\text{l}}$, and $E_0 = 376.73 \text{ GV}$ as

$$c_{\text{r}} = \frac{e^2 Z_0 \lambda B^2}{20 \sqrt{2} \pi m c^2 a v_{\text{r}}}.$$  

We have checked Equation 3 against our numerical simulation results at constant $\phi = 40^\circ$ and for various values of $\lambda$, $E_0 T$, $N_{\text{ion}}$, and $c_{\text{r}}$ for prolate bunches. We have not yet determined the scaling of emittance growth with respect to $\phi$. Equation 3 is consistent with the earlier observation by Pabst and Bongardt.  

**SUMMARY**

We have studied longitudinal emittance growth in RFQ and DTL linacs. For the RFQ bunching, we find that (1) nonlinear rf and space-charge fields are both important, (2) $c_{\text{r}}$ scales as the product of zero-current separatrix area times a current-dependent factor that decreases with increasing current, and (3) the decrease of $c_{\text{r}}$ with current is correlated with transverse emittance growth. For the DTL, both rapid and slow charge-redistribution emittance growth mechanisms are observed. The rapid growth is consistent with the charge redistribution mechanism studied previously. The slow growth is caused by a gradual weakening of the longitudinal focusing force with increasing beam velocity and can be controlled if the accelerating field can be ramped to compensate.

**REFERENCES**