IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

THE EFFICIENCY OF ACCELERATION BY PHASE DISPLACEMENT IN THE PRESENCE OF RF NOISE

Stephen Myers and Ernst Peschardt*

Summary

During acceleration by phase displacement the particle synchrotron motion is non-linear around the exterior of the RF bucket. Simultaneous numerical evaluation of this motion for a large number of particles allows calculation of relevant beam statistical parameters which can be used to define the relative efficiency of the acceleration technique. This procedure has been used in a computer program which calculates the reduction in efficiency of acceleration by phase displacement in the presence of RF noise and when the bucket parameters vary during the RF sweep. The results show that the increase of \( \gamma_t \) with \( \Delta p/p \) produces a significant increase in the momentum blow-up per accelerating sweep. It is also shown that RF noise (frequency modulation) causes particles to penetrate the bucket separatrix and become decelerated within the bucket. This results in a significant reduction in the efficiency of acceleration and greatly contributes to the intensity losses. Extensive experimental results have confirmed the computed results.

1. Introduction

In the ISR beams of very high intensity (up to 35 A) are routinely accelerated by phase displacement from 26.6 to 31.4 GeV/c\(^2\). The efficiency of this acceleration process may be defined by three principal parameters:

- (i) The increase of the rms momentum width of the beam (\( \delta_{\text{rms}} \)) per sweep,
- (ii) The momentum increase \( \Delta p \) of the center of gravity of the stack per sweep and
- (iii) The intensity loss per sweep due to particles which become trapped inside the RF buckets.

A computer simulation\(^3\) of phase displacement on a beam with zero initial momentum spread showed

\[
(\delta_{\text{rms}})^2 = \frac{1}{2} \sigma_{\theta}^2 \quad \Delta p = C \delta_{\text{rms}}
\]

where \( \sigma_{\theta} \) is the stable phase angle and \( C \) is the mean momentum spread of a stationary bucket having otherwise the same RF parameters. It is also well known that the ideal momentum gain resulting from a phase displacement sweep is

\[
\Delta p = \frac{p_c}{2\pi} \quad \Delta p = C \delta_{\text{rms}}
\]

Recent measurements showed significant discrepancies with calculations based on equations (1) and (2). Preliminary experiments indicated that the momentum blow-up (\( \delta_{\text{rms}} \)) and the intensity loss per sweep were strongly affected by the injection of band-limited white noise to the RF system.

2. Computation of Efficiency Parameters

The motion of a particle in longitudinal phase space is described by

\[
\frac{\partial r}{\partial t} = \Omega \sin \phi, \quad \frac{\partial \phi}{\partial t} = \omega + \sin \phi - \sin \phi \sin \phi
\]

where \( \phi \) is the phase of the RF waveform and \( \Omega = p / a \) is the synchrotron frequency for oscillations of small amplitude.

Solution of the above equation is normally restricted to small amplitude oscillation inside the RF bucket and with \( \omega \) and \( \Omega \) being constant. However, for the case of phase displacement of stacks with a large relative momentum spread the synchrotron motion may involve both large and small amplitude oscillations as well as time-dependent coefficients \( \omega(t) \) and \( \Omega(t) \). For this case, equation (3) is analytically insoluble but may be solved (as a function of time) numerically. The effect of frequency modulation (RF noise) may be computed by making \( \omega(t) \) time dependent.

Frequency spectra measurements made in the ISR showed that the inherent modulation due to 'noise' was very narrow band at the mains frequency (50 Hz) and its harmonics. Hence a sinusoidal noise variation was used for the analysis

\[
\phi_n = \phi_{n0} + \phi \sin \omega t
\]

where \( \omega \) is the frequency of the 'noise'.

Phase displacement acceleration is particularly advantageous for stacks with a large momentum spread. As a consequence of the large range (in \( \Delta p/p \)) covered by the RF bucket, the variations of bucket parameters with \( \Delta p/p \) must be taken into account. In particular measurements have shown\(^6\) that the \( \gamma_t \) varies in the ISR as

\[
\frac{\partial \gamma_t}{\partial t} = - \frac{\Delta p}{p}
\]

Such a variation causes all RF parameters to vary during the scan of the beam, e.g. the bucket area \( A_{b} \), the synchrotron frequency for small amplitudes \( \Omega_{b} \), the \( f(\sin \phi) \) also changes due to its dependence on \( df/dt \).

It is also important to know when a particle crosses the bucket separatrix either into or out of the bucket. This can be done by comparing the position of the particle in phase space with the position of the separatrix,

\[
\phi_{n0} + \phi \sin \omega t = \phi_{n0} + \phi_{b} \sin \omega t = - \phi_{b0} \sin \omega t
\]

where \( \phi_{b0} \) is the unstable fixed point \( (3\pi - \phi_{b}) \).

The number of particles which remain inside the bucket as it is decelerated across the lower momentum aperture limit gives the intensity losses caused by the sweep.

The computations are initialized by representing the coasting beam by a large number of points in the \( \phi, \theta \) plane. The points are distributed randomly between \( 0 \) and \( 2\pi \) in \( \phi \) and distributed in \( \theta \) according to the initial density distribution of the beam. For the ISR a uniform distribution is a reasonable approximation. After initialization, the position of each particle \( \{ \phi_i, \theta_i \} \) is calculated as a function of time by solution of equation (3) by a 4th order Runge-Kutta numerical method. At regular time intervals the statistical parameters related to efficiency are calculated by converting the \( \bar{\phi}_i \) to the more measurable quantity \( \bar{\phi}_i \)

\[
\bar{\phi}_i = - \frac{\Delta p}{2\pi} \left( \phi_{i0} - \phi_{b0} \right)
\]
where $p_0$ is the momentum at the centre of the RF bucket.

