null field in this region. Of course, only the first regime is of interest. The two corresponding resonant frequencies can be approximately determined from the lumped circuit representation shown in fig.2. If $L_1$ represents the inductance of the two opposite magnetic regions, $L_2$, the inductance of the central, i.e., coupling region and $C_1$, the capacitance of the two identical electric regions, these parameters can be related to the different dimensions shown in fig.1 by:

$$L_1 = \frac{\mu_0 \omega L_1}{2 \pi R}$$

$$L_2 = \frac{\mu_0 \omega (r + 1/2 \lambda \tan \delta)}{2 \pi R}$$

$$C_1 = \frac{\omega \mu_0 R L_2}{d}$$

Then, the two resonant frequencies are given by:

$$f_1 = \frac{1}{2 \pi} \sqrt{\left(1 + \frac{\omega_0}{L_1 C_1}\right) \cdot \frac{1}{L_1 C_1}}$$

$$f_2 = \frac{1}{2 \pi} \sqrt{\frac{1}{L_1 C_1}}$$

By studying the current distribution without losses in the equivalent circuit, one can verify that $f_1$ and $f_2$ correspond respectively to maximum and null current in $L_2$. Thus, the only difference between the classical biperiodic structure and the proposed structure lies on the existence of a second resonance in the coupling cell. In order to see the effect of this resonance, let us examine the dispersion relation of the structure, by using a lumped circuit model. It can be shown that:

$$\cos \phi = -1 + \frac{1}{k^2} \left(1 - \frac{L_0^2}{L_2^2}\right) \left(1 - \frac{L_1^2}{L_2^2}\right) \left(1 - \frac{L_2^2}{L_1^2}\right)^{-1}$$

Where $\phi$ is the phase shift of a period, $f_0$ the resonant frequency of the accelerating cell and $k$, the coupling coefficient to which the relative bandwidth can be related by:

$$BW = \frac{k L_1}{k^2 + (B - 1/2)^2} \left(1 - \frac{1}{B^2}\right)^{1/2}$$

Where $k$ is defined as $(1 + L_1^{-2})^{-1}$ and then smaller than 1, or:

$$BW = \frac{k L_1}{1 + \frac{1}{B^2} \left(\frac{L_1}{L_1 + L_2^{-1}}\right)^2} \left(1 - \frac{1}{B^2}\right)^{1/2}$$

provided $K^2$ is small compared to 1.

As $K$ is defined as $n/(2 L_0 L_2)$, a representing the mutual inductance between coupling and accelerating cells and $L_0$, the equivalent inductance of this latter relation (5) shows that, in order to obtain a wide bandwidth, $L_2$ must be chosen small enough compared to $L_1$, but large enough however to provide sufficient space for coupling apertures.

By the same analysis, it can be shown that the ratio between the frequency of the lowest mode of the pass-band and the second resonant frequency of the
coupling cell is approximately equal to $k^{-1/2}$ and so, the existence of this resonance has no influence on the pass-band as long as $L_2$ is not too large compared to $L_1$. This condition is furthermore consistent with a large band-width requirement. In the practical case of fig. 1 where $k$ is chosen equal to 0.59, this ratio is 1.3 and the BW very close to $k$. These above considerations can help for the optimal choice of the structure geometry.

**Triperiodic arrangement**

The design shown in fig. 1 corresponds to a biperiodic cell of $S = 1$. When small overall diameter is desired and especially in case of low-$\beta$ structure, the triperiodic arrangement may be used. The structure is then made from elements of two kinds as shown in fig. 3. Elements with coupling cell will have identical geometry and length while elements without coupling cell will have lengths variable according to the $S$-value. In this arrangement indeed the length of the element of the first kind can be chosen longer than a half wave length, leading to a small $r$-value figured in relation (1) and hence to an overall reduced diameter.

**Practical design**

According to size and weight, two fabrication techniques can be used. For $S$-band or $X$-band structure, the whole structure or sections can be assembled by piling elements inside a cylindrical envelop and brazing at one time. As the envelop is vacuum tight, only low temperature brazing will be needed. Each element will need only a lathe machining. An alternative technic consists in realizing the envelop by electroforming, the ridged geometry of the coupling cell having been filled up previously with wax. For $L$-band structure, because of large dimension and weight, the best way would be to achieve tooling, assembling, brazing and tuning cavity by cavity and to assemble the whole structure by a second brazing. In the first case, cooling circuits can be realized by tubings welded at low temperature along the envelop and, in the second case, by hollow rings welded around the structure in grooves housed between elements.

A typical device for rf-power input is shown in fig. 4. **Model test results**

After the principle had been well verified on a simple shaped model with two periods, a second three period model with actual shape, as shown in fig. 1, was realized recently in view of testing the rf-behavior of structure and also the fabrication technique described above. For commodity of measurement equipments available, the model has been designed in $S$-band. Fig. 5 shows a picture of a disassembled cavity and an assembled one. **RF-Test**

On of the most interesting feature of the proposed structure is its capacity of wide band-width. Fig. 6 shows the dispersion curve. A typical band-width of $13\%$ is obtained with two diametrically opposite coupling apertures of 10 mm wide and 70° long. This figure might be increased by choosing larger aperture, but possibility of direct coupling between accelerating cells and mechanical stiffness of the elements must be kept present in mind.

**Fabrication technics**

The model has been made following the first technic described above for $S$-band structure. Some precautions have to be taken to prevent mechanical deformation during tooling and assembling. The coupling apertures, for instance, must be cut only at the last moment. In order to take into account the envelop roundness defects, sufficient clearance has to be provided between elements and envelop, unless these elements had been frozen before introduction.

**Conclusion**

These first tests show that this type of structure, can answer to the requirements stated above. The overall diameter does not exceed 12 cm at 3 GHz, the band-width is large and the fabrication processes are proving easy. In order to test the actual behavior of this kind of structure against mechanical errors, a long section is being designed. As the mechanical tolerance is getting tight at high frequency, the frequency will be chosen in $X$-band.

**References**

Fig. 2 Lumped-circuit representation of the coupling cavity.

Fig. 3 Triperiodic arrangement: $B = 0.7$.

Fig. 4 Power-input port.

Fig. 5 Photograph of a disassembled and an assembled cavity.

Fig. 6 Dispersion curve.