Self-Consistent Computation of Electromagnetic Fields and Phase Space Densities for Particles on Curved Orbits

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1. Vlasov-Maxwell with Shielding: Planar Case
2. Maxwell in Lab Frame with fixed vertical density Field Formula and its computation
3. Vlasov in Beam Frame
   Beam Frame Phase Space Density and Relation to Lab Charge/Current Densities
4. Two Numerical Approaches:
   Method of Local Characteristics, in progress
   Self-Consistent Monte Carlo, code developed
5. Self-Consistent Monte Carlo Results
Planar Vlasov-Maxwell System in Lab Frame

\[(\triangle - \partial_u^2)\mathcal{E} = H(Y)S(R, u),\]
\[\partial_u\psi + \dot{R} \cdot \nabla_R \psi + \dot{P} \cdot \nabla_P \psi = 0,\]
\[\mathcal{E}(Y = \pm h/2) = 0,\]
\[\mathcal{E} = (E_Z, E_X, B_Y)\]
\[u = ct, \ R = (Z, X)\]
\[P = m\gamma V\]

- Vertical Charge Density $H(Y)$ is fixed
- $\dot{R}$ and $\dot{P}$ are given by the Lorentz Equation
Planar Vlasov-Maxwell Continued

- Continuity Equation $\iff$ Vlasov Equation
- Poynting Theorem and Energy Conservation, and CSR vs. ISR
  Heinemann, et. al. FRPMN101 Friday
- 1D charge/current model
  Warnock FRPMS083 Friday
- Self Consistent Monte Carlo Results
  Bassi, et. al. THPAN084 Thursday
Solution of 3D wave equation with Shielding BC

Solve: \((\triangle - \partial^2_u) \mathcal{E} = H(Y)S(R, u), \quad \mathcal{E}(Y = \pm h/2) = 0\)

1. Eigen Expansion in \(Y \rightarrow 2D\) Klein-Gordon with no BC
2. Trick \(\rightarrow 3D\) Wave Equation with no BC
3. 3D wave equation solved with retarded Green’s function
4. Make temporal argument of source a variable of integration

Solution:

\[
\mathcal{E}(R, Y, u) = -\frac{1}{4\pi} \int_{-\eta}^{\eta} d\eta G(\eta, Y) \int_{0}^{u-|\eta|} dv \int_{0}^{2\pi} d\theta S(R + \sqrt{(u - v)^2 - \eta^2 e_{\theta}}, v)
\]

where \(e_{\theta} = (\cos \theta, \sin \theta)\) and no singularity.

Only assumption: Planar Motion
Approximate Field Formula

Approximations (to decrease computational time):

- Average fields over $H(Y)$ and assume $Y$ extent of beam small compared to $h$

Without shielding the formula is:

\[ E(R, u) \approx -\frac{1}{4\pi} \int_{0}^{u} dv \int_{\theta_{\min}}^{\theta_{\max}} d\theta S(R + (u - v)e_{\theta}, v) \]

1. Need to find reasonable $\theta$ limits
2. $\theta$ integration: superconvergent trapizoidal rule
3. $v$ integration: adaptive Gauss-Kronrod
\[ \mathcal{E}(R, u) \approx -\frac{1}{4\pi} \int_0^u dv \int_0^{2\pi} d\theta S(R + (u - v)e_\theta, v) \]

- circle is domain of dependence of OP at \( R \) due to source at time \( v \)
Beam Frame Coordinates

\[ R = R_r(s) + xn(s) \]
Beam Frame EOM

- **Lab to Beam** \((Z, X, P_Z, P_X; u) \leftrightarrow (z, x, p_z, p_x, ; s)\)

\[
z = s - \beta r u, \quad p_z = \frac{\gamma - \gamma r}{\gamma r}
\]

- **EOM**

\[
\begin{align*}
z' &= -\kappa(s)x \quad p'_z = F_z \\
x' &= p_x \quad p'_x = \kappa(s)p_z + F_x
\end{align*}
\]

where the self forces are

\[
F_z \approx \frac{q}{P_{rc}} (E\parallel(z, x, s) \cdot t(s) + p_x E\parallel(z, x, s) \cdot n(s))
\]

\[
F_x \approx \frac{q}{P_{rc}} (E\parallel(z, x, s) \cdot n(s) - cB_Y(z, x, s))
\]

