The sideband instability in free electron laser

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Abstract

The one-dimensional fast-time averaged Hamiltonian \( H \) for an electron passing through a constant parameter wiggler and a radiation field is described. Integration of the linearized Vlasov equation with perturbing sidebands over the unperturbed orbits found from \( H \) yields the sideband growth rates including both trapped and untrapped particles. The growth rates for the upper and lower sidebands are found absolutely symmetric, while stability is determined by the sign of \( \frac{df}{d\omega_n} \), where \( \omega \) is the wave number distribution of the asymptotic wave particle equilibrium evolving along the unperturbed motion from some given initial distribution, and \( \omega \) is the bouncing frequency in the ponderomotive well.

Introduction and Summary

The growth of parasitic modes at frequencies near the main signal frequency during high power FEL operation was theoretically predicted [1,2] in early 1980's. Since then there has been ample numerical [3,4] and experimental [5,6] evidence of sideband excitation in constant wiggler FELs. Unstable modes in variable wiggler FELs have also been observed in simulations [7,9] and recently in experiment [10]. Sidebands degrade the main signal efficiency and optical quality by channeling a considerable fraction of the power into parasitic frequencies. The performance of the mirrors in an oscillator can be harmed from the modulation of the wave envelope caused by the sidebands. Last, but not least, interaction among nearby sidebands above a certain amplitude may lead to chaotic particle motion, loss of trapping and incoherent radiation.

The above have stimulated a considerable amount of theoretical work focused on sideband growth. Simple one-dimensional configurations that are analytically tractable have been used to model the situation. Two lines of approach have been considered. The single particle picture regards the particle trajectories as functions of the initial conditions and computes the gain by ensemble averaging over initial distributions [7-9]. The alternative approach assumes some adiabatic equilibrium between the particles and the main signal and examines the stability of the perturbations induced by the sidebands, solving the kinetic equation [11,12]. Because of the equilibrium assumption the kinetic method is more appropriate for FEL operation as an amplifier. In both treatments so far, analytic results have been obtained only for particles localized near the bottom of the ponderomotive well. This implies the following limitations. The sideband spectrum becomes discrete

\[ \omega = \omega_r \pm (k_x/k_y) \omega_n(0), \quad k_x/k_y \sim 2, \]

where \( \omega_n(0) \) is the bounce frequency at the bottom of the bucket, \( k_x \) and \( k_y \) are the radiation and wiggler wave numbers respectively and \( \gamma = \sqrt{1 - \omega_n^2/c^2} \). The contribution from untrapped particles is trapped particles away from the bottom is neglected. The offset \( \pm \omega_n(0) \) is determined by the action of the perpendicular \( J_0 \), and \( \omega_n(0) \) is the constant.

Here canonical formalism is introduced by expressing the unperturbed particle orbits in terms of action angle variables. The unperturbed orbits, shown in Fig. 1(a), are the fast time averaged "synchrotron" oscillations of the electrons in the potential well formed by the combined action of the wiggler and the radiation signal. The perturbed kinetic equation is solved in action space, starting from an equilibrium extending over all trapped and untrapped electrons. We find that the normalized growth rate \( g^* = (d/dz) \ln n_{\omega_n} \) is given by

\[ g^* = \frac{1}{2} \sum \frac{\omega^2}{\omega_n^2} \frac{\omega_n^3}{\omega_s^3} \sum \left[ 1 + \frac{1}{(1 + \left| \frac{1}{\omega_n} \right|)^2} \right] \frac{d\omega_n}{d\omega} \]
Figure 1. Time averaged motion without the sidebands. (a) Plots in phase space of the unperturbed orbits $H(P,\psi)=K$. The intersections with the horizontal line $P=\text{const.}$ mark the initial conditions for each orbit. (b) The normalized bounce frequency $\omega_b$ and the first two harmonics as functions of the trapping parameter $\lambda^2$. The intersections with the horizontal line $\delta=(\omega_0-\omega)/\omega$ determine the position $J_n$ of the resonant orbits for a given $\omega_0$.

The normalized gain $g/\omega_0$ is plotted against the percentage mismatch $(\omega_0-\omega)/\omega_0$ for both upper and lower sidebands in Fig. 2. The contribution up to the third harmonic $n=3$ in Eq. (2) is included in these plots. The parameters chosen correspond to a wigglr wavelength $\lambda_w=3$ cm, $s_w=3$, main signal strength $s_w=5\times10^6$, beam energy of $11.63$ MeV ($\beta_0=0.999$, $\gamma=47.37$) and current density $j=100$ A/cm$^2$ (beam density $6.25\times10^{10}$ cm$^{-2}$). We have chosen two types of equilibrium distributions $f_0(J)$: (i) Two Gaussian $f(J)=(1/2\pi D^2)\exp[-(J/\lambda)^2]$ centered at the center of the island and of characteristic lengths $D$ equal to half the island width $D=J/2$ in Fig. 2(a) and the island width $D_0$ in Fig. 2(b). (ii) Two "step-like" distributions of the form $f(J)=(1/\pi D^2)\exp[-(J/\lambda D)^2]$ with $\lambda=1$. Selecting $\omega=(\omega_0-1)^2$ places the sharp gradient at $J=\lambda D$ and we plot the case $D=J/2$ in Fig. 2(c) and $D=J_0$ in Fig. 2(d).

The limit of a $\delta$-function distribution $f=\delta(J-J_0)$, examined elsewhere [13], yields the fastest growth but is of small practical interest, because even the case of a monoenergetic beam distribution $p_n(J)$ is described in $J$-space by a smooth distribution $f(J)$ of finite width $\Delta J$ (see Fig. 1).
Figure 2. Normalized gain for monotonic distributions centered at the bottom of the well J=0 including the first three harmonics n≤4 in Eq. (2). (a) Gaussian distribution of width D equal to half the island width \( w, D=J_s/2 \) (b) Gaussian distribution with \( D=J_s \). (c) Step-like distribution with \( D=J_s^{1/2} \) and (d) Step-like distribution with \( D=J_s^{1/2} \).

References


*Work supported by U.S. ARMY STRATEGIC DEFENSE COMMAND.
+Science Applications International Corp., McLean, VA.