The SPS is the first hadron collider operating with bunched beams and head-on collisions. In this respect, it is more similar to electron storage rings than to the ISR, but without the natural beam cooling due to synchrotron radiation damping. Therefore it was expected that the nonlinear beam-beam effect should play an important role in limiting the beam lifetime at a value of the beam-beam tune shift at least an order of magnitude below that achievable in electron machines. Experience has shown that this is, indeed, the case. With the design tune shift of $3 \times 10^{-3}$ per intersection, dramatic effects are observed.

Introduction

The SPS started operation as a proton-antiproton storage ring at the end of 1981. This was followed by a second experimental period at the end of 1982, when the machine was run continuously for about 2 months for physics data-taking. During this time access to the machine for accelerator physics was extremely limited, so most of the results had to be obtained on a parasitic basis.

Up to now, the machine has operated entirely in the strong-weak regime due to a lower antiproton collection rate than expected. Initially, the collider was operated with two proton bunches of around $7 \times 10^{10}$ particles per bunch (ppb) together with a single antiproton bunch of up to $10^{10}$ ppb in order to simultaneously provide collisions in both experimental areas, which are in adjacent straight sections.

Recently, the luminosity has been improved by colliding 3 proton bunches of $10^{11}$ ppb with 3 antiproton bunches. In this configuration the full design beam-beam tune shift $\Delta Q_m = 3 \times 10^{-2}$ per intersection is experienced by the antiprotons at each of the 6 intersections. Under these conditions the amount of space available in the tune diagram is extremely limited throughout the early part of the acceleration cycle as well as during storage.

Injection and Acceleration

The protons and antiprotons are injected on a magnetic flat-bottom at 26 GeV/c, which is above transition energy to avoid instabilities encountered with such dense bunches. The low-S insertions are detuned to a value of $\delta Q = 7\,\text{m}$, $\delta S = 1.5\,\text{m}$ in order to liberate injection aperture and facilitate chromaticity correction.

The working point must be carefully chosen in order that the particles survive the acceleration process.

Space-charge effects have important consequences, as illustrated in Figure 1. The high frequency broadband impedance of the machine ($27 - 10^{-2}\,\text{m}^{-1}$) introduces a coherent tune shift $\Delta Q_H = 0.013$, $\Delta Q_Y = -0.027$ for $10^{11}$ ppb, measured from the variation of the coherent tune with intensity. In addition, the dense proton bunches have a large incoherent detuning ($0.05$) which pushes them down towards the 3rd integer resonance. On the other hand the much weaker $\bar{p}$ bunches are pushed upwards by the beam-beam tune shift toward the (beam-beam driven) 4th order resonance. The measured tune $Q_m$ must be set to high precision and the amplitude and intensity carefully controlled in order that antiprotons are not lost during the acceleration.

During the injection platform and acceleration cycle the antiprotons straddle 7th and 10th order resonances as well as cross the main diagonal. About 25% of the antiprotons are lost, some of which can be attributed to imperfect capture, but most of the loss is slow and is most likely due to resonances.

Storage

During the first second of the 270 GeV/c flat top, the $S^*$-values are squeezed to their final configuration. Most experience has been gained with $S^* = 2\,\text{m}$, $Q^* = 1\,\text{m}$, although other configurations have been successfully tried for short periods ($7\,\text{m}$, $1.5\,\text{m}$, $0.75\,\text{m}$, $100\,\text{m}$). The chromaticity is adjusted to be as close to zero as possible (but slightly positive). Generally $Q^* = -\left(Q/Q^*\right)/\left(C/C^*\right) < 0.1$ can be achieved in both planes.

Figure 1 also shows the working diamond during storage. The incoherent tune spread is now negligibly small, and most of the space is occupied by the beam-beam tune spread in the antiprotons. The antiproton beam lifetime is now critically dependent on the working point. If they touch the 10th order resonance, the lifetime falls dramatically from about 40 hours to 7 hours. However, if the proton beam is debunched, retaining the same beam-beam tune spread whilst eliminating the resonance excitation, the antiproton lifetime is high and much less
sensitive to the working point.

