EFFECT OF LONG RANGE BEAM-BEAM INTERACTION ON THE STABILITY OF COHERENT DIPOLE MOTION

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The coherent effect of beam-beam forces on the stability of the motion of beams as rigid bunches in a collider is studied by means of simulation. The number of bunches per beam is taken to be large, with many bunches colliding simultaneously within each interaction region. It is also assumed that they populate the beams uniformly, and that they are equally spaced. The interaction regions are all identical and evenly spaced. The collision forces are assumed to be one-dimensional linear kicks. Results for stability limits are presented, as a function of tune, for various beam configurations and several values of the crossing angle.

Introduction

The SSC has one feature that may have an important effect on its design: the fact that the beams have several thousand bunches makes the interaction between beams significant, so that beam stability is potentially affected. In this, in turn imposes a restriction on the choice of tune, and may weaken the conclusions of tracking studies which ignore the beam-beam interaction. Basically, the effect arises from the fact that there are many bunches in a given interaction region simultaneously, so that they interact several times (with different strengths, which depend on the value of the crossing angle) before leaving. The induced transverse motion gets quickly compounded, and this has a potential effect on the stability and the choices of tune and crossing angle.

Ideally, one would want to include, in beam-beam interaction studies, the effect of the forces on each particle produced by the electromagnetic fields of the other particles within its own bunch and of those with which it collides, in addition to the forces produced by the magnets and the walls of the beam pipe. This is a formidable task from the programming point of view, and, in any case, no computer exists now nor will exist within the SSC design time scale which would be able to produce significant results from such a program.

In this note we present first results on the effect that the beam-beam interaction has on the stability of the beams. We make (so far) many simplifying assumptions, the key ingredients of many bunches per beam and multiple, simultaneous, bunch-bunch interactions within all interaction regions are kept. We present results for several beam configurations and crossing angles. While these results are preliminary, they do suggest an important effect on beam stability.

Analysis

Assumptions

We consider here only the motion of the center of charge of the bunches. In practice, this amounts to treating them as rigid, disk-like objects. In this sense the effect is "coherent" since all the particles within a bunch move together. This dipole approximation to the charge distribution may be taken as the starting point of a more realistic calculation in which higher multipoles are treated. Furthermore, we study the motion in one dimension only, so that each bunch is fully described by its transverse coordinate and its slope. The bunches are evenly spaced and populate the beams uniformly. Both beams are identical. All the interaction regions (IR's) are also identical, as are the arcs between them. Thus the superperiod of the machine equals the number of IR's.

The configuration of the machine is therefore described by the number of IR's, the number of bunches within an IR, and the number of bunches within an arc. Thus we have

\[ N_d = N_{bf}(m+m') \]  

where \( N_b \) is the number of bunches per beam, \( N_{bf} \) is the number of IR's, \( m \) is the maximum number of bunches that can fit simultaneously within an IR, and \( m' \) is the minimum number of bunches that can fit simultaneously within an arc.

The bunches are assumed to interact only within the IR's. A given bunch interacts every time it moves a distance \( L/2 \), where \( L \) is the interbunch distance. Within the IR, the bunch is drifted between interactions by a simple 2x2 drift matrix \( D(0/L) \). In the arcs, it is transported by a phase advance matrix \( T \) from the end of an IR to the beginning of the next one. The matrix \( T \) satisfies the relation

\[ D \left( \frac{m-1}{2} \right) T D \left( \frac{m-1}{2} \right) = \begin{bmatrix} \cos(\mu^*) & \beta^* \sin(\mu^*) \\ -\sin(\mu^*) & \cos(\mu^*) \end{bmatrix} \]  

where \( \mu^* \) is the tune between the centers of two neighboring IR's and \( \beta^* \) is the value of the beta function at the center of the IR.

