In electron Linacs, the radial component of the rf accelerating electromagnetic wave and space charge both tend to increase the radius and divergence of the electron beam. These defocusing forces are commonly counteracted by the use of magnetic lens and solenoids. The use of Linacs as portable, high energy radiographic sources, however, necessitates that the weight and power requirements of each component be reduced to a minimum. This has led to the concept of periodic permanent magnet focusing as a replacement for the conventional magnetic lens and solenoids.

In this study the conventional copper discs of the accelerating waveguide have been replaced by a magnetic material which is plated with copper to provide the required resistivity. Permanent magnets are placed around the outside of the accelerating structure to provide a periodic magnetic system capable of focusing the injected beam.

A computer program has been developed for the iterative solution of the relativistic equations of motion which give the electron trajectories in the periodic field during the acceleration process.

The influence of periodicity and axial magnetic field strength on the beam radius for accelerators operating at 6,000 Mc/sec, and 10,000 Mc/sec are discussed.

Materials considerations and mechanical design techniques are discussed briefly.

Introduction

In order to overcome the effects of the rf radial defocusing forces, space charge and initial angular divergence of the electron beam in the Linac some form of focusing is required. This is commonly provided by an axial magnetic field provided by a solenoid formed around the outside of the accelerating structure. While, in general, this is a perfectly satisfactory method of providing the necessary focusing, portable accelerators necessitate serious consideration of alternate techniques. The main disadvantages associated with solenoids in this application are their weight, electrical power demand, and cooling. One method that can be used to overcome this disadvantage of large weight and power demand is to use permanent magnet focusing. It has been shown by Clegeton and Hoffner that it is possible to overcome the effects of space charge and focus with periodic fields. In addition other early work in this field was carried out by Pierce, Tien, and Mendel. All these analyses however pertain to microwave tubes in which the beam energy is low (< 100 kV) and consequently essentially nonrelativistic. This allows the theory to be handled in terms of a series of lens in a manner analogous to an optical problem. In the case of the accelerator no such simplifications can be made since the beam becomes relativistic after traveling a very short distance. Relativistic mechanics must therefore be used throughout and the coupling between longitudinal, radial and azimuthal momentum must be carefully considered. The question still remains however, can this periodic focusing be used to focus a highly relativistic electron beam subjected to radial defocusing electric fields. Assuming that this, in fact, can be accomplished what weight savings will accrue?

In order to focus a given beam with a periodic magnetic field it has been shown that it is necessary to use an rms value of the alternating field which is equal to the Brillouin field required to focus the same beam. Therefore since the effective field strength is not changed, only a minor improvement could be expected if a series of electromagnets were used instead of a solenoid. However, Pierce and Mendel have pointed out that the weight of a permanent magnet required to produce an alternating field of a given rms value is much less than the weight of the magnet required to produce a uniform field. Theoretically the weight saving which accrues from the use of periodic focusing instead of solenoidal focusing is \( N^2 \), where \( N \) is the number of sections in the periodic field. In practice the savings is somewhat less than \( N^2 \) and lies between \( N \) and \( N^2 \). Clearly, however, a substantial savings in weight can be obtained if the periodic magnet focusing system can be employed. Furthermore the savings in weight can be further enhanced by the simple expedient of using the discs in the accelerating structure to guide the magnetic field. A permanent magnet material can be plated with copper or silver to provide the necessary low resistivity for the accelerator walls. Since the discs must be present whatever system of focusing is used we are able to provide the necessary magnetic field at very little increase in the weight of the system over the case where no magnetic focusing at all is provided. The final question that must be answered is what is the amplitude of the magnetic field to produce the necessary degree of focusing?

To attack the problem of periodic focusing and in order to determine the periodicity and magnetic field strength required to contain an electron beam the radial defocusing forces must first be determined. These radial forces result from the radial electric field of the accelerating rf wave, from the mutual repulsion of the electrons within the beam (space charge) and from any centrifugal forces which arise from the angular momentum which the electrons acquire in the process of acceleration and focusing. In the analysis developed in the following section space charge forces have been neglected since they only become important for large (ampere) values of the beam current. The effect can be included, however, with no basic change in the approach.
It should be recalled that once the operating frequency and characteristics of an accelerator have been chosen the inner diameter of the discs in the structure are fixed. In the case of the X-band accelerator this diameter is approximately 7 mm, and in the case of a C-band accelerator it is approximately 12 mm. To obtain good beam transmission the radius of the beam at the end of the accelerator must be less than these dimensions. In order to determine the radial dimensions of the beam under varying conditions of applied magnetic field as a function of position along the accelerator axis a computer program FOCUS has been developed. The theory on which the program is based is presented in the following sections.

