CHARACTERISTICS OF MAGNETIC FOCUSING AND CHROMATICITY
CORRECTION SYSTEM FOR TARN

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Summary

TARN is designed by a separated function FOD0 lattice. Its main magnet system consists of eight dipole and sixteen quadrupole magnets. The relatively lower injection energy of TARN ($\beta = 0.134$) leads to large transverse coupling impedance and the relatively smaller intensity limit from transverse coherent instability ($6 \times 10^8$ ions/pulse). In order to overcome the instability by Landau damping, a chromaticity control system with twelve sextupole magnets is designed and fabricated. The field properties of all the magnets are measured precisely before alignment and the result is taken into account by the calculation with the computer code SYNCH. The closed orbit is obtained by iteration and the nonlinear elements such as sextupole magnets are linearized in the neighborhood of the closed orbit. The work line is also experimentally studied by an RF knock-out method. The chromaticities of TARN without correction sextupole magnets were measured at -1.59 and -0.47 in horizontal and vertical directions, respectively.

1. Introduction

TARN (Test Accumulation Ring for the NUMATRON project) has been constructed to test the feasibility of a beam accumulation method which uses the combination of a multi-turn injection into the transverse phase space and an RF stacking into the longitudinal phase space. The beam dynamics are also to be studied by the ring. It is designed to be able to accumulate heavy ions up to $N^n_+$ with the kinetic energy of 8.5 MeV/u. The mean radius of the ring is 5.06 m, which is determined considering the synchronization between the RF system of TARN and that of the injector SF cyclotron.

It is important to choose an optimum operation point to accumulate ions in the storage ring. For the case of TARN, an RF stacking is applied and the momentum spread of the accumulated beam is large ($7.5\%$) and from the point of view of avoiding the lower order single particle resonances, it is desirable to make the chromaticity as small as possible, while some amount of chromaticity is needed to surmount the transverse coherent instability by Landau damping.

For the purpose of controlling the work line, the chromaticities in both horizontal and vertical directions are to be tunable and two families of correction sextupole magnets are needed.

The number of betatron oscillations per revolution is calculated by the computer code SYNCH, and it is also experimentally studied by an RF knock-out method.

2. Transverse Coherent Instability

The transverse resistive instabilities are found in various high energy accelerators in the world. The stability condition against the transverse coherent instability (called as TCI hereafter) is given by the relation:

$$\frac{2\beta}{Z_0} < 2 F \frac{A}{q^2 n R^2} \frac{1}{N R_C} \left( \frac{n + \nu}{\nu}, \frac{n + \nu'}{\nu'} \right) \frac{\Delta p}{p} ,$$  

where the notation is as follows:

- $Z_0 (= 120 \pi a)$ is the impedance of space,
- $F$ is a form factor depending upon the shape of momentum distributions, a value of 0.45 is appropriate for the present case,
- $N$ is the total number of accumulated ions,
- $\nu$ and $\nu'$ are the number of betatron oscillation per revolution,
- $\gamma$ is the chromaticity ($= \frac{A}{\nu}$),
- $A$ and $q$ are the mass number and charge state of the accumulated ion, respectively.

For the ideal case where the beam and the vacuum chamber have constant circular cross sections, transverse coupling impedance $Z_1(0/m)$ is given by the formula:

$$Z_1 = \frac{i R Z_0}{\left( \frac{1}{A^2 \nu} + \frac{1}{A^2 \nu'} \right) - (1 + i) \frac{\Delta p}{p} \gamma},$$

where $R$ is the mean radius of the machine, $a$ and $b$ are the radii of the beam and the vacuum chamber respectively, and $\Delta p$ is the skin depth of the chamber wall. As known from Eq. (2), the coupling impedance $Z_1$ becomes larger for lower value of $\beta$ as is the case of TARN ($\beta = 0.134$). Then the intensity limit of the TCI is estimated to be $6 \times 10^8$ ions/pulse without the correction sextupole magnets. The e-folding growth rate, $\tau$, is given by

$$\frac{1}{\tau} = N \delta \frac{\gamma^2}{2} \frac{A}{2\pi} \frac{2R}{n \gamma} \frac{2b}{n \gamma} \frac{Z_0}{\sigma b} \gamma \nu ,$$

where $\delta$ is the conductivity of the chamber wall ($1.37 \times 10^6 \Omega^{-1} m^{-1}$ for stainless steel) and $\tau$ is estimated to be 0.2 sec for $Z \times 10^{10}$ ions of N$^{16}_+$ are stored. It is
known from Eq. (1) that the intensity limit can be raised by increasing the size of chromaticity \( v' \).

3. Chromaticity Correction System

The arrangement of the magnets of main lattice for TAR is shown in Fig. 1 by solid lines. It consists of 8 dipole and 16 quadrupole magnets and has a separated function FDDG structure. Its superperiodicity is 8.

The rather higher superperiodicity is preferred so as to avoid the lower order sector resonances. The \( v \)-value is chosen around 2.25 both in horizontal and vertical directions. The beta and dispersion functions along the central orbit calculated by the computer code SYNCH is shown in Fig. 2.

In order to control the work line in the tune diagram, it is necessary that chromaticities in \( dv \) horizontal and vertical directions (\( E, f v'x = \frac{dv}{d(\theta p)} \) and \( E, f v'y = \frac{dv}{d(\theta p)} \)) can be varied independently.

The contributions of sextupole magnets to the chromaticities are given by

\[
\begin{align*}
\xi_x &= \frac{1}{4\pi} \int B'' \eta \theta_p \ ds \\
\xi_y &= \frac{1}{4\pi} \int B'' \eta \theta_p \ ds
\end{align*}
\]

(4)

In the present case, the wavelength of betatron oscillation is about 14 m, which is long enough compared with the core length of the sextupole magnet (0.1 m).

