INTRODUCTION

Several methods for the particle beam cooling are in hand now: (i) radiation cooling, (ii) electron cooling, (iii) stochastic cooling (SC), (iv) optical stochastic cooling (OSC), (v) laser cooling, (vi) ionization cooling, and (vii) radiative (stimulated radiation) cooling [1-3]. Recently OSC and EOC extend the potential for fast cooling and can be successfully used in proton and ion colliders. OSC and EOC extend emittance exchange and OSC [8-10]. These ideas have to the maximal frequency of the first harmonic of the undulator radiation, \(\omega \sqrt{c^2 - K^2/M} \) in the smooth directions to do away with the unwanted part of URWs. The precision \(\delta \psi_{\text{ph}}^{\text{kick}}\) of the phase advance \(\psi_{\text{ph}}^{\text{kick}}\) is limited by the equation

\[
\langle \delta \theta \rangle_{\text{bet}} < \langle \Delta \theta \rangle_{\text{bet}} ,
\]

where \(\langle \delta \theta \rangle_{\text{bet}} = (2\pi A_{\text{z},\text{bet}} / \lambda_{\text{z},\text{bet}}) \sin(\delta \psi_{\text{ph}}^{\text{kick}})\) is the change of the angle between the electron average velocity in the kicker undulator owing to an error in the arrangement of undulators, \(A_{\text{z},\text{bet}}\) and \(\lambda_{\text{z},\text{bet}}\) are the amplitude and period of betatron oscillations of the electron, in the smooth approximation \(\delta \psi_{\text{ph}}^{\text{kick}} = 2\pi \Delta s / \lambda_{\text{z},\text{bet}}\). \(\Delta s\) is displacement of the kicker undulator from optimal position.

The number of photons in the URW emitted by electrons in suitable cooling frequency and angular ranges (1) is defined by the following formula

\[
N_{\text{ph}} = \frac{\Delta E_{\text{ph}}}{\hbar \omega_{\text{max}}} = \frac{\alpha \pi K^2}{1 + K^2} ,
\]

where \(\Delta E_{\text{ph}} = (dE_{\text{ph}}/d\omega)\Delta \omega = 3E_{\text{ph}}/2M (1 + K^2)^2\), \(\omega_{\text{max}} = 2\pi c / \lambda_{\text{ph}}\), \(\lambda_{\text{ph}} = \lambda_{\text{ph}} (1 + K^2 + \theta^2) / 2\theta^2\) is the wavelength of the first harmonic of the UR, \(\alpha = e^2/\hbar c \approx 1/137\) [11].

If the density of energy in the URWs is approximated by a Gaussian with a waist size higher then electron one \(\sigma_{\text{w}} > \sigma_{\text{ph}}\), \(Z_{\text{w}} > L_{\text{w}} / 2\), the R.M.S. electric field strength of the wavelet in the kicker undulator defined by

\[
E_{\text{ph}} = \sqrt{2\Delta E_{\text{ph}}} / \sqrt{\pi \sigma_{\text{ph}}\lambda_{\text{max}}} = \frac{2e \sqrt{c^2 - K^2}}{\sqrt{(1 + K^2)^2} \sqrt{\pi \sigma_{\text{ph}}}} ,
\]

where \(Z_{\text{w}} = 4\pi \sigma_{\text{w}} / \lambda_{\text{max}}\) is the Rayleigh length, \(L_{\text{w}} = M \lambda_{\text{w}}\).

The electric field value (4) is valid for \(N_{\text{ph}} >> 1\). Such case can be realized only for heavy ions with atomic number \(Z > 10\), \(K > 1\). If \(N_{\text{ph}} < 1\) then, according to quantum theory, one photon emitted with the probability \(P_{\text{em}} = N_{\text{ph}}\), and the energy \(\Delta E_{\text{ph}} = \hbar \omega_{\text{max}} = 1\) eV.

The maximum rate of energy losses for the electron in the fields of the kicker undulator and amplified URW is

\[
P_{\text{loss}} = -eE_{\text{ph}}^n / f / \beta_{z,\text{bet}}/N_{\text{av}} \Phi(\Phi(N_{\text{ph}})N_{\text{kic}}) / \alpha_{\text{amp}}^2 ,
\]

where \(\beta_{z,\text{bet}} = K / \gamma\); \(f\) is the revolution frequency; \(N_{\text{kic}}\) is the number of kicker undulators; and \(\alpha_{\text{amp}}\) is the gain in the optical amplifier. The function \(\Phi(N_{\text{ph}})\) is defined as

\[
\Phi(N_{\text{ph}}) \big|_{N_{\text{ph}} = 1} = 1 ,
\]

for \(N_{\text{ph}} > 1\).

