Although an accelerator waveguide may be "tuned" under the conditions of intended use that procedure is quite inconvenient; usually microwave engineers tune waveguides in air at room temperature, correcting the tuning frequency for evacuated waveguide at a specified temperature and intended frequency of operation.

From Helmholtz's equation it follows that an electromagnetic wave frequency $f_0$ that propagates at the velocity of light in vacuum will propagate at the velocity of light in a medium of dielectric constant $K$ at frequency $f$ (in the same waveguide) where

$$f = f_0 / K$$  \hfill (1)

The frequency correction clearly may also be written

$$\Delta f = f - f_0 = f_0 \left(1 - \sqrt{\frac{1}{K}}\right)$$  \hfill (2)

From ref. 1 the dielectric constant of air is

$$K = 1 + 10^{-6} \frac{P_a(T + 180) \times 10^{-6}(1 + 5580/T)}{P_w/T}$$  \hfill (3)

where $P_a$ is the partial pressure of the air (about 760 mm Hg), $T$ is the room temperature (about 293 deg K) and $P_w$ is the partial pressure of the water vapor (about 14 mm Hg for 60% relative humidity, ref. 2). For example, for the figures given, the dielectric constant of the air is 1.000717.

For a change in temperature, the correction is merely geometric. The running frequency correction is given by

$$\Delta f / f_0 = \alpha \Delta T$$  \hfill (4)

where $\alpha$ is the thermal coefficient of expansion of the metal (about 17 ppm/deg C for copper).

Thus, a waveguide intended to be operated at say 1300.5 mc/s, evacuated and at 40 deg C but to be tuned at 20 deg C in air ought to be tuned at a frequency 0.466 mc/s lower for temperature compensation, or at 1300.477 mc/s.

Coupler Matching

It is, doubtless, obvious that owing to machining tolerances, sketchy cold-test modeling data, etc., that acceptably precise accelerator waveguide cannot be readily manufactured. Nevertheless, the situation is not hopeless; it is possible by perturbation methods to adjust the waveguide to an intended mode of operation. The principal difficulty in getting started is that what is needed is either a matched input coupler or a perfectly "tuned" line.

If a matched termination (load) for the periodic structure is available (Appendix 1) a match point taken with respect to the de-tuned coupler will fall approximately on the $Q$-circle of the coupler. In reality this point is the center of the impedance circle of the load, and the "Smith" center is displaced slightly more; however, the approximation becomes increasingly better as a coupler match is approached. This match point (VSWR and phase) is assumed to lie on the $Q$-circle of the coupler, which can easily be drawn, since the $Q$-circle passes through the match point, the zero of the Smith chart and its diameter is the real axis. If the match point implies that the $Q$-circle is too small (does not pass through unity), the coupling must be increased, and vice versa. If the match point lies above or below the real axis the cavity is de-tuned.

Since by Foster's Theorem (extended to lossy networks), the locus of impedance points passes through the match point in a clockwise direction with increasing frequency, the direction in which to perturb the cavity to tune it is evident. (Note that when the operating frequency is below resonance, the cavity is resonant at a higher frequency and must be lowered.) But as a check, before deforming the cavity, it may be perturbed by metallic or dielectric slugs to simulate the intended correction.

If no matched termination is available but the line is known to be good, the following method of matching couplers may be used. In any case, after tuning a section, this technique may be used to review the coupler match. Referring to Fig. 1, note that the impedance of the sliding short (viewed after mismatch #2) is always on the rim of the Smith chart (pure reactance). At mismatch #2 a constant reactance is added to that arising from the length of line to the short. This only redistributes the reactance points, which still lie on the rim of the Smith chart. Now, the attenuation of the line shrinks the impedance locus into what is called an impedance (or admittance) circle that is centered on the Smith chart. Next, after some arbitrary rotation of the impedance circle, owing to the coupling coefficient between the two waveguides. This further shrinks the impedance circle size and moves its Smith center off the unit circle. The important point in reviewing this sequence of operations is that only the attenuation of the waveguide and input coupler mismatch determine the size of the impedance circle; the output coupler was not involved. The location of the center of the circle is determined by the input coupler only.

Consider now the case of data taken with the shorting plane moved successive one-eighth guide-wavelengths with the impedance circle fitted through the four points, $P_1, P_2, P_3$, and $P_4$ plotted with respect to a de-tuned coupler reference, Fig. 2.

If the impedance circle contains the center of the Smith chart, the Smith center may be located as having a VSWR 6 ,
\[ \varepsilon = \sqrt{\frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{min}}} \cdot \frac{\varepsilon_{\text{min}}}{\varepsilon_{\text{max}}}} \]  \hspace{1cm} (5)

where \( \varepsilon_{\text{max}} \) and \( \varepsilon_{\text{min}} \) are the maximum and minimum VSWR of the impedance circle on the line from the center of Smith chart through the center of the impedance circle. If the impedance circle does not enclose the center of the Smith chart.

\[ \varepsilon = \sqrt{\varepsilon_{\text{max}} \times \varepsilon_{\text{min}}} \]  \hspace{1cm} (6)

The point thus located is on the Q-circle of the input coupler (marked SC). The Q-circle may be drawn since it passes through zero on the Smith chart, the above Smith Center and has the real axis as a diameter. In the example shown, the Q-circle indicates that the input coupler is under-coupled and is tuned high with respect to the driving frequency.