The optimum tune value is such that the antiproton bunches stay clear of the 10th order resonance. This is more critical for 3 x 3 bunches than for 2 x 1 bunches when carefully adjusted the antiproton lifetime is equal in both cases. Further increase of the beam-beam tune shift per interaction or of the number of bunches per beam will almost certainly cause a degradation of the antiproton lifetime at this working point unless the total beam-beam spread can be reduced.

The lifetime of the antiproton beam depends strongly on emittance. If it is larger than the proton emittance a self-scraping process occurs, where the large amplitude particles are rapidly lost and consequently the emittance decreases over the first hour or so of storage. As a result the initial lifetime is low and the initial background in the experiments is very high. Gradually, the emittance approaches that of the protons and finally the two beams blow up at the same rate.

Figure 2 shows the emittance growth measured under a number of different conditions. Shot 29 shows the vertical emittance growth of the proton and antiproton beams for 2p x 1p. The self-scraping of the 1 beam is clearly seen. The emittance growth rate is 1.1 x 10^{-3} mm.mrad/h, which is not significantly higher than that calculated from the measured gas pressure. Shot 71 shows that the growth rate with about 70% more intensity is identical, so is independent of bunch current. Shot 239 shows the situation with 3p x 1p. The emittance grows 3 times faster than in the previous cases. This behaviour is very reproducible from store to store and has been measured both with the wire scanner and with the synchrotron light detector. It is thought to be due to the proximity of the 3rd order resonance, known to be strongly excited by the chromaticity correction sextupoles. In the case of 3p x 3p the protons must be moved closer to the resonance in order to clear the 10th with the antiprotons (figure 1). With full design intensity in the proton bunches (10^{11} ppb) and nominal initial emittance (\epsilon = 17 mm.mrad), the mean luminosity lifetime is about 40 hours. This is roughly equally divided between antiproton bunch lifetime, (r = 40 hours) and this anomalous proton beam blow-up.

Other limitations

Clearly the need to stay clear of resonances of such high order imposes a severe limitation on the maximum proton intensity tolerable either in few high intensity bunches or in many bunches of lower intensity. In addition, there are several other effects which limit the beam intensity and phase-space density.

Microwave instability

Radio-frequency capture loss rises sharply for exceeding bunch intensities above 1.2 x 10^{11} ppb. This corresponds almost exactly to the prediction of the local peak intensity Kell-Schnell-Boussard criterion\(^2\) for the microwave instability threshold using the measured SPS longitudinal coupling impedance Z/\(\mu = 16\)\(^3\). Bunches of up to 2 x 10^{11} ppb could possibly be stabilized by a pre-emptive longitudinal density dilution in the injector (CPS), the limit being set by the available RF voltage in both the CPS and SPS.

Coherent modes

Low order head-tail modes are stabilized by a slightly positive chromaticity and potentially unstable high-order modes have not been seen up to 10^{11} ppb. Longitudinally, Landau damping is effective at present intensities only for modes 3 (sextupole) and above. Consequently, dipole and quadrupole modes must be damped by individual feedback loops. With increased intensities, higher modes would need to be damped either by increasing the longitudinal emittance or using the fourth harmonic cavities.

Intra-beam scattering

In ultra-relativistic beams there is a large unbalance between longitudinal and transverse temperatures which leads to a blow-up of the energy spread and radial emittance. Calculations for the SPS\(^4\) indicate growth times of around 20 hours, which would contribute to a reduction of luminosity lifetime. No clear observation of this effect has yet been made.

A model of beam-beam induced diffusion

Repeated crossings of high order resonances would be expected to produce beam growth only if successive crossings occur with a random phase distribution. For the case of coherent modulation due to synchrotron motion this is not the case unless the 'stochastic limit' is reached, resulting in the overlap of synchro-betatron islands in phase space and chaotic behaviour\(^5\). We first assume that this is the case, and compute the growth rate due to multiple fast crossings.

