RESEARCH ON ELECTRON LINEAR ACCELERATORS WITH TRAVELING WAVE RESONATORS (2)

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Abstract

In the first paper, under beam loading conditions, the formula of the energy gain of the electron linac with the traveling wave resonator was given by:

\[ V_b = \frac{M_b E_0 L}{2} \left( 1 - e^{-\tau} \right) - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}) \]  

(1)

In this paper, under beam loading conditions the field multiplication factor \( M_b \) is derived. It makes this type of accelerators (including accelerators with feedback) design according to the same way as general electron traveling wave linacs. Under test of the electron linac with the traveling wave resonator, unstable phenomena are described. Their causes are discussed. They have been overcome.

1. Field multiplication factor

Under beam loading conditions, the field multiplication factor \( M_b \) not only depends on parameters of the accelerating structure, such as the shunt impedance per unit length \( r \), the field attenuation factor \( \gamma \) and the length of the structure \( L \), but also on the beam current and the input rf power. It can be obtained by repeating the iteration, before the expression of \( M_b \) is derived. But it takes a long time to design linacs with traveling wave resonators.

In order to derive \( M_b \) we start from the power diffusion equation:

\[ \frac{dP}{dz} = -2\pi P - iE \]  

(2)

where \( P \) is the rf power density, \( E \) is the electric field strength and \( i \) is the beam current in the pulse.

From eq. (2) we obtain an expression for the electric field \( E \) as a function of \( z \),

\[ E(z) = E_0 e^{-az} - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}) \]  

(3)

where \( z \) is the axial distance of the accelerating structure. If the length of the accelerating structure is \( L \), then its total attenuation is \( \gamma \), \( \gamma = 2\pi L \). Let \( E_{in} \) and \( E_{out} \) express electric fields at the input and output of the accelerating structure respectively. We have:

\[ E_{out} = E_{in} e^{-\gamma} - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}) \]  

(4)

For the linac with the traveling wave resonator, we assume that the attenuation of the waveguide in the resonator is neglected as compared with the attenuation of the accelerating structure. We can obtain:

\[ E_{in} = M_b E_0 \]  

(5)

\[ E_{out} = M_b E_0 e^{-\gamma} - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}) \]  

(6)

and the voltage transmission coefficient \( T \),

\[ T = \frac{E_{out}}{E_{in}} = e^{-\gamma} - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}) \]  

(7)

Using the definition of the field multiplication factor, at resonance and under optimal coupling, we can obtain:

\[ M_b = \frac{1}{\sqrt{1 - e^{-2\gamma}}} = \frac{1}{\sqrt{1 - (e^{-\gamma} - \frac{1}{2} \frac{1}{r L} (1 - e^{-r}))^2}} \]  

(8)

Eq. (8) can be converted into a quadratic equation

\[ (1 - e^{-2\gamma}) M_b^2 + 2 \frac{1}{r L} (1 - e^{-r}) M_b - \frac{1}{r^2 L^2} (1 - e^{-r})^2 = 0 \]  

(9)

Eq. (9) has two solutions. Abandoning an unsuitable solution, we can obtain the expression of \( M_b \).

\[ \frac{M_b}{E_0} = \sqrt{\frac{12\gamma^2 (1 - e^{-\gamma})^2 + (1 - e^{-2\gamma})}{1 - e^{-2\gamma}}} - \frac{1}{1 - e^{-2\gamma}} \]  

(10)

If the attenuation of the waveguide in the resonator is not neglected as compared with the attenuation of the accelerating structure, let \( \gamma' \) express the attenuation of the waveguide in the resonator except the attenuation of the accelerating structure. We can obtain:

\[ \frac{M_b}{E_0} = \sqrt{\frac{12\gamma^2 (1 - e^{-\gamma})^2 + (1 - e^{-2\gamma})}{1 - e^{-2(\gamma + \gamma')}}} - \frac{1}{1 - e^{-2(\gamma + \gamma')}} \]  

(11)

