DESIGN CONSIDERATIONS ON PEAK ELECTRICAL FIELDS AND MAXIMUM BEAM CURRENTS FOR HEAVY ION RFQ LINACS*

P. Junior, H. Deitinghoff, K.D. Halfmann, W. Neumann, N. Zoubek
Institut für Angewandte Physik der Universität Frankfurt am Main,
Robert-Mayer-Str. 2-4, D-6000 Frankfurt am Main, Germany

Abstract

Aperture and frequency determine voltage limits with respect to sparking. For a corresponding application of Kilpatrick's criterion exact knowledge of peak electrical fields in the RFQ is required. Results of calculations for Bi^{2+} fusion injectors considered in our institute are presented. As a general result maximum beam current turns out approximately proportional to electrode voltage. Some notes on the influence of space charge models shall be reported.

Introductory Remarks

A certain RFQ structure type presently investigated in our institute consists of rotational rods with trapezoidal profile manufactured on a lathe^{1,2,3}. Our principle has recently been confirmed by the successful operation of a proton accelerator^{4,5}. Especially for the acceleration of heavy ions at very low frequencies the applied zero mode operation has proven rather useful. Apart from the simplicity in construction the suitability of such electrodes with rotational symmetry is obvious particularly for a multichannel system, as fig. 1 demonstrates. In this connexion the following points need detailed discussion: Information associated with exact solutions of Laplace's equation satisfying the RFQ boundary conditions is required for the purpose of establishing correct relations between electrode geometry and acceleration parameters within the beam area and in the profound knowledge of limits set by peak surface fields what sparking is concerned.

Thus the first section of this paper reviews results in two dimensional calculations of optimum beam current, phase advance, acceptance corresponding to several accelerator designs at various electrode voltages neglecting boundary conditions transversely given by the rotational electrodes. The second section then extends to three dimensions and in principle presents some information on higher field harmonics necessarily occurring with such electrodes (fig. 2). These studies have just been started, however.

a) Two Dimensional Calculations

Our considerations are based on the GSI design B for a heavy ion linac⁶ with ion $^{209}\text{Bi}^{2+}$; rf 13.56 MHz, injection voltage 365 kV (input energy 730 keV), initial synchronous phase 77° , final one 30° at buncher output with 5.6 MeV⁷. In this paper we restrict our attention to the buncher, as this accelerator part features all points of interest for the moment. In fig. 3 the optimum beam current I_{max} (where optimum is understood as that state of undepressed transverse phase advance, where longitudinal and transverse current limits agree), undepressed phase advance σ_0 , undepressed transverse acceptance ϵ and the linac length are plotted

versus electrode voltage. These quantities are given by the following expressions⁸, which are a generalization of Reiser's formula⁹

$$I_{\text{max}} = I_0 \frac{\omega^3 a_u^2 a}{4\pi c^3 \kappa} \frac{\chi - \frac{\alpha^2}{2} q}{1 + \frac{q}{2}} \quad (1)$$

$$\sigma_0^2 = \pi^2 \frac{\chi - \frac{\alpha^2}{2} q}{\kappa} \left(\frac{3a_u}{a} \left(1 + \frac{q}{2} \right) - 1 \right) \quad (2)$$

$$\epsilon = \frac{\sigma_0}{c} \frac{fa^2}{\left(1 + \frac{q}{2} \right)^2} \quad \text{with abbreviations} \quad (3)$$

$$I_0 = \frac{4\pi\epsilon_0 mc^3}{e}; \quad \alpha = \frac{\omega}{v} a; \quad \chi = \frac{eV}{mv^2}; \quad \kappa = \frac{1 + \frac{c\hbar}{\sin\phi_s}}{\sin\phi_s}$$

$$a_u = \frac{2v}{\omega} \sqrt{1 - \phi_s \text{ctg}\phi_s}; \quad q = P \left[\sqrt{1 + \frac{0}{p^2}} - 1 \right]$$

$$0 = \frac{4\chi}{\frac{2\alpha\kappa}{3a_u} + \alpha^2}; \quad P = \frac{\alpha^2 - \chi}{\frac{2\alpha\kappa}{3a_u} + \alpha^2}$$

Here a resp. a_u stands for transverse resp. longitudinal aperture, v is velocity at buncher output, ϕ_s final synchronous phase, ϵ_0 dielectric constant.

On the abscissa of fig. 3 there is a further subdivision for the Kilpatrick¹¹ factor concerning the peak surface field usually occurring in the first section behind the buncher, where the modulation is largest. Inspecting fig. 4 the equipotential lines (fig. 4b) at coordinates $z = 0, 8\lambda/2$ (fig. 4a) indicate maximum enhancements, where the electrodes are thinnest. In fig. 5 field enhancements are plotted versus modulation, besides showing a rather good agreement between exact calculations for four electrodes and the approximation by a two electrode formula¹⁰. When the enhanced fields are submitted to Kilpatrick's criterion¹¹ in the approximation¹²

$$E_p^3 \cdot g \left(1 - e^{-\frac{9.33 \cdot 10^{-11} E_p^2}{g}} \right) e^{\frac{1.7 \cdot 10^7}{E_p}} = 1.8 \cdot 10^{18} \quad (4)$$

(E_p in V/m, gap g, λ in m)

Kilpatrick factors of fig. 3 result.