There is one "head-on" collision at the center of the IR plus several "long-range" interactions away from the center. We assume these interactions between bunches to be kicks which can be linearized as follows: consider two opposing bunches, one from each beam, whose coordinates are \( (x_1, x'_1) \) and \( (x_2, x'_2) \). The \( x \) and \( x' \) are measured relative to the respective design trajectories. They "collide" at a point where the distance between the design trajectories is \( d \). If the distribution is Gaussian, the slopes change according to \( x'_1 \rightarrow x'_1 + \Delta x' \), and \( x'_2 \rightarrow x'_2 + -\Delta x' \), where

\[ \Delta x' = \frac{1}{f} \left[ E(d+\Delta x, x) - E(d, x) \right] \]  

\[ E(x, \alpha) = \frac{1}{2} \left[ 1 - e^{-x^2/(2\alpha^2)} \right] \]  

\[ x' = \frac{x}{\alpha} \]  

\[ \Delta \alpha = \frac{1}{\alpha} x \]
The parameter \( \alpha^* \) is the effective transverse
size of the beam at the center of the IR. The effective
"focal length" \( f \) determines the strength of the kicker.
For the on-head collision, \( f = 0 \), \( \alpha = \alpha^* \) and
\( f = f^* \). For the long-range interactions, \( \alpha = \alpha^* \)
\((1 + s^2/\beta^2)\), where \( s \) is the distance from the collision
point to the center of the IR; \( f \) is related to \( f^* \) by
\( f = f^*(\alpha^*/\alpha^*)^2 \), (see eq. (5)).

For the purposes of our simulation, we linearize
the above expressions about \( \Delta x_0 \), and assume the
following for the parameters: \( \alpha^* = 7 \mu m, \beta^* = 1 \mu m, \) \( \beta = 1.5 \mu m \). Then the expressions for the kicks
are well approximated by the following:

- **head on:** \( \Delta x' = \frac{1}{f^*} \Delta x \)

- **long range:** \( \Delta x' = - \frac{1}{f^* \alpha} \Delta x \)

where the parameter \( p = (2\alpha^*/d)^2 \). The distance \( d \)
between the design trajectories at the collision
point is determined by the beam configuration,
crossing angle and interbunch distance.

The difference in sign between the head-on and
long-range kicks arises from the shape of the force function \( E \), eq. (3): close to the origin, the slope
is positive, but it is negative out in the tail. In
the linearized form of the kicks, it is obvious that
\( \Delta x' \) is propotional to this slope.

Instead of the parameter \( f^* \), we use a dimension-
less one,

\[ \xi = \frac{\Delta x}{r_0} = \frac{B^* N r}{4 \alpha L} \]

where \( N \) is the number of particles per bunch, \( r_0 \)
is the classical radius of the particle and \( \gamma \) is the usual relativistic factor.

**Method**

In practice we start the simulation by assigning values to \( N, \xi, \beta^*, L, \alpha^* \) and the crossing angle \( \alpha \). Then we assign random values, within a certain range, to \( \Delta x \) and \( \Delta x \) for all the bunches. We go around the
ring bunch by bunch either kicking it, drifting it
within the IR's, or transporting it through the arcs;
this constitutes one step. At the end of this step,
all bunches have moved a distance \( L/2 \), and we repeat
the process until a full turn is completed. After
each turn we evaluate the maximum amplitude \( x \) for
all the bunches at the center of the IR's; if this
exceeds a certain value, we call the motion unstable.
If no instability is found after a large number of
turns, we call the motion stable. In this way we can
find a stability boundary in the \( (\xi, \mu^*) \)-plane for a
given beam configuration.

For linear kicks it is also possible to study
the stability of the beams by finding the full trans-
sfer matrix for one turn, and diagonalizing it. If
there is an eigenvalue greater than one in absolute
value, the motion is unstable; otherwise it is stable
(the simplicity of the matrix ensures that it is
not possible to have all eigenvalues less than one
in absolute value, so there can not be damping). We
have used this method as a check for one beam con-
figuration only, as explained below.

