The paper deals with analysis of output characteristics of beams, accelerated by electron linacs. Being the major parameter electron density in a bunch is calculated from data published for linacs in operation. The value of this parameter is shown to be within the limits $5 \times 10^8 - 3 \times 10^{10} \text{cm}^{-3}$. The reasons, limiting its maximum value are discussed. The obtained results will be useful for the electron linacs design.

Introduction

Considerable volume of data concerning parameters of linacs in operation makes it possible to study general laws defining their output characteristics thus providing further application when designing a linac.

One of the main parameters is density in accelerated bunches. Quantitative knowledge of this parameter typical for linacs permits to select accelerated current value and other beam parameters correctly.

The paper is concerned with the density of electron bunches at the output of linear resonant accelerator. This characteristic was calculated by the authors on the basis of data obtained for several tens of linacs [1-8]. Calculations were made for known values of peak beam current, rf generator frequency and phase and radial bunch sizes. Electron density was supposed to be constant both along the radius and axis of the bunch.

The difficulty of electron density determination consists in almost complete absence of bunch size data and especially those of phase length. Under the circumstance bunch phase length was determined from the value of relative energy spread at the linac output using the relation from

$$\Delta W/N = K \cdot \Delta \Phi^2$$

(1)

Where coefficient $K = \frac{1}{2}$ corresponds to accelerating position on the wave crest, and $K = \frac{1}{2}$ - when any other position. When no beam transverse size data, it was roughly estimated as equal to 30-50% waveguide aperture.

The results are presented in the table and in the diagram. The analysis of the table data obtained from literature shows that electron density is in the limits of $10^9 - 3 \times 10^{10} \text{cm}^{-3}$. Higher values correspond to linacs with medium energy and considerable (hundred mA) pulse current. High density was also gained at the single bunch experiments at SLAC [3]. In the case of large electron energies the density is close to $(1 - 1,5) \times 10^9 \text{cm}^{-3}$.

Diagram, illustrating achieved values of electron density (calculated both known and approximated beam sizes) also shows that upper and lower boundaries are $2 \times 10^{10}$ and $5 \times 10^9 \text{cm}^{-3}$ accordingly. It's necessary to note that density value used is an average one per the bunch.

Let's consider factors, defining electron density. These are electron density near cathode, phase bunching in accelerating system and Coulomb repulsion in a bunch. Now we shall discuss each of them separately.
Near cathode electron density

If no longitudinal bunching occurs and assuming beam radius constancy the next relationship is valid

\[ j = \frac{e \pi n}{\sqrt{2}} \]  

(2)

where \( j \) is electron current density, \( e \) - electron charge, \( n \) - electron density and \( \sqrt{2} \) - electron flow velocity.

Equation (2) shows that bunches being accelerated, their density necessarily decreases defined by the position of the bunch in the accelerating system, namely by corresponding velocity, and cathode current density.

For example, take initial energy of electrons equal 0.1 - 0.5 ev for thermionic cathode. Cathode current density values achieved are varied from 1 A cm\(^{-2}\) to 100 A cm\(^{-2}\). Consequently, maximum initial electron density is \( 3 \times 10^{11} - 3 \times 10^{13} \) cm\(^{-3}\) depending on current density. Then after the acceleration of electrons to energies, higher than 10MeV, their density will decrease up to \( 2 \times 10^8 - 2 \times 10^{10} \) cm\(^{-3}\). Note, that indicated interval almost covers the range of density variation presented in the table and the diagram.

RF bunching

Let's estimate increase in electron density at the expense of longitudinal bunching in RF fields. Bunching coefficient, defined as the ratio of bunch phase length at the input and output of the system considered, changes from 6 for accelerating structure with wave phase velocity equal to velocity of light up to 15 for structure with slowly varying phase velocity. In the last case compressing coefficient for real space decreases as compared with bunching coefficient in \( \Phi \) times, where \( \Phi \) in is relative phase velocity at the input of accelerating system. Then maximum compressing value is near 7. We suppose that longitudinal length shortening of bunch is not accompanied by perceptible increase of radial size.

These estimations can be used for the analysis of one of the well described linacs - the injector of Stanford 3-km linear accelerator. Electron density of accelerated beam at the output achieves the value of \( 1.8 \times 10^9 \) cm\(^{-3}\) when no bunching occurs and the cathode current density is equal to 0.8 A cm\(^{-2}\) the output density reaches the value of \( 1.67 \times 10^9 \) cm\(^{-3}\). Taking into account bunching action of accelerating field we obtain the value of density equal to \( 1.24 \times 10^9 \) cm\(^{-3}\).

Coulomb repulsion

At high cathode current densities (greater than 10 A cm\(^{-2}\)) further increase of input electron density to \( (1 - 5) \times 10^{10} \) cm\(^{-3}\) result in growth of beam space charge influence.

Longitudinal Coulomb repulsion of electrons decreases bunching action of accelerating field. Its influence can be estimated regarding the bunch as a sufficiently long, homogeneous charged cylinder. Then field at the front or back edge of such cylinder equals

\[ \frac{E}{Q} = \frac{10^{-14}}{2 \pi \bar{\varepsilon} a^2} \cdot \left( 1 + \frac{a}{l} \right) \left( 1 + \frac{E \bar{\varepsilon}}{a^2} \right)^{-1} \]  

(3)

where \( E \) - electric field at the edge of the bunch, kV/cm;
\( Q \) - bunch charge, nC;
\( a \) - bunch radius, m;
\( l \) - bunch length, m;
\( \bar{\varepsilon} \) - reduced energy of electron;
\( \bar{\varepsilon} \) - dielectric permeability of vacuum.

In case of beam injection with \( \varepsilon = 1.1 \), \( a = 0.5 \) cm, \( l = 5 \) cm at electron density
\( n_e = 5 \times 10^{10} \text{cm}^{-3} \) Coulomb field value is near 20 kV/cm.

In transverse direction Coulomb repulsion in electron linacs is usually compensated by longitudinal magnetic field or other focusing systems. Now we calculate electron density of beam, kept in equilibrium by some magnetic field. Balance of forces in radial direction gives the following relation between density \( n_e \) and longitudinal magnetic field \( B_z \):

\[
\frac{\varepsilon_e B_z^2}{2m_e} = \frac{1}{4} n_e \quad (4)
\]

where \( m_e \) - rest mass of electron.

With magnetic field being 0.1T electron density equals \( 5 \times 10^{10} \text{cm}^{-3} \).

Adducing analysis of factors, which determine the electron density in accelerated bunches, shows that at low cathode current density parameter considered is conditioned by both electron density near cathode and phase bunching in a linac. When cathode current density increases and electron density at the input of accelerating system reaches approximately \( 5 \times 10^{10} \text{cm}^{-3} \) the value of parameter under consideration at the linac output is determined mainly by initial near-cathode electron density for bunch compressing is difficult because of strong Coulomb repulsion.

**Conclusion**

The following conclusions can be made:

1. Electron density of bunches, accelerated in linacs, was found not to exceed \((2 - 3) \times 10^{10} \text{cm}^{-3}\).

2. Experimentally received value of electron density was explained by means of the following factors: electron density near cathode, longitudinal beam bunching and Coulomb repulsion influence.

References

The diagram characterizing electron density of accelerated beams for linacs in operation.