Therefore beta and dispersion functions can be assumed to be constant in the sextupole magnets with good approximation. Then Eq. (4) can be written as

\[
\begin{align*}
\xi_x &= \frac{1}{4\pi} \int \frac{\eta}{B_p} \theta_p \ ds \\
\xi_y &= \frac{1}{4\pi} \int \frac{\eta}{B_p} \theta_p \ ds
\end{align*}
\]

(5)

where \( \langle \eta \rangle_x, \langle \eta \rangle_y \) and \( \langle \eta \rangle \) are beta and dispersion functions in the sextupole magnet. The sextupole magnets should be placed at the two places where \( \langle \eta \rangle_x, \langle \eta \rangle_y \) and \( \langle \eta \rangle \) take different values from each other so as to introduce two degrees of freedom and are aligned at the places shown by dashed lines in Fig. 1.

Numerical calculation is made for various work lines with the use of computer code SYNCH. It calculates the closed orbits for various fractional momenta \( \langle \eta \rangle \) and then the non linear elements as sextupole magnets are linearized in a neighbourhood of the closed orbit. The sextupole magnet is treated as a thin lens by the relation

\[
\begin{align*}
\frac{dx}{ds} \text{out} - \frac{dx}{ds} \text{in} &= -\frac{B''}{(B_0)^2} \left( \frac{1}{1 + \frac{\eta}{\theta_p}} \right) \frac{1}{x^2} \left( x^2 - z^2 \right) \\
\frac{dz}{ds} \text{out} - \frac{dz}{ds} \text{in} &= \frac{B''}{(B_0)^2} \theta_p x .
\end{align*}
\]

(6)

\[
\begin{array}{|c|c|c|}
\hline
\text{Line} & \text{Configuration} & \text{Chromaticity} \ (v'_x/v'_y) & N_{\text{max}} \\
\hline
\text{A} & \text{No Correction} & -4.35/-1.07 & 1.2 \times 10^9 \\
\text{B} & \text{No. Correction} & -5.74/-0.25 & 5.9 \times 10^8 \\
\text{C} & \text{Realistic Magnet} & -5.03/-0.50 & 3.8 \times 10^9 \\
\text{D} & \text{N} & -7.13/-6.56 & 4.4 \times 10^9 \\
\text{E} & \text{P} & -8.49/-1.45 & 30.0 \times 10^9 \\
\hline
\end{array}
\]

Fig. 3 Candidates of Work Line for TAR.

where \( (B_0) \) is the magnetic rigidity of the ion with the central momentum and \( B'' \) represents integrated sextupole component, \( \int B'' ds \). Some candidates of recommended work lines are shown in Fig. 3, where lines A and B are work lines without correction sextupole magnets with and without the effect of the deviation of real magnetic field from the ideal one, respectively. In the table, the estimated maximum number of \( N_{\text{max}} \) with the kinetic energy of 8.5 MeV/u which can be accumulated in the ring without TCI is given for these work lines. It is expected that the intensity limit without instability can be raised up to \( 4 \times 10^9 \) ions/pulse with the use of sextupole magnets.

4. Characteristics of Magnets for TAR

Dipole Magnet

Dipole magnets for TAR are designed with window-frame type because of compactness and good field property and their edges are designed to be perpendicular to the central orbit (no edge focusing). The magnet is fan-shaped so as to avoid the so-called sagitta which amounts to 5.1 cm if the magnet is made with a straight rectangular shape.

The magnetic field was measured by a temperature-controlled Hall-probe which was precisely positioned by a two-dimensional driving system.\(^{10}\) The uniformity of the field is better than \( z \times 10^{-7} \). The sextupole components of the dipole magnets are found to be 0.233 kG/m2.\(^{2} \) The sextupole components of the dipole magnets are found to be 0.233 kG/m2 for full excitation.

Quadrupole Magnet

The pole shape of the quadrupole magnet is designed by a hyperbola which extends to its tangential lines at both sides.\(^{11}\) The magnet is designed to preserve the fourfold symmetry to suppress the octupole component.\(^{12}\)

The field structure of the magnet was measured by twin coils which translate in a horizontal plane. The octupole components of the quadrupole magnets are
of molecular hydrogen (H₂⁺) and α-particle with kinetic energy 7 MeV/u. The upper signal of Fig. 6(a) is the sweep signal of RF frequency of the RF field, fRF, applied to the accelerating cavity for stacking. The frequency is 7.987 MHz at the base line and 7.841 MHz at the flat top. The lower signal represents the pulsed transverse RF applied to the electrodes (500 V/div.). The upper signal in Fig. 6(b) is the beam signal picked up by an electrostatic monitor (5 mV/div.) and the lower signal is the same as Fig. 6(a). It is clearly observed that when the pulsed transverse RF field with an adequate frequency, fKO, is applied, the beam is lost at the corresponding timing.

From the value of fKO and Eq. (7), the fractional part of v-value, c, is known, because the revolution frequency f, is given by

\[ f = \frac{1}{h} f_{RF} \]  

where h is the harmonic number and is chosen at 7 for the case of TARN. The measured v-values for various fractional momenta are shown in Fig. 7 together with the calculated line. Without correction sextupole magnets, the experimentally obtained chromaticities are -1.59 and -0.47 for horizontal and vertical directions, respectively.

References
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11) A. Noda et al., "Quadrupole Magnet for TARN", INS-NUMA-23 (1980).