In a proton linac, a change-over to a shorter-wavelength accelerating field is made at a certain energy (of the order of 100 or 200 MeV). With the wavelength reduced by a factor of $n$, the phase width of bunch if scaled to this shorter wavelength becomes expanded by the same factor of $n$ while the longitudinal phase acceptance remains essentially equal to that in the initial, i.e., longer wavelength stage. In view of the rather high beam intonics in modern proton linacs this necessitates that the most careful consideration be given to the problem of beam loss reduction.

The requirement of beam loss minimization calls for complementing the classical methods of beam dynamics analysis by the more sophisticated Monte-Carlo method of computer modeling to understand better the effect of random perturbations in longitudinal motion on beam parameters.

The results of such modeling have revealed that both automatic correction of bunch center of mass position in the phase-space plane and phase-space volume transformation must be provided in a high-energy linac.

### Mathematical Model

Two computer programs were used for mathematical modeling, one to compute accelerating channel parameters and the other to compute particle trajectories of a sufficiently large particle ensemble. The representative points in the phase plane corresponding to the linac input either filled uniformly a given region of the plane or were formed from a continuous beam by a single-gap buncher into a bunch with a given velocity spread.

Particle trajectories were computed in a single-particle approximation taking into account the non-linear and non-conservative nature of longitudinal oscillations. A uniform field amplitude within cavity gaps and a zero field inside drift tubes were assumed.

Random perturbations of longitudinal motion were investigated by the statistical method (Monte-Carlo method). Fluctuations of RF field parameters and manufacturing and misalignment errors would be simulated within given tolerances by a random number generator. This would produce one possible channel realization. Then a new channel realization would be obtained from a new set of random numbers and so on. Output particle distributions for a large number of realizations were then processed statistically.

A 200-MeV proton linac consisting of seven drift-tube-loaded cavities operating at a 1.5-m wavelength has been chosen for modeling. A designed synchronous phase of $26^\circ$ and a normalized acceleration rate of $2.7 \times 10^{-3}$ were taken. Also the channel errors as listed in the Table below were assumed. These are conventional manufacturing tolerances for a strong-focusing linac.

### Table

<table>
<thead>
<tr>
<th>Error Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift tube length tolerance, $%$</td>
<td>1</td>
</tr>
<tr>
<td>Drift tube misalignment, m</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>RF field nonuniformity along cavity $%$</td>
<td>1</td>
</tr>
<tr>
<td>Field tilt along cavity, $%$</td>
<td>1</td>
</tr>
<tr>
<td>RF phase mismatch between cavities, grad</td>
<td>1</td>
</tr>
</tbody>
</table>

### Results of Modeling

Figs. 1 and 2 show variation of bunch parameters along the accelerator for an input bunch formed by a single-gap buncher from a continuous beam with a velocity spread of $\pm 0.5 \%$. The buncher parameters have been chosen to provide the maximum capture ratio. Similar curves were obtained for a beam filling uniformly the bucket at the accelerator input. The phase and velocity distributions at the output of the seventh cavity (summed over 40 random realizations) are compared with those in an ideal channel in Fig.3. With the tolerances assigned and uniform initial filling of bucket the width of distribution increases by a factor of 1.8 along each of the coordinate axes. Distribution curves in a perturbed channel differ from those in an ideal channel in that their slope is quite small due to which fact only 20-25 per cent of particles eventually come out of the ideal distribution width despite that the latter becomes increased by as much as 80 per cent.

Thus the modeling has shown that the strong tolerances for RF amplitudes and phases (of the order of 1 $\%$ and $4^\circ$, respectively) were insufficient and that the bunch c.o.m. displacement at the model output might become comparable with the bunch size. Because of the n-fold change of the phase scale at the transition between the two stages the effective bunch phase width may appear greater and the momentum spread may appear less than the longitudinal phase acceptance. Therefore it becomes necessary:

1. to reduce the effective phase-space area of bunch by suppressing bunch c.o.m. longitudinal oscillations,
2. to compress the bunch in phases.
(by stretching it in momenta) in order to match it better with the bucket shape corresponding to the second stage of the linac.

Coherent-phase-oscillations suppression system

The first of these problems has been solved by a system designed to suppress coherent-phase (longitudinal) oscillations. This coherent-phase-oscillations suppression system operates as follows. Suppose that the bucket center of mass at the output of the last but one cavity has phase and momentum displacements \( \Delta \Phi \) and \( \Delta P \), respectively (Fig.4a). A phasemeter located between the last but one cavity and the last one of the first stage measures \( \Delta \Phi \), and its output is used to shift the RF phase in the last cavity of the first stage by \(-\Delta \Phi\). This corresponds to a translation of the phase plane coordinate origin (Fig.4b). The cavity length is taken to be a quarter of the longitudinal oscillation wavelength and the representative point while moving along a phase trajectory (an ellipse in the case of small oscillations) comes to a point on the abscissa axis at the time corresponding to the bunch leaving the last cavity (Fig.4c).

The output of another phasemeter measuring the \( \Delta P \) displacement is used to shift the RF phase in the cavities of the second stage by \(-\Delta P\). As a result of the latter correction the representative point hits the coordinate origin (Fig.4d) and the bunch center ceases to move. In fact, since the last cavity of the first stage continues to store phase errors and since the longitudinal oscillations are nonlinear the c.o.m. momentum will differ from an ideal one by a relatively small value.

There had been some apprehensions before modeling works began that the coherent-oscillations suppression system would appear ineffective because of the fact that the nonlinearity of longitudinal fields should change the bunch configuration and increase its effective phase area.