EOM with \(F = 0\) have been linearized
Relation Between Lab and Beam Frame Densities

- **Subtle** relation between phase space densities

\[(Z, X, P_Z, P_X; u) \leftrightarrow (z, x, p_z, p_x, ; s)\]

\[F_{Lab}(Z, X, P_Z, P_X; u) = \frac{\beta_r^2}{p_r^2} f_{Beam}(z, p_z, x, p_x; s)\]

- Lab frame charge and current densities from \(f_{Beam}\)

\[\rho_{Lab}(\mathbf{R}; u) \approx \int dp_z dp_x f_{Beam} = \rho_{Beam}(z, x; s),\]

\[J_{Lab}(\mathbf{R}; u) \approx Q\beta_r c [\rho_{Beam}(z, x; s) t(s) + \tau(z, x; s) n(s)]\]

where \(\tau = \int dp_z dp_x p_x f_{Beam}\) and \(s = z + \beta_r u \rightarrow \beta_r u.\)
Method of Local Characteristics-I

2D Lab Frame Fields and 4D Beam Frame Vlasov are

\[ E(R, u) \approx -\frac{1}{4\pi} \int_0^u dv \int_0^{2\pi} d\theta S(R + (u - v)e_\theta, v) \]

\[ \partial_s f + z' \partial_z f + x' \partial_x f + p'_z \partial_{p_z} f + p'_x \partial_{p_x} f = 0, \]

Basic Idea

1. \( f, E \) and history of charge/current density are known at \( s \)
2. Freeze fields at \( s \), then Vlasov equation becomes a Liouville equation on \([s, s + \Delta]\)
3. Find \( f \) at \( s + \Delta \) by integrating backward along characteristics
4. Calculate the fields at \( s + \Delta \) from \( f \) at \( s + \Delta \) and history
5. iterate

Remarks

- Interpolation needed at step 3
Remarks

- It’s hard to imagine a better approach, since this approach preserves the geometry of the solutions.
- Method developed by Warnock and extended by Venturini in 2D
- Extended by Sobol to 4D in collective beam-beam interaction
- Less “noise” than Monte Carlo
- Our 4D work here is in progress
Outline and comparison with PIC for Vlasov-Poisson

1. Monte Carlo generation of initial conditions from $f_{Beam}(z, x, p_z, p_x; 0)$ (Similar in VP PIC)

2. Create a smooth Lab Frame charge density from scattered beam frame phase space points. We use a Fourier method used in statistical estimation. (Charge deposition in VP PIC)

3. Calculate fields from history of Fourier coefficients using our field formula (Solve Poisson Equation in VP PIC)

4. Use 3) to move the phase space points (Same in VP PIC)

5. Go to 2) (Same in VP PIC)

Unperturbed Source Model

Fields calculated up front from source evolved with no self forces, $F$.

Very fast in Gaussian case, but not self-consistent.
A parallel code has been developed and results for the Zeuthen benchmark bunch compressor will be presented in Bassi, et. al. THPAN084 Thursday.

Can follow 2D densities; 4D probably beyond current computing capability.

A few results follow (Thanks to the UNM High Performance Computing Center and NERSC)
Gaussian/ 5GeV/ 1nC/ 200 → 20 µm
Gaussian/ 500MeV/ 1nC/ 200 → 20 μm

Distance (m)
Gaussian/ 5GeV/ 1nC/ 200 → 20 µm

SCMC: 1.70 mm-mrad
UPS: 1.69 mm-mrad
Gaussian/ 500MeV/ 1nC/ 200 → 20 µm
Gaussian/ 5GeV/ 1nC/ 200 → 20 µm

\[
\rho(z_n, x_n, s)
\]

\(s=15m\)

\[
\begin{align*}
4.1 & \quad 3.6 \\
3.1 & \quad 2.6 \\
2.1 & \quad 2.1 \\
1.6 & \quad 1.1 \\
1.1 & \quad 0.6 \\
0.6 & \quad 0.1
\end{align*}
\]
\( \rho(z_n, x_n, s) \)

Gaussian/ 500MeV/ 1nC/ 200 \( \rightarrow \) 20 \( \mu \)m

\( s=12m \)
- 4.5
- 3.9
- 3.3
- 2.7
- 2.1
- 1.5
- 0.9
- 0.3

\( z_n \)
\( x_n \)

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