We consider only the weak-strong case of a round Gaussian beam with \(N\) particles per unit length interacting with a test particle. The linear beam-beam tune shift is \(\Delta \nu = \frac{N e^2}{L} \cdot \frac{1}{(8\pi B_{35} B_{20})}\). The resonant invariant near an nth order resonance is

\[\dot{\eta} = \left(\frac{\nu - \nu_{0}}{\nu_{0}^2}\right) \cdot N \Delta v_{0} U_{n}(\phi) \cdot \overline{\Delta \nu} \cdot V_{n}(\phi) \cos \phi\]

where \(\overline{\Delta \nu}\) is the coefficient of the \(n\)th azimuthal harmonic of the beam-beam force and \(\overline{\Delta \nu}\) is the normalized emittance.

\[\sigma_{\nu} = \frac{r^{2}}{\nu \Delta \nu}\]

The functions \(U_{n}\) and \(V_{n}\) are given by \(6\).
The amplitude equation is

\[ \frac{d \varphi}{d t} = -\frac{\pi}{\pi_0} \Delta \varphi \sin \varphi \]  

Following Schoch\(^7\) we compute the maximum amplitude growth \( \Delta \varphi = \sqrt{\frac{1}{\Delta \varphi_0} \int \Delta \varphi^2 \, d\varphi} \) per crossing by integrating \( 3 \) through the resonance.

Where \( \pi_0 \) is the strong beam emittance and \( \Delta \varphi_0 \) is the tune change/revolution. Assuming a modulation of the form \( \varphi = \varphi_0 \sin \varphi \), treating repeated crossings as a random walk process we obtain for the means square increase in radius

\[ \Delta r = \sqrt{\frac{1}{4 \varphi_0^2} \frac{\int \Delta \varphi_0^2 \, d\varphi}{\Delta \varphi_0^2}} \]

where \( \Delta r \) is a strongly amplitude dependent diffusion coefficient. This can be compared with the diffusion rate due to multiple Coulomb scattering with the residual gas.

\[ \Delta r = 0.08 \pi_0 / \sqrt{\Delta \varphi_0^2} \]

From the measured growth rate \( d \varphi / dt = 1.1 \times 10^{-3} \, \text{mm.mrad/h} \), we obtain

\[ D_0 = 3.82 \times 10^{-14} \, \text{m.s}^{-1} \]

Figure 3 shows a comparison of this rate with that predicted by equation \( 5 \). We see that emittance growth at large amplitude (above 1.6\( \pi \) for a 10th order resonance) is dominated by resonance excitation. This is at least in qualitative agreement with the observation of fast loss of large emittance particles.

The implicit assumption of the theory underlined above is that successive resonance crossings are made with random phase. This is equivalent to the condition for stochastic motion due to the overlapping of synchrobetatron resonances\(^5\). In terms of the resonance functions, the stochastic conditions are\(^6\)

\[ \Delta \varphi_0 \geq \frac{1}{4} \left( \pi_0 \right)^{1/4} \left( \frac{\varphi_0}{\pi_0} \right)^{1/4} \]

When this criterion is applied to the SPS conditions ( \( \varphi_0 = 4.5 \times 10^{-3} \), \( \Delta \varphi_0 = .003 \) it is found that stochastic motion should only occur above \( 3 \pi \) for a 10th order resonance, which is in contradiction with observations. This may be due to the many simplifying assumptions made, particularly the one-dimensional nature of the model. In addition, in the real world many other effects (e.g., noise, power supply ripple) conspire to lower the stochastic threshold.

### Conclusions

The SPS has operated in the weak-strong regime where the antiprotons experience the full design beam-beam tune shift of \( 3 \times 10^{-3} \) per intersection and with 5 intersections. Several single beam effects limit the present maximum achievable intensity of the proton bunches to around 10\(^11\) ppm. Further increase in bunch current would require a controlled blowup of the longitudinal emittance in the CPS. A more fundamental intensity limitation is set by the maximum allowable beam-beam tune spread such that the beams stay clear of resonances up to and including 10th order.

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### References

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