However, a study of bunch deformation in the phase plane carried out by the Monte Carlo method has shown the bunch shapes in the phase plane to differ little from an elliptical one and their center-of-mass to be close to the synchronous particle even with the tolerances more than twice as high as in the Table above, with further increasing the tolerances a bunch shape in the phase plane is becoming less and less elliptical and in some cases it eventually fails to be a compact formation. This in general the apprehensions concerning potential bunch spreading over the phase plane seem to have grounds; nevertheless, with the present-day tolerances these effects do not manifest themselves and the coherent-phase-oscillations suppression system operates quite effectively. Fig.5 shows several typical bunch shapes obtained in some random realizations of channel with different tolerances.

To estimate the efficiency of the coherent-phase-oscillations suppression system the linear model included in the model of accelerator channel. The modeling program simulated operation of this system and output phase and velocity distributions for 40 random channel realizations, each with 125 accelerated particles, have been obtained and compared with the distributions of the absence of this system. The comparison of the distributions of Fig.6 shows that with conventional tolerances, the oscillation suppression system eliminates 50% of the total increase in the momentum spread and 80% of the increase in the bunch phase width.

Matching the channels of the two stages

The second of the above problems, i.e., that of bunch transformation is solved by choosing the parameters of the last cavity in the first stage so that to convert it essentially into a matching device. The longitudinal oscillation frequency in this cavity is increased with respect to the frequency in the last but one cavity by about a factor of 1.5 due to an increase of the synchronous phase up to 60°. The operation principle of the matching device illustrated in Fig.7. Consider a coordinate plane \((\psi, \phi, \Phi, \phi_m, \Phi_m)\) where \( \psi \) and \( \phi \) are particle phase and momentum, respectively, and \( \Phi \) and \( \phi_m \) are bunch c.o.m. phase and momentum, respectively. Suppose that the bunch particles at the input of the last cavity are positioned inside the shaded ellipsis A. Due to the high frequency of longitudinal oscillations in this cavity the representative points move in the course of acceleration along phase trajectories in the form of ellipses that are stretched more along the ordinate axis than the bunch envelope ellipsis. This is due to the nonconservative nature of the longitudinal oscillations in this cavity. The bunch c.o.m. momentum will differ from an ideal one by a relatively small value.

The modeling has shown that the nonlinear and nonconservative nature of the longitudinal motion as well as its random perturbations do not affect essentially the transforming properties of the matching cavity.

Various devices for matching accelerating channels have been known. However, they required a third intermediate wavelength. Therefore, additional RF amplifiers, accelerating systems, and other hardware operating at the intermediate wavelength were necessary for these devices. In this respect, the above-described matching device is advantageous in that it operates at the wavelength of the first stage.

Conclusion

Modeling of a perturbed longitudinal motion has confirmed the idea that it would be reasonable to use beam-based control systems as well as matching devices not used before in proton linacs.

From the results obtained is expedient to say that a coherent oscillations suppression system may be useful to solve at least two problems in a proton linac. First, in a
high-energy accelerator in which a change -
over to a shorter RF field wavelength is to
take place at a certain energy (of the order
of 100 MeV) this suppression system will make
it possible to reduce the effective phase
width of bunch at the second stage input, to
improve the capturing conditions, and to re-
duce particle losses. Secondly, this suppres-
sion system may be used at the linac output
with a resulting improvement in the beam
energy spectrum.

A matching device similar to that consid-
ered above makes it possible to vary the lon-
gitudinal bunch size and the momentum spread.
In principle, this device is applicable in all
those cases where a necessity arises of a li-
near transformation of bunch phase-space vo-
lume. This device can be used for matching the
accelerating channels of the two stages of a
two-stage high-energy proton linac. In an ac-
celerator with a high time-averaged current
(for a meson factory or a neutron generator),
channel matching will result in smaller par-
cicle losses, a better operation reliability,
and a higher acceleration efficiency. Also,
the matching of accelerating channels seems
potentially helpful in a reduction of moment-
um spread at the output of a linac injector
for a proton synchrotron.

References
1. V.G. Andreev, E.L. Burshtein, V.G. Kulman,
   on High Energy Accelerators, Frascati,
   1965, p. 634.
2. A.D. Vlasov, Theory of Linear Accelerators,
   Atomizdat, Moscow, 1965.

Fig. 1 - Bunch phase length at outputs of dif-
ferent cavities: 1 - maximum bunch ef-
effective size, 2 - rms bunch effective
size, 3 - maximum bunch width, 4 - average
bunch width, 5 - minimum bunch width,
6 - maximum bunch phase spectrum displa-
cement, 7 - rms bunch c.o.m. displace-
ment (in Figs. 1 to 3 and 5 - averaged
over 40 realizations of channel).

Fig. 2 - Velocity spread at outputs of differ-
ent cavities: 1 - maximum effective velo-
city spread, 2 - rms effective velocity
spread, 3 - maximum velocity spread, 4 -
average velocity spread, 5 - minimum ve-
locity spread, 6 - maximum bunch c.o.m.
velocity deflection from synchronous
particle velocity, 7 - rms bunch c.o.m.
velocity deflection.

Fig. 3 - Velocity and phase distributions in
accelerating channel with random per-
turbations (solid curve) and in ideal
channel (dashed curve), 200 MeV energy
Fig. 1- Block-diagram and operation principle of coherent-phase-oscillations suppression system.

Fig. 5- Phase plane bunch configurations at model output in different random realizations of channel:
   a) errors as given in Table,
   b) errors doubled,
   c) errors increased by a factor of 4;
   @ - synchronous particle

Fig. 6- Velocity and phase distributions in accelerating channel with random perturbations with (solid curve) and without (dashed curve) coherent-phase-oscillations suppression system, 200 MeV energy.

Fig. 7- Matching the accelerating channels of the two stages of linac.