Radio-frequency quadrupole (RFQ) linacs are becoming widely accepted in the accelerator community. They have the remarkable capability of simultaneously bunching low-energy ion beams and accelerating them to energies at which conventional accelerators can be used, accomplishing this with high-transmission efficiencies and low-emittance growths. The electric fields, used for radial focusing, bunching, and accelerating, are determined by the geometry of the vane tips. The choice of the best vane-tip geometry depends on considerations such as the peak surface electric field, per cent of higher multipole components, and ease of machining.

We review the vane-tip geometry based on the "ideal" two-term potential function and briefly describe a method for calculating the electric field components in an RFQ cell with arbitrary vane-tip geometry. We describe five basic geometries and use the prototype RFQ design for the Fusion Materials Irradiation Test (FMIT) accelerator as an example to compare the characteristics of the various geometries.

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**Vane-Tip Geometry from Two-Term Potential Function**

As a starting point for obtaining electric fields and vane-tip geometry in RFQ linacs, we take

\[ U(r,\theta,z) = \frac{V}{2} \left( \frac{r_0}{r} \right)^2 \cos \theta + A I_0(kr) \cos k z \]  

(1)

as the time-independent portion of the two-term potential (TTP) function. In this expression, \( V \) is the intervan potential difference, and \( k = \pi/L \), where \( L = \lambda/2 \) is the length of one "cell" of the RFQ. Although the cell length and other geometrical characteristics change gradually throughout the linac, the field analysis is done as if each cell were one element in a completely periodic structure.

Let \( z = 0 \) at the beginning of a cell in which the horizontal vanes (centered at \( \theta = 0 \)) are at the minimum displacement, \( a \), from the z axis; \( z = L \) at the end of the cell where the horizontal vane-tip displacement is \( ma \), and \( m \), the modulation parameter, is \( >1 \). The boundary conditions

\[ U(a,0,0) = U(ma,0,0) = V/2 \]

are used for calculating \( A \) and \( r_0 \) from Eq. (1):

\[ A = \frac{m^2 I_0(ka) + I_0(mka)}{m^2 I_0(ka)} \]  

(2)

\[ r_0 = a(1 - A I_0(ka))^{-1/2} \]  

(3)

If \( a \) and \( m \) are specified, \( A \) and \( r_0 \) can be calculated directly. However, because focusing and acceleration depend upon \( r_0 \) and \( A \), these quantities usually are determined by beam dynamics requirements, and \( a \) and \( m \) are calculated by iterating Eqs. (2) and (3).

\[ U(r) = \int G(r;\xi) \sigma(\xi) \, d\xi \]  

(6)

where \( G(r;\xi) \) is the potential produced at point \( r \) by a unit charge located at point \( \xi \) on a vane-tip surface; \( \sigma(\xi) \, d\xi \) is the amount of charge in the infinitesimal
area \( ds \); and the integration is over all vane-tip surfaces. Any surface point, \( s \), can be derived from two independent variables, \( u \) and \( v \). We assume that \( \sigma \) can be approximated by a bicubic spline function of \( u \) and \( v \), where the locations of the knots of the spline, \( u_i \) and \( v_j \), are specified; but the values of \( \sigma \) at the knots, \( \sigma_{ij} \), are unknown and determined by minimizing

\[
\int \left( \frac{V}{2} - U(s) \right)^2 ds
\]

with respect to the unknown \( \sigma_{ij} \)'s. The resultant system of equations appears extremely well conditioned and yields very satisfactory results. This allows us to analyze the properties of many types of vane-tip geometries.

**Alternate Vane-Tip Geometries**

Five vane-tip geometry types have been analyzed using the technique described above. The results are tabulated in Ref. 4. Types 1-3 have the same longitudinal vane-tip profile as specified by Eq. (4). Type 1 has a variable transverse radius of curvature given by Eq. (5); Types 2 and 3 have transverse radii of curvature equal to \( r_0 \) and 0.75 \( r_0 \), respectively. Types 4 and 5 also have \( r_0 \) and 0.75 \( r_0 \) for their transverse radii of curvature but have sinusoidal longitudinal profiles given by

\[
x_p = r_0 \left( 1 - \frac{m - 1}{m + 1} \cos k z \right)
\]

(7)

These geometry types are summarized in Table I.

For each type, the geometry is specified completely by two parameters, \( m \) and \( L/r_0 \). Field characteristics have been calculated for each type over a two-dimensional array of \( m \) and \( L/r_0 \). An interpolation procedure is used to find a particular characteristic at any value of \( m \) and \( L/r_0 \) within the range of the arrays.

**Comparison for a Particular Example**

To compare the properties of these five geometries, it is helpful to take a particular example for an RFQ linac and to see how various characteristics depend on the geometry. The characteristics include the field enhancement factor, the minimum aperture, the modulation parameter, the intervane capacitance per unit length, and the amplitudes of the various multipole components. For the particular example, we take the prototype design for FMIT, an 80-MHz RFQ that accelerates deuterons from 0.075 to 2.0 MeV.

**Table I**

<table>
<thead>
<tr>
<th>Type</th>
<th>( x_p )</th>
<th>( r_t )</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TTPd</td>
<td>TTPd</td>
<td>□</td>
</tr>
<tr>
<td>2</td>
<td>TTPd</td>
<td>( r_0 )</td>
<td>○</td>
</tr>
<tr>
<td>3</td>
<td>TTPd</td>
<td>0.75 ( r_0 )</td>
<td>△</td>
</tr>
<tr>
<td>4</td>
<td>sinusoidal</td>
<td>( r_0 )</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>sinusoidal</td>
<td>0.75 ( r_0 )</td>
<td>x</td>
</tr>
</tbody>
</table>

\( ^{d} \)Two-term potential function.
The complete multipole expansion for the potential is

\[ U(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left( \frac{r_0}{r} \right)^{2m} \cos 2\theta \cos 2\theta n r z + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left( \frac{r_0}{r} \right)^{2m} \cos 2\theta \cos nkz \] \[ \text{(8)} \]

In the first sum, the even \( m \) coefficients are zero because of symmetry; in the second sum, the coefficients are zero when \( m \) and \( n \) are both even or both odd. The eight lowest order, nonzero coefficients are shown in Figs. 2 and 3. Because of the procedure that was used, \( A_{10} \) is the same for all geometries. The quadrupole coefficient, \( A_{01} \), deviates by a few percent from its ideal value of unity. The quadrupole gradient could have been maintained at its design level by changing \( r_0 \) slightly to compensate for \( A_{01} \).

**Conclusions**

We conclude that vane-tips having \( \rho_L \) proportional to \( r_0 \) can offer the advantages of a reduced peak surface electric field and an intervane capacitance independent of longitudinal position. The latter property should be beneficial in the tuning process. The higher order multipoles will not have serious effects on small beam radii. Our latest RFQ design for FMIT uses geometry Type 3 (Table I). According to a study using a multiparticle program, the beam quality produced by this RFQ was unaffected by the higher order multipoles.

**References**


5. G. P. Boicourt, "Comparison of the FMIT Round Vane RFQ and the FMIT F-17 RFQ," Los Alamos National Laboratory, Accelerator Technology Division, Group AT-6 memorandum No. AT-6:82-36 (August 4, 1982).