Abstract

A particle simulation code was used to study the effect on transverse beam dynamics of nonlinearities of the focusing field in a linear accelerator transporting a multiple beam array. Nonlinear field strengths for various multiple-beam design geometries were calculated by relaxation codes for use in the simulation calculation. Nonlinearities due to asymmetry of the electrode array with respect to a single beam were found to be negligible. For misaligned beams, a dodecapole field caused significant emittance growth. This was not seen in single particle tracking calculations. Fields due to induced beam array. Nonlinear field strengths for various multiple-beam design geometries were calculated by an integration of the KV envelope equations. Nonlinearities due to asymmetry of the electrode array with respect to a single beam were found to be negligible. For off-axis beams, though for the geometry of MBE-4, this was negligible. For misaligned beams, a dodecapole field can be eliminated by proper choice of the electrode radius.

1. Introduction

An array of quadrupoles can be used to transport multiple charged particle beams through a linear accelerator. The geometry of one such array, that of the 4 beam Multiple Beam Experiment (MBE-4) presently under construction at Lawrence Berkeley Laboratory, is shown in Fig. 1. This experiment will accelerate high current heavy ion beams through an alternating gradient electrostatic focusing system. We examine here the effect of focusing field nonlinearities on the transverse dynamics of the transported beams. Such field nonlinearities result from a variety of sources of field error. Those studied here can be divided into three classes: (1) imperfections in the field of a single quadrupole, (2) nonlinearities due to the asymmetry of the complete electrode array with respect to the beam of interest, and (3) end effects of the quadrupoles.

Of particular interest is the effect of these nonlinearities on the space-charge-dominated beams needed to transport the required power for heavy ion fusion applications. Therefore beam stability has been explored using a two-dimensional Transverse Plasmaparticle-in-cell (PIC) computer simulation code, as well as an "envelope-type" code, which tracks single particles through the potential of the beam, generated (non-self-consistently) by an integration of the KV envelope equations.

2. Imperfections of the Quadrupole Field

The focusing field potential can in general be written as a sum of multipoles:

$$V(r, \theta) = V_0 \sum_{m=1}^{\infty} \left[ A_m \left( \frac{r}{d} \right)^m \cos m\theta + B_m \left( \frac{r}{d} \right)^m \sin m\theta, \right.$$  \hspace{1cm} (1)

where r and \( \theta \) are polar coordinates about an origin at the center of a focusing channel, \( d \), the aperture radius, is the distance to the surface of the nearest focusing electrodes; and \( \theta \) is measured from the x axis, which passes through the origin and the center of one of these electrodes.

We consider in this section only the field of the single quadrupole centered at the origin. Assuming that the four-fold quadrupole symmetry is maintained, we find that \( B_m = 0 \) for all \( m \), and \( A_m = 0 \) unless \( m = 4n+2 \), with \( n = 0,1,2, \ldots \). Neglecting the terms for \( n \geq 1 \), since it is feasible in practice to make the electrodes such that these high order terms are completely negligible, we find that the nonlinearity of concern is due to a dodecapole potential: \( V = V_0 A_5 (r/d)^5 \cos 2\theta \). If \( R \) is chosen to have the value \( 1.1457d = R_d \), where \( R \) is the electrode radius, then \( A_5 = 0.24 \). However it may be of interest to change this ratio for a number of reasons, e.g., to increase the aperture size, or to decrease the transverse dimensions of the machine.

The PIC code was used to determine the stability of a high current ion beam to the dodecapole field. In the code, the quadrupole focusing force was applied using a thin lens approximation. Consistent with this, the dodecapole was also applied as a momentum kick to each particle at the axial position of the center of each lens -- i.e., once per lattice half-period. The relaxation code POISSON was used to determine \( A_5 \) for a given \( R \). The simulation boundary conditions were periodic in \( x \) and \( y \), but the beam was kept far enough from the calculation boundaries that it did not interact with its periodic images. The beam was taken to be uniform in density and elliptical in shape, with radii matched to the focusing system using a KV envelope code. The velocity distribution was Maxwellian, with a uniform temperature throughout the beam. No acceleration was applied, and it was assumed that \( v/c << 1 \).

Fig. 1. Transverse cross section of the MBE-4 experiment.

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Fig. 2. y rms emittance (normalized to its initial value) vs. z, with a dodecapole force acting. Parameters are given in the text. \( a = 11 \) mm.

The simulation showed no effect of the dodecapole on the beam when the beam was centered on the axis of the transport channel. For misaligned beams, however, emittance growth occurred in both the x and y dimensions. Figure 2 demonstrates this effect. Here \( R/d = 0.573 \), giving a value of \( A_6 = 5.77 \times 10^{-2} \). The beam major radius was \( a = 0.523 \) d, with \( \sigma = 12^\circ \), \( \alpha_0 = 60^\circ \) (where \( \sigma \) and \( \alpha_0 \) are the particle phase advance per lattice period with and without the space charge force), and an initial offset of the beam centroid of \( \Delta x = a/2 \) at the center of a focusing lens. The rms emittance grows, and accompanying the growth is an oscillation of the emittance with frequency slightly greater than the coherent betatron frequency. The growth is linear for \( \sim 10 \) periods, then slows or saturates, and subsequently grows again linearly, but with reduced growth rate. Single particle calculations, using the code mentioned above, show no sign of developing stochasticity for the same nonlinearity strength.