There is a method of determining the mismatch of the output coupler, which cannot be included here but is given in ref. 3. As an alternative, a practical procedure is to reverse the equipment and adjust the output coupler as described above.

**Nodal Shifting in Periodic Structures.**

When the input coupler has been matched it is convenient to proceed with adjustment of the slow wave structure. The most practical method of obtaining the data is to draw a "shorting" or de-tuning plunger through the structure plotting the input impedance (with respect to the de-tuned coupler reference) on a Smith chart.

Just as in a uniform line, if a shorting plane at mid-cavity is moved one period the phase pattern will move the same electrical distance (and in the same direction as the plunger, if there are no reflections in the line. However, the phase shift information refers to the cavity ahead of the de-tuned cavity and not to the one containing the plunger.

In a nearly ideally tuned line, image points of successive positions of the shorting plunger will, when plotted with respect to the de-tuned coupler reference, lay on the same phase reference, Fig. 3. In the event there is not the intended phase shift in a cavity, it is necessary to correct the phase shift by perturbation of the cavity.

There are, occasionally, desperate situations (arising from design mistakes, machining errors, accidents, etc.) that require 'heroic' methods to correct.

For example, when deformation of the outer wall of a waveguide will not permit sufficient perturbation to correct the phase shift over a period of the structure, it becomes necessary to bend (or move) the loading obstacle (disc). This technique, which obeys the usual perturbation rules, causes the structure quite effectively; that is, a very small movement changes the phase shift without seriously changing the periodicity. But the process has the disadvantage that the problem is transferred to the following cavity rather than being solved.

Still further, it is not unheard of to remachine the interior of a waveguide after completion otherwise. Clearly, this procedure can only 'conveniently' be done when construction errors are near one end of the waveguide.

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**References**

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**Appendix 1**

A practical method of terminating a periodic structure without reflection is the insertion of sufficient lossey material to absorb all the incident power, for example, a lossey strip of some specified DC resistance per unit length. However, the lossey material must be deposited on a substrate to support it inside the accelerator; this support is a dielectric and therefore detunes the cavity, causing a reflection. It is conveniently placed another mismatch in the next previous cavity to cancel that of the load.

A test of the success of the cancellation is whether the dielectric (without coating of lossey material) produces the same null (on a slotted line in the drive line) as when it is removed from the shorting plunger. That condition may conveniently be determined by inserting the de-tuning or shorting plunger into two adjacent cavities. The null position on the plotted line being noted as a reference. The dielectric strip (with compensating paddle) is then inserted on the head of the plunger. If the null, when the assembly is placed in the same cavities, is not identical the nature of the reactance error is apparent. Either the strip or the paddle may be corrected. When the null has the same position whether or not the paddle and strip are inserted on the plunger the compensation is acceptable.

Lossey material, such as acquadag, may then be painted on the strip. If successive trials are plotted on a Smith chart, for example, it is obvious whether or not enough lossey material is being used. (Note that more lossey material is less resistance.)

A suggested design for a load is given in Fig. 4.

**Appendix 2**

The equivalent of a sliding short in non-periodic waveguide is essential in periodic structure measurement. This consists of a "plunger" of the sort shown in Fig. 5. This device extends from
half way through a cavity, through a following cavity and half-way through a further following cavity. This "bridge" exists primarily to facilitate drawing the plunger through the periodic structure; there is no microwave need of such a thorough "shorting". Positioning grooves (in the case shown to position the plunger on radiused discs) are used to locate the plunger midway through the cavity to be de-tuned.

The effect of the de-tuning plunger on a cavity is somewhat of a surprise; it does not short the waveguide in an expected manner. In periodic structures, successive impedance points on a Smith chart move oppositely to the direction of plunger movement. This is a consequence of the de-tuning nature of periodic structures. But trials will make it evident to the experimenter that near the middle of the cavity the null position moves slowly on the Smith chart as the plunger moves uniformly. That is, the plunger position is not critical near mid-cavity, so this position is useful to establish periodic shorts.

Fig. 1 Arrangement of apparatus and idealization of accelerator section with couplers.

Fig. 2 Impedance and Q-Circles on Smith Chart

Fig. 3 Nodal Shift pattern when the shorting plunger is placed in successive cavities and impedance is plotted with respect to the de-tuned coupler reference for the $2\pi/3$ mode.

Fig. 4 Load termination for $2\pi/3$-mode at 10.5cm ($V_0/C = 1$) DC resistance approximately 25,000 ohms.

Fig. 5 Detuning Plunger. The plunger grooves rest on disc hole edges when correctly positioned. The 0.050" diam. hole is used to position a resistive termination. The 10-32 tapped hole is used for extension rods to insert the plunger into the structure. ($p$ = periodic length.)