In eqs. (10) and (11) if we substitute \( 2P_{in} \) for \( E_0 \), then they become:

\[ \frac{M_b}{E_0} = \sqrt{\frac{12\gamma^2 (1 - e^{-\gamma})^2 + (1 - e^{-2\gamma})}{1 - e^{-2(\gamma + \gamma')}}} - \frac{1}{1 - e^{-2(\gamma + \gamma')}} \]  

(12)

and

\[ \frac{M_b}{E_0} = \sqrt{\frac{12\gamma^2 (1 - e^{-\gamma})^2 + (1 - e^{-2\gamma})}{1 - e^{-2(\gamma + \gamma')}}} - \frac{1}{1 - e^{-2(\gamma + \gamma')}} \]  

(13)
2. Computations of energy spectra and captures

For the general electron linac (the constant impedance accelerator) energy gain is given by:

\[ v_n = E_0 \frac{1 - e^{-\gamma}}{\gamma} - 4\pi \rho (1 - \frac{1}{2}) \quad (14) \]

Comparing (1) with (14) we can find that if we substitute \( M_0 E_0 \) for \( E_0 \) then two formulae are the same.

Therefore, so long as we obtain \( M_0 \), we can design the linac with the traveling wave resonator according to the same way as the general linac.

In order to acquire good results we have computed energy spectra and captures for various parameters of the accelerating structure, different injecting voltages, electric field distributions and the interaction distances from the entrance fringing field to the cavity mid-plane.

In the following, for the constant impedance structure, 2/3 mode, \( \gamma = 0.179 \) nepers/m, \( \rho = 55 \) M/O/m, we give three groups of computing results. The first accelerating structure, it consists of nine identical cavities in which the phase velocity is the velocity of light. The computing results are shown in fig.1. The second structure, it consists of one bunching cavity in which the phase velocity is 0.7 times as fast as the velocity of light and other eight identical cavities in which the phase velocity is the velocity of light. The results are shown in fig.2. The third structure, it consists of one bunching cavity in which the phase velocity is 0.5 times as fast as the velocity of light and other eight identical cavities in which the phase velocity is equal to the velocity of light. Its results are shown in fig.3.

It is seen that in the first case the energy spectra and captures get worse and worse with decreasing the injecting voltage.
In the second case, the energy spectrum is still very good as the injecting voltage decreases to 2kv. The capture can be accepted although already decreased. In the third case, the energy spectra are bad. Therefore, we can adopt the second structure, the injecting voltage will be chosen between 10kv and 20kv. The energy spectrum and capture of this accelerator both are satisfactory.

3. Unstable phenomena under test

Under test, when the microwave power enters the traveling wave resonator (i.e. the linac with the traveling wave resonator), unstable phenomena appear on the envelope of the microwave in the resonator at about resonance. They are shown in fig.4. When we change the frequency or the phase in the resonator, sometimes the unstable region appears in the front of the wave top, sometimes in the back of the wave top and at other times in the whole of the wave top.

These phenomena, as we know, result from reflection of the resonator. The reflection affects the oscillatory frequency of the source. It makes the source oscillate unstable. We observe the envelope of the reflected wave at the input. It is not a square pulse. Sometimes the high crest appears in the front of the pulse, sometimes in the back and at other times in the whole.

They correspond to those unstable regions respectively. Therefore, we want to decrease the reflection from the resonator at first. It is a radical method, but difficult especially at resonance. As we know, when there is the reflection in the accelerating structure, the reflection of the linac with the resonator is M^2 times as large as without the resonator. The measured band characteristics of the linac with and without the traveling wave resonator are shown in fig.5 for M=3. The curve 1 is the band of the linac without the resonator. It is seen that the linac has been well tuned. But when this linac is with the resonator, its band becomes the curve 2. We have to use an isolator between the source and the resonator. In this case these unstable phenomena are eliminated. It is a simple and effective method.

![Diagram of unstable phenomena](image)

Fig.4. Unstable phenomena

![Diagram of V.S.W.R.](image)

Fig.5. V.S.W.R. of the linac without and with the traveling wave resonator as a function of frequency.

References