Experiments on Transverse Bunch Compression on the Princeton Paul Trap Simulator Experiment*

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2007 Particle Accelerator Conference

Albuquerque, NM

June 26th, 2007

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*This work is supported by the U.S. Department of Energy.
PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

Purpose: Simulate the nonlinear transverse dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems in a compact experiment.
Scientific Motivation

- Beam mismatch and envelope instabilities
- Collective wave excitations
- Chaotic particle dynamics and production of halo particles
- Mechanisms for emittance growth
- Effects of distribution function on stability properties
- Quiescent propagation over thousands of lattice periods

**Transverse compression techniques**
Magnetic Alternating-Gradient Transport Systems

\[ B_{q}^{\text{foc}}(x) = B_{q}'(z)(y\hat{e}_x + x\hat{e}_y) \]
\[ F_{\text{foc}}(x) = -\kappa_{q}(z)(x\hat{e}_x - y\hat{e}_y) \]
\[ \kappa_{q}(z) \equiv \frac{ZeB_{q}'(z)}{\gamma m \beta c^2} \]
PTSX Configuration – A Cylindrical Paul Trap

\[ e\phi_{ap}(x, y, t) = \frac{1}{2} \kappa'_q(t)(x^2 - y^2) \]

\[ \kappa'_q(t) = \frac{8eV_0(t)}{m\pi r_w^2} \]

\[ \omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2f} \]

\[ \xi = \frac{1}{2\sqrt{2\pi}} \]

\[ \tilde{\xi} = \frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}} \]

When \( \eta = 0.572 \), they’re equal.

- Plasma length: 2 m
- Maximum wall voltage: \( \sim 400 \text{ V} \)
- Wall radius: 10 cm
- End electrode voltage: \( < 150 \text{ V} \)
- Plasma radius: \( \sim 1 \text{ cm} \)
- Frequency: \( < 100 \text{ kHz} \)
- Cesium ion mass: 133 amu
- Pressure: \( 5\times10^{-9} \text{ Torr} \)
- Ion source grid voltages: \( < 10 \text{ V} \)
Transverse Dynamics are the Same Including Self-Field Effects

If…

• Long coasting beams
• Beam radius << lattice period
• Motion in beam frame is nonrelativistic

Then, when in the beam frame, both systems have…

• Quadrupolar external forces
• Self-forces governed by a Poisson-like equation
• Distributions evolve according to nonlinear Vlasov-Maxwell equation

Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system in the beam frame.
Electrodes, Ion Source, and Collector

Broad flexibility in applying $V(t)$ to electrodes with arbitrary function generator.

Increasing source current creates plasmas with intense space-charge.

Large dynamic range using sensitive electrometer.

Measures average $Q(r)$.
If \( p = n kT \), then the statement of local force balance on a fluid element can be integrated over a radial density distribution such as,

\[
n(r) = n(0) \exp \left[ - \frac{m \omega_q^2 r^2 + 2q \phi^s(r)}{2kT} \right]
\]

to give the global force balance equation,

\[
m \omega_q^2 R^2 = 2kT + \frac{N q^2}{4 \pi \varepsilon_0}
\]

\[
s \equiv \frac{\omega_p^2}{2 \omega_q^2} < 1
\]

\[
\frac{\nu}{\nu_0} = (1 - s)^{1/2}
\]

\[
\omega_p^2 = \frac{n(0)q^2}{m \varepsilon_0}
\]

for a flat-top radial density distribution

<table>
<thead>
<tr>
<th>( s )</th>
<th>( \nu / \nu_0 )</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
<td>0.95</td>
</tr>
<tr>
<td>0.2</td>
<td>0.90</td>
</tr>
<tr>
<td>0.3</td>
<td>0.84</td>
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<tr>
<td>0.4</td>
<td>0.77</td>
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<tr>
<td>0.5</td>
<td>0.71</td>
</tr>
<tr>
<td>0.6</td>
<td>0.63</td>
</tr>
<tr>
<td>0.7</td>
<td>0.55</td>
</tr>
<tr>
<td>0.8</td>
<td>0.45</td>
</tr>
<tr>
<td>0.9</td>
<td>0.32</td>
</tr>
<tr>
<td>0.99</td>
<td>0.10</td>
</tr>
<tr>
<td>0.999</td>
<td>0.03</td>
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</table>

PTSX-accessible
Transverse Bunch Compression by Increasing $\omega_q$

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

If line density $N$ is constant and $kT$ doesn’t change too much, then increasing $\omega_q$ decreases $R$, and the bunch is compressed.

$$\omega_q = \frac{8eV_{0\text{ max}}}{m\pi r_w^2 f} \xi$$

Either
1.) increasing $V_{0\text{ max}}$ (increasing magnetic field strength) or
2.) decreasing $f$ (increasing the magnet spacing) increases $\omega_q$
Instantaneous Changes in the Voltage and Frequency Do Not Compress the Bunch When $\omega_q$ is Fixed

\[ m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_0} \]

- $s = 0.2$
- $kT \sim 0.7 \text{ eV}$
- $N \sim \text{constant}$

Baseline case

$V_{0\text{max}}$ & $f$ up to 1.5X

- $\sigma_v = 33^\circ$
- $\sigma_v = 50^\circ$
- $\sigma_v = 75^\circ$

$V_{0\text{max}}$ & $f$ down to 0.66X

- $\sigma_v = \frac{\omega_q}{f}$

- Baseline case
- $\omega_q = \text{const., } \sigma_v \text{ down}$
- $\omega_q = \text{const., } \sigma_v \text{ up}$

- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $\nu/v_0 = 0.88$
- $V_{0\text{max}} = 150 \text{ V}$
- $f = 60 \text{ kHz}$
- $\sigma_v = 49^\circ$
Adiabatic Amplitude Increases Transversely
Compress the Bunch

20% increase in $V_{0\text{ max}}$

90% increase in $V_{0\text{ max}}$

Baseline

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$R$</td>
<td>0.83 cm</td>
</tr>
<tr>
<td>$kT$</td>
<td>0.12 eV</td>
</tr>
<tr>
<td>$s$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\varepsilon \sim R\sqrt{kT}$</td>
<td>$\Delta \varepsilon = 10%$</td>
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</tbody>
</table>

Adiabatic

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>0.63 cm</td>
</tr>
<tr>
<td>$kT$</td>
<td>0.26 eV</td>
</tr>
<tr>
<td>$s$</td>
<td>0.10</td>
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<tr>
<td>$\Delta \varepsilon = 10%$</td>
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</table>

Instantaneous

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$R$</td>
<td>0.93 cm</td>
</tr>
<tr>
<td>$kT$</td>
<td>0.58 eV</td>
</tr>
<tr>
<td>$s$</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Delta \varepsilon = 140%$</td>
<td></td>
</tr>
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</table>

$\sigma_v = 63^\circ$

$\sigma_v = 111^\circ$

$\bullet s = \omega_p^2/2\omega_q^2 = 0.20 \quad \bullet \nu/\nu_0 = 0.88 \quad \bullet V_{0\text{ max}} = 150 \text{ V} \quad \bullet f = 60 \text{ kHz} \quad \bullet \sigma_v = 49^\circ$
Less Than Four Lattice Periods Adiabatically Compress the Bunch

\[ \sigma_v = 63^\circ \]
\[ \sigma_v = 81^\circ \]
\[ \sigma_v = 111^\circ \]

\[ s = \frac{\omega_p^2}{2\omega_q^2} = 0.20 \]
\[ \nu / \nu_0 = 0.88 \]
\[ V_{0_{\text{max}}} = 150 \text{ V} \]
\[ f = 60 \text{ kHz} \]
\[ \sigma_v = 49^\circ \]
2D WARP PIC Simulations Corroborate Adiabatic Transitions in Only Four Lattice Periods

Instantaneous Change.

Change Over Four Lattice Periods.
Peak Density Scales Linearly With $\omega_q$

\[ m\omega_q^2 R^2 \sim 2kT \]
\[ \varepsilon \sim R \sqrt{kT} \]
\[ \omega_q R^2 \sim \text{const.} \]

\[ n(0) R^2 \sim N = \text{const.} \]

\[ n(0) \sim \omega_q \]

\[ \omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2 f} \]

(a) Constant emittance

(b) Constant energy

Exact vacuum phase advance

Experimental results
- Instantaneous transition
- Adiabatic transition

Analytical estimates
- Instantaneous transition
- Adiabatic transition
Increasing $\omega_q$ by Adiabatically
Decreasing $f$

\[ V(t) = V_{0\text{max}} \sin \phi(t) \]

\[ \omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2 f} \xi \]

\[ \phi(t) = \frac{f_i + f_f}{2} t + \frac{f_f - f_i}{4} \tau \ln \left[ \cosh \left( \frac{t - t_{1/2}}{\tau/2} \right) \right] - f_0 \]
Increasing $\omega_q$ by Adiabatically
Decreasing $f$

$$V(t) = V_{0\text{max}} \sin \phi(t)$$

$$\omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2 f}$$

$$\phi(t) = \frac{f_i + f_f}{2} t + \frac{f_f - f_i}{4} \pi \ln \left( \cosh \frac{t-t_{1/2}}{\tau/2} \right) - f_0$$

$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} \left[ \tanh \frac{t-t_{1/2}}{\tau/2} \right] + 1$$
Adiabatically Decreasing $f$ Compresses the Bunch

- $s = \frac{\omega_p^2}{2\omega_q^2} = 0.2$
- $\nu / \nu_0 = 0.88$
- $V_{0\text{max}} = 150$ V
  - $f = 60$ kHz
  - $\sigma_v = 49^\circ$

Good agreement with KV-equivalent beam envelope solutions.

$\omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2 f} \xi$

On-axis charge when no transition is made.

33% decrease in $f$
Transverse Confinement is Lost When Single-Particle Orbits are Unstable

\[ \frac{\phi(t)}{2\pi} \]

\[ \tau = \tau_c \]

\[ f(t) = f_i t + \frac{f_i - f_f}{t_{f/2}} \left[ \tanh \frac{t - t_{f/2}}{\tau/2} + 1 \right] \]

\[ \sigma_v = \frac{\omega_q}{f} \propto \frac{1}{f^2} \]

Measured \( \tau_c \) (dots)

Set \( \sigma_{v_{\text{max}}} = 180^\circ \) and solve for \( \tau_c \) (line)
Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions

\[ f_0 = 60 \text{ kHz} \]

- \( \tau_{f_0} = 0 \)
- \( \tau_{f_0} = 19.9 \)
- \( \tau = \tau_c \)
- \( \tau_{f_0} = 26 \)
PTSX is a compact and flexible laboratory experiment.

PTSX has performed experiments on plasmas with normalized intensity $s$ up to 0.2.

Instantaneous changes can cause significant emittance growth and lead to halo particle production.

Adiabatic increases in $\omega_q$ approximately 100% can be applied over only four lattice periods.

The charge bunch will “follow” even non-monotonic changes in $\omega_q$. 