**Results**

We present here results for only three beam con-
figurations. The simpler one has \( N_r = 2, m = 2 \)
and \( m' = 0 \). Thus, there are 4 bunches per beam. Each bunch interacts three times within each IR: there
is one head-on collision at the center of the IR, and
two long-range interactions at either side of
the center. Figure 1 shows the stability limits for various crossing angles. We plot the maximum value
of \( \xi \) for which the beam is stable vs. the angle \( \alpha \) of the entire machine. The periodicity of the graph
is one unit of tune, so we plot only two cycles. For
50 \( \mu rad \) crossing angle the limit is given by the
curve ACE. The parameter \( p \) takes on the value 6.97x
10^{-4} for the long-range interaction closest to the
center of the IR. The region above the curve is
unstable, below it is stable. For 10 \( \mu rad \) the
corresponding curve is ABE. In this case \( p = 1.74 \times
10^{-2} \) for the long-range kick closest to the center.
As the crossing angle is increased, the right side
of the curve becomes steeper, until it becomes
vertical. All this means in our calculation is
that, in this limit, the long-range interactions
have zero strength, and only the head-on collision
is present. Thus the curve AD is also the stability
limit for the 2-bunch per beam configuration \( N_r = 2, m = 1, m' = 0 \). In this case there is a simple analytic
expression for the curve [2,3], namely,
\( \tan(\omega u/2)/4\xi \), with which our simulation agrees. The right side of the
curves (BE and CE), with negative slope, is
caused by the destabilizing effect of the attractive
long-range interactions, while the rising edge (AB,
AC, AD) is due to the repulsive head-on kicks (for
beams with oppositely charged particles the curves
are reversed, i.e., points A and E are interchanged).

Figure 2 shows the corresponding results for the
36-bunch per beam configuration \( N_r = 6, m = m' = 3 \).
In this case each bunch interacts a total of 5 times
within each IR. In this case the periodicity of the
graph is 3 units of tune, and we show only one cycle
(the tune is, again, that of the entire ring). For
50 \( \mu rad \), the stability curve is ADA'C'AC'CE'. For 10 \( \mu rad \), the attractive long-range interactions have a
stronger destabilizing effect, and the stability
curve is, in this case, ADA'B'A'D'CE. In the limit
of infinite crossing angle there is only a head-on
kick per IR, so the stability curve corresponds to the
8-bunch per beam configuration \( N_r = 6, m = 1, m' = 0 \), which is given by ADAB'AB'. In this case
there is also a simple analytic expression [11] for
the stability curve, with which our result agrees.

Figure 3 is similar to the previous one except
that it corresponds to the beam configuration \( N_r = 6, m = 5, m' = 1 \). The larger number of bunches
within the IR's have a stronger destabilizing
effect, which manifests itself in downslope curves
farther away from the vertical line. This effect
will be even more pronounced for the SSC, which is
expected to operate at \( m = 26 \).

**Remarks**

As mentioned above, we have also studied the
beam stability by finding the full transfer matrix
and diagonalizing it. If there is at least one
eigenvalue greater than unity in absolute value, the
beam is unstable; otherwise it is stable. This is
not a convenient method of programming because each
beam configuration requires a different matrix.
Besides, the matrices, of dimension \( 4N_r \times 4N_r \),
become
too large in most cases of interest. However, we have used this method to verify the simulation results for the particular beam configuration \( n_{IR} = 2, m = 3, m' = 0 \). In this case, the results of both methods agree within the precision of the computer (these results are not shown here).

We have also done another "experiment" on our simulation program: we have replaced the drifts within the IR's by appropriate phase advance matrices corresponding to the motion between collisions. In this case there is no clear distinction between arcs and IR's, so the stability pattern should reflect the configuration of a beam with many more IR's. This is indeed what we observed: the sawtooth shape of the curve remains, but the periodicity is increased. For the configuration \( n_{IR} = 2, m = 0, m' = 0 \), the pattern takes on a periodicity of 2 units in tune, while for the configuration \( n_{IR} = 6, m = 3, m' = 3 \) the periodicity becomes 36 units of tune.

**Conclusions**

Even though we have made many simplifying assumptions, our results show the effect of the long-range coherent beam-beam interaction on the stability of the beam. Generally the stop-band width is increased significantly for small crossing angle, and therefore has a potential effect on the choice of tune.

Admittedly, the parameters used here are not realistic for the SSC. For instance, the SSC is expected to operate at a value of \( q = 0.005 \), and therefore our simulation, if taken at face value, does not restrict the choice of tune significantly except near integers. We have taken into account only the dipole motion of the bunches. However, each higher multipole approximation to the motion is expected to introduce its own stop-band. The nonlinear character of the beam-beam force is not expected to change significantly the stability of the dipole motion, but it will excite higher order multipole motion. If the effect we have observed and described here persists to higher order multipoles, it may seriously restrict the working point of the SSC. We are presently extending our simulation to include these effects, and the results will be presented elsewhere in the near future.

**References**