To study the effect of the dodecapole a series of simulation runs was done for beams whose centroids were offset in the x direction. The results suggest that the rms emittance growth is roughly proportional to the emittance oscillation amplitude. The growths of emittance in x and y are comparable. As the emittance grows, the amplitude of the centroid motion damps (by \( \sim 2\% \) for the case shown in Fig. 2). The oscillation amplitude appears to be linearly proportional to \( A_6 \), where \( h \) is the amplitude of the centroid misalignment. It increases with increasing beam radius or \( \alpha_0 \), or decreasing \( \sigma \). Phase plots suggest the occurrence of an even mode of oscillation as the dominant mode (see Figs. 3,4), perhaps the \( j = 0, m = 4 \) mode. In the beam frame the dodecapole potential has an octupole component whose magnitude is proportional to \( h^2 \cos \omega_4 \), where \( \omega_4 \) is the coherent betatron frequency. This component may be driving the \( j = 0, m = 4 \) mode, whose potential (as \( r^3 \cos 4\theta \)) is of the same form, and whose frequency is close to the coherent betatron frequency for low \( a/\alpha_0 \).

The emittance growth due to this mode gives a limit on the acceptable alignment for the accelerator, given the current density one desires to transport. The effect can be reduced, of course, by setting the ratio \( R/d \) to reduce the strength of the dodecapole force. We have also found, however, that the effect of the "image force" due to induced charge on the electrodes is to suppress the emittance growth due to the dodecapole. This effect is also discussed in another paper at this conference. The ratio of \( R/d = 0.744 \) used in MBE-4 was chosen because for this geometry the simulation predicts that the image force suppresses emittance growth due to the dodecapole, and vice versa.

3. Asymmetry Nonlinearities

We next consider nonlinearities which occur due to the asymmetry of the complete electrode array with respect to a single beam. The code POISSON was used to calculate
the strength of the coefficients \( A_m \) and \( B_m \) for the potential due to the electrode array of MBE-4. The electrode array was surrounded by a grounded cylinder at radius \( R = 0.5 \) in. and \( R/d = 0.764 \), as in MBE-4, and the electrodes were at potentials of \(-V\) and \(0\). Results showed that, except for the dodecapole component discussed in Section 2, all nonlinear forces except the dipole were less than 1% of the quadrupole force, for \( r/d \). This was assumed to be negligible, and no simulation code studies were done.

The dipole term provides a force which displaces the equilibrium position of the beam. The dipole strength calculated would displace the beam by approximately 0.05 mm.

Similar calculations were done with the electrode centers at the same positions as in MBE-4, but with larger \( R \), so that \( R/d = 1.1457 \). In this case for \( r/d \) and \( m > 2 \) all nonlinear forces were less than 0.06% of the quadrupole force. The beam displacement due to the dipole force was 7 \( \mu \)m. This configuration therefore shows much less nonlinearity than the version with the smaller electrode radius and therefore larger aperture discussed above. This is due to the greater shielding of the beam by the electrodes of its focusing channel. However, neither geometry appears to present nonlinearity of sufficient strength to cause collective oscillations of the beam or significant beam misalignment.

4. End Effects

In order to study quadrupole end effects, a 3-D relaxation code was used to determine the multipole expansion of the associated nonlinearity. The fringe field was found to consist mainly of an octupole component, with a \( z \) dependence about the center of the lens which was odd in \( z \). (The force is defocusing at the entrance to a focusing lens.) This was applied in the simulation as momentum kicks of alternating sign one eighth of a lattice period before and after each lens, approximating an occupancy factor of 0.5.

The simulation showed no change in the beam for the nonlinearity strengths calculated for MBE-4, which has a half-period of 5 inches. For values of \( A_4' \) approximately twice as large, an oscillation of the rms emittance was seen, which was approximately 5% for a beam misalignment of 5 mm. For higher nonlinearity strengths, for misaligned beams, some rms emittance growth occurred in beams with low \( \sigma_0 \) along with an oscillation of the emittance. An example is shown in Fig. 5. (\( A_4' \) for this figure is 6.4 times its value for MBE-4.) The emittance growth increased with \( A_4' \) and the misalignment magnitude, and decreased with increasing \( \sigma_0 \). The rms momenta increased, while the rms beam radius remained approximately constant. As with the dodecapole nonlinearity discussed above and the image force nonlinearity discussed elsewhere, the oscillation of the rms emittance occurred at a frequency of approximately twice the coherent betatron frequency. As the rms emittance grew, the amplitude of the coherent betatron oscillation increased - by approximately 25% for the case shown in Fig. 5. This occurred for the MBE-4 design parameters also, but was of negligible magnitude for the 50 lattice period length of the experiment.

The transverse dynamics of a space-charge dominated beam in the presence of various focusing field nonlinearities. A dodecapole potential was found to cause an oscillation and growth of the rms emittance for misaligned beams and low particle phase advance, though the simulation predicts that in MBE-4 this will be suppressed by image forces. Nonlinearity due to the asymmetry of the electrode array as seen by a single beam was calculated to be of negligible magnitude. Finally, quadrupole end effects caused no significant beam changes for the parameters of the MBE-4 experiment, but appear to amplify coherent betatron motion and lead to rms emittance growth for much higher nonlinearity strengths and longer accelerators.

Summary

A particle simulation code has been used to study the transverse dynamics of a space-charge dominated beam in the presence of various focusing field nonlinearities. A dodecapole potential was found to cause an oscillation and growth of the rms emittance for misaligned beams and low particle phase advance, though the simulation predicts that in MBE-4 this will be suppressed by image forces. Nonlinearity due to the asymmetry of the electrode array as seen by a single beam was calculated to be of negligible magnitude. Finally, quadrupole end effects caused no significant beam changes for the parameters of the MBE-4 experiment, but appear to amplify coherent betatron motion and lead to rms emittance growth for much higher nonlinearity strengths and longer accelerators.

References