Profound gains in beam current (1), stability (2) and acceptance (3) together with a shortened linac become obvious, when the threshold (4) is considerably crossed.

The agreement with equ. (1) is confirmed by PARMTEQ simulations, where as an example in case of electrode voltage 100 kV a beam of 19.5 mA was transported. This is about 20 % less than the limit of fig. 3, thus corresponding to a tune depression of about 0.45.

b) Prospects for Three Dimensions

Calculations leading to fig. 3 are based on two dimensional boundary conditions $x-z$, where in the potential expansion with hyperbolic functions the transversely rotational shape of the electrode is neglected.

* Work supported by the BMFT

$$\phi(x, y=0, z) = \frac{V}{2} \left(C_0 \frac{x^2}{a^2} + \sum_{N=1}^{\infty} A_N \cos Nkz (\cosh Nkx - (-1)^N) \right) \quad (5)$$

When the potential is calculated in the general three dimensional case, however, the representation¹³ with modified Bessel-functions

$$\phi(r, \phi, z) = \frac{V}{2} \left(\sum_{M=1}^{\infty} A_{0,M} r^{2M} \cos 2M\phi + \sum_{M=0}^{\infty} \sum_{N=1}^{\infty} A_{N,M} I_{2M}(Nkr) \cos 2M\phi \cos Nkz \right) \quad (6)$$

should be applied.

Characters $A_{N,M}$ vanish in case of RFQ symmetry, when $N + M$ is an even number, thus series (6) usually starts with the longitudinally constant quadrupole term $A_{0,1}$, which corresponds to C_0 of (5), followed by the constant dodekapole term $A_{0,3}$. Then the monopole term $A_{1,0}$ causing the acceleration and corresponding to A_1 of (5) appears with key wave length $\beta\lambda$. A quadrupole term $A_{2,1}$ (corresponding to A_2 of (5)) succeeds together with a dodekapole term $A_{2,3}$ both periodic with half the wave length $3\lambda/2$. Octupoles start with $A_{1,2}$ again at key wave length. Some evidence on their order of magnitude in case of rotational electrodes may be derived from still two dimensional calculations, when potential distributions of fig. 4b are analyzed with respect to field harmonics. Here the aperture a is taken for the radius vector and calculations refer to situations at $z = 0$ and $z = \beta\lambda/4$ (fig. 4a). According to table 1, where 2 examples are comprehended, participation of octo- and dodekaipoles compared to the quadrupole term seem hardly severe for beam dynamics, so predictions of fig. 3 with respect to the ideally milled electrodes

Table 1 Estimations of higher field harmonics for different modulations (A_1 calculated with (5) for $ka \ll 1$ in parentheses)

Character	Symbols in (6)	$z=0, r=a$	$z=\beta\lambda/4, r=a$	$z=0, r=a$	$z=3\lambda/4, r=a$
Monopole	$A_{1,0} I_0(kr) \cos kz$	0.67 [0.61]	0	0.8 [0.75]	0
Quadrupole	$A_{0,1} r^2 + A_{2,1} I_2(2kr) \cos 2kz$	0.35	0.44	0.21	0.32
Octopole	$A_{1,2} I_4(kr) \cos kz$	-0.02	0	-0.016	0
Dodekapole	$A_{0,3} r^6 + A_{2,3} I_6(2kr) \cos 2kz$	$-0.3 \cdot 10^{-2}$	$0.9 \cdot 10^{-3}$	$-0.23 \cdot 10^{-2}$	$0.36 \cdot 10^{-3}$
Modulation		2		2.5

Fig. 1 Scheme of multichannel RFQ

remain valid for rotational electrodes as well however, deviations of $A_{0,1}$ from A_1 e. g. rather suggest a replacement of our two dimensional design code based on (5) by a generalized three dimensional one using (6).

These computations have just been started, studies of their influence on optimal design and to a minor extend on beam dynamics become inevitable. They shall be reported as soon as there is more evidence.

Computations have been carried out at the Hochschulrechenzentrum.

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Fig. 2 Realization of our RFQ profile⁵

Fig. 3 Optimum beam current limit I_{max} , underpressed phase advance σ_0 , underpressed transverse acceptance ϵ (normalized) and length of gentle buncher versus electrode voltage and Kilpatrick factor

Fig. 5 Field enhancements versus modulation. Curve 1 formula for two-wire line, curve 2 exact calculation

Fig. 4 Cross sections of electrodes
a) longitudinal scheme
b) transverse scheme and potential distribution calculated with the Herrmannsfeldt code¹⁴ at $z = 0$ in the case of modulation 2