**Theory**

It will be recalled without derivation that the radial rf defocusing force in an electron Linac is given by,

\[ F = \frac{eE\pi}{\lambda} \left[ 1 - \beta \right] \cos \Delta \]

where \( \Delta \) is the phase angle between the wave and the electron. With no azimuthally varying magnetic field, it can be shown that

\[ F_\theta = -e(v B_x - v z B_z) \]
\[ F_r = -e(v e B_z - v B_r) \]
\[ \psi = r v z \frac{d \theta}{dz} \quad \text{since} \quad \theta = v \theta / r \]
\[ \beta = \frac{d(m v z)}{dt} - m z \beta^2 \]
\[ \psi = m v o z \frac{d \psi}{dz} - \frac{m o \psi v^2}{r} \]

since \( m = \gamma m_0 \) equating 2 and 4 gives

\[ -e(v \theta B_z + B_r) = m o v \frac{d}{dz} (v \psi) - \frac{m o \psi v^2}{2} \]

similarly,

\[ \frac{d^2 \psi}{v z \frac{d}{dz} + m v z \psi} \]
\[ \frac{d \psi}{r} \quad \text{since} \quad \theta = v \theta / r \]
\[ \beta = \frac{d(v z B_z - B_r)}{d} \]

and

\[ \frac{d \psi}{v \theta z d \phi + v v m \frac{d \theta}{dz}} \]
\[ \frac{d \psi}{r} \quad \text{since} \quad \theta = v \theta / r \]

in addition we have that

\[ \frac{dm}{dz} = \frac{m o}{\beta} \left( \frac{\frac{d}{v r d z} + \frac{d \theta}{d z} + \frac{d v}{z d z}}{1 - \left( \frac{v + v z + v^2}{\beta} \right)^2} \right) \]

so

\[ \frac{dv}{dz} = \frac{3}{e^2} \left[ \frac{dv}{v r d z} + \frac{dv}{\theta d z} + \frac{dv}{z d z} \right] = 0 \quad \text{[8]} \]

Now,

\[ E_z = E_o e^{-az} \sin \left( \frac{2\pi o}{\lambda} \right) \int_0^z \left( \frac{c}{v z} - \frac{c}{v \text{phase}} \right) dz \quad \text{[9]} \]

and

\[ F_r = \frac{\pi E o e^{-az}}{\lambda} \cos \left( \frac{2\pi o}{\lambda} \right) \int_0^z \left( \frac{c}{v z} - \frac{c}{v \text{phase}} \right) dz \quad \text{[10]} \]

Equations 5, 6, 7, and 8 together with Eqs. 9 and 10 are the fundamental equations from which the electron trajectories in the magnetic field are calculated. The initial beam conditions of velocity and radius and the accelerator parameters of wavelength, field strength and attenuation must be specified. The required functional dependence of the magnetic focusing field on distance and radius is then provided as the input parameter. The step-by-step solutions of Eqs. 5, 6, 7, 8, 9, and 10 then provide the solution to the electron orbits.

**Results**

The first test program to be run involved the input of a dc magnetic field (i.e., a solenoidal field) with no radial defocusing forces. The electrons were then given an initial radial momentum with a particular value of the initial radius. The trajectory of the electron under these conditions is well known and can be determined analytically. The orbit is of course a helix whose pitch angle is determined by the value of the radial momentum and magnetic field. This trial run has been completed and the results found to be compatible with the analytical predictions.

Data has now been obtained for accelerators operating at X-band and C-band. In summary for the X-band accelerator, for an initial radius of 2.5 mm and a periodic field of maximum amplitude 1500 gauss the maximum radius attained was 4.4 mm. While this is somewhat greater than the allowable radius the field configuration was not optimized. This is presently being undertaken and preliminary results indicate that no difficulty will be encountered in transmitting the beam through the structure at this frequency.

To ensure that the C-band accelerator was also capable of transmitting the electron beam with reasonable values of axial magnetic field a run was made in which a field strength parameter of \( q = 0.97 \) was used. From an initial radius of 2.5 mm the maximum value that the beam radius reached was 2.8 mm for a peak magnetic field of 1500 gauss. Since the inner diameter of the discs in this accelerator is approximately 12 mm it can be seen that no difficulty would be encountered in transmitting the beam with this periodic focusing system.