At each time interval, the mean and rms of the momentum distribution are calculated as well as the number of particles inside and outside the bucket. The momentum blow-up is calculated from the usual quadratic addition

$$\Delta p^2(t) = \Delta p^2(u) + \Delta p^2(t)$$

The momentum gain is simply

$$\Delta p(t) = \bar{p}(t) - \bar{p}(0)$$

By defining the time (or $\Delta p/p$) at which the end of the scan occurs, the statistical parameters for a complete sweep are evaluated.

The experimental measurements of the pitch momentum width and centre of gravity were performed by computer analysis of the longitudinal Schottky scan. The measurement of the number of particles which are inside the RF bucket at the end of the scan was performed by suitable triggering of the dc beam intensity monitor just before and just after the RF bucket had reached the inner aperture limit. Each experimental point is the result of many phase displacement scans. The RF 'noise' was input as shown in Fig. 1.

The variation of the efficiency parameters with the amplitude of the RF noise is shown in Fig. 3.

The computed results are in good agreement with those measured. The momentum blow-up increases linearly with the noise amplitude. Fig. (a) shows the relative momentum increment ($\Delta p/p$) decreasing with noise amplitude.

3. Discussion of Results

In Fig. 2 the computed momentum blow-up is plotted as a function of $\gamma$, also shown is the theoretical momentum blow-up expressed by equation (11). It can be seen that for the case of no noise and no $\gamma$, variation the computed results agree very well with the results from existing theory. In addition the computed momentum gain per sweep ($\Delta p$) corresponds to the theoretical value with a high degree of accuracy. These results confirm the accuracy of the computational technique. Also shown in Fig. 2 is the computed variation of $\Delta p_{\text{rms}}$ when the $\gamma$ varies across the machine momentum aperture. It can be seen that for lower values of $\gamma$ (the operational value is 0.1) a significant increase in the momentum blow-up is produced. Blow-up rates corresponding to the computed values have been measured during normal acceleration in the ISR. At higher $\gamma$ values the computed blow-up rates converge towards the theoretical value even when $\gamma$ varies. A $\gamma$ value of 0.25 was chosen for all tests with RF noise. Higher values of $\gamma$ are of no real importance.

The computed results are in good agreement with those measured. The momentum blow-up increases linearly with the noise amplitude. Fig. (a) shows the relative momentum increment ($\Delta p/p$) decreasing with noise amplitude. Both of these effects may be explained qualitatively by the fact that the RF noise causes particles to enter and spend some time inside the RF bucket. During the time spent inside the bucket these particles are decelerated rather than accelerated. This behaviour will obviously increase the momentum width artificially and bias the centre of gravity of the stack towards lower momentum. Plot (c) shows the increase of internally losses per sweep with the noise amplitude. The agreement between computed and measured results is surprisingly good in view of the poorer statistics. More interesting is the behaviour of the efficiency parameters with the frequency of the 'noise' (Fig. 4). Both measured and computed results show that the reduction in efficiency is maximum at noise frequencies just below the synchrotron frequency for small amplitudes (around 0.8 $f_1$). Based on these measurements, a large effort was concentrated on the reduction of the inherent noise at 50 Hz by building a completely new low-noise electronic control for phase displacement. At the same time the RF parameters for phase displacement were chosen so as to place the dangerous frequencies (6.6 $f_1$) as far as possible from the prevailing (50 MHz) main frequency. These measures resulted in a significant improvement in the efficiency of acceleration.

It is also interesting to observe the notion of individual particles which penetrate the separatrix. An example of a particle which remained inside the RF bucket for a significant number of phase oscillations is shown in Fig. 5 and Fig. 7.

Fig. 5(a) shows the trajectory of a particle as it approaches and becomes trapped inside the bucket. This particle spirals inwards until it approaches the centre of the bucket, then its motion changes and the particle spirals outwards again (Fig. 5(b)). The same motion is shown in Fig. 7 as a function of time. The amplitude
the synchrotron oscillations decreases as the frequency of the synchrotron oscillations increases.

The bucket is in the exact middle of the stack. In the absence of RF noise (Fig. 6(a)) the momentum width stabilises after the bucket leaves the stack. However, in the presence of RF noise (Fig. 5(b)) the particles continue to drop out of the bucket for some time after exit from the stack, causing an increased momentum width.

4. Conclusions

The efficiency with which acceleration by phase displacement is performed can be greatly influenced by FM of the RF ('noise') and by variations in $\gamma_f$ with $d\gamma_f/d\phi$. RF 'noise' at frequencies around 0.8 times the synchrotron frequency for small oscillations is particularly harmful. These effects can be evaluated with good accuracy by performing numerical solutions to the equations of synchrotron motion. RF 'noise' causes some particles to enter the RF bucket, then spiral towards its centre and back towards the exterior of the bucket. This type of motion can continue for several periods and produces a reduction in the efficiency of the acceleration.

5. References

2. C. Fischer et al; "Performance of the ULMN ISR at 31.4 GeV/c" (to be presented at this Conference).
6. E. Ciapala, A. Hofmann and S. Myers; "The variation of $\gamma_f$ with $d\gamma_f/d\phi$ in the CERN ISR" (to be presented at this Conference).