The damping times for the longitudinal and transverse degrees of freedom are

\[
\tau_{\text{damp}}(z) = \frac{P_{\text{loss}}}{Mz^2} ,
\]

\[
\tau_{\text{damp}}(\gamma) = \frac{P_{\text{loss}}}{Mz^2} ,
\]
\[ \tau_{s,E0} = \frac{6\sigma_{s,0}}{P_{\text{loss}}} \], \[ \tau_{s,EXC} = \frac{\sigma_{s,0}}{P_{\text{loss}}} \eta_{s, \text{kick}} \]

where \( \sigma_{s,0} \) is the initial energy spread of the electron beam, \( \sigma_{s,0} = \sigma_{s,0} \) are the initial radial beam dimensions determined by betatron and synchrotron oscillations, \( \eta_{s, \text{kick}} \) is the dispersion function in the kicker undulator. The damping time for the longitudinal direction does not depend on \( \eta_{s, \text{kick}} \) and for the transverse one is inverse to \( \eta_{s, \text{kick}} \). Factor 6 takes into account that the initial energy spread is \( \sigma_{s,0} \), electrons do not interact with their URWs every turn, the jumps of the electron closed orbit lead to lesser jumps of the amplitude of synchrotron and betatron oscillations [6].

The average power of the optical amplifier is

\[ P_{\text{amp}} = \varepsilon_{\text{sample}} \cdot f \cdot N_e + P_n, \]

where \( \varepsilon_{\text{sample}} = h \alpha_{\text{max}} N_{\mu B} \Delta \omega_{\text{amp}} \) is the average energy in an URW, \( N_e \) stands for the number of electrons in the ring, \( P_n \) is the noise power.

The difference \( dt \) in the propagation time of the URW and the traveling time \( T_{\text{p,k}} \) of the electron between pickup and kicker undulators depends on initial conditions of electron’s trajectory which can be expressed as

\[ \begin{align*}
    dcdt & = c_1 - R_{11} (s,s_1) \cdot \lambda - R_{12} (s,s_1) \cdot \lambda_0 - \\
    & \Rightarrow R_{\eta}(s,s_1) \Delta E \beta^2 \varepsilon \left| \eta_{\text{pick}} \right| \Delta E \beta^2.
\end{align*} \]

The initial phase of an electron in the field of amplified URW propagating through kicker undulator is \( \varphi_{m} = \omega_{\text{max}} \Delta t \) and the rate of the energy loss

\[ P_{\text{loss}} = -|P_{\text{loss}}| \sin(\varphi_{m}) \cdot f(\Delta E), \]

where \( f(\Delta E) = \begin{cases} -1, & |\Delta E|/2\pi M, \text{if } |\varphi_m| < 2\pi M, \\
1, & |\Delta E| > 2\pi M, \end{cases} \)

if \( |\varphi_m| > 2\pi M \), \( \Delta E = E - E_d \). The function \( f(\Delta E) \) takes into account that electron with some energy \( E_d \) and its URW enter kicker undulator at the phase \( \varphi_{m} = 0 \) and passing together all undulator length at zero rate of the energy loss if \( c_1 = 0 \).

According to (9), electrons with different initial phases are accelerated or decelerated and gathered at phases \( \varphi_m = \pi + 2\pi m \left( -M \leq m \leq M, \ m = 0, \pm 1, \pm 2 \ldots M \right) \) and at energies

\[ E_n = E_d + \frac{(2m + 1)\pi b^2}{\alpha_{\text{max}} T_{p,k} \eta_{l}}, \]

if RF accelerating system is switched off (see Fig.2).

The energy gaps between equilibrium energy positions have magnitudes given by

\[ \delta E = E_{n+1} - E_n = \frac{\lambda_{\text{max}}}{\lambda_{p,k}} \eta_{l} b^2 E_d, \]

The power loss \( P_{\text{loss}} \) is the oscillatory function of the energy \( |E - E_d| \) with the amplitude linearly decreasing from the maximum value \( |P_{\text{loss}}| \) at the energy \( E = E_d \) to a zero one at the energy \( |E - E_d| \geq M \cdot \delta E \). If the particle energy falls into the energy range \( |E - E_d| < M \cdot \delta E \), it is drifting to the nearest energy value \( E_m \).

The variation of the particle’s energy looks like it produces aperiodic motion in one of \( 2M \) potential wells located one by one. The depth of the well decreases with their number \( |m| \). If the delay time in the optical line is changed, the energies \( E_m \) are changed as well.

![Figure 2: In the EOC scheme electrons are grouping near the phases \( \varphi_m = \pi + 2\pi m \) (energies \( E_m \)).](image)

Two variants of the EOC can be suggested [12].

1. If the local slippage factor \( \eta_{l} = 0 \), betatron oscillations are small, dispersion function in the pickup undulator \( \eta_{\text{pick}} \neq 0 \). In this case \( \delta \theta = \text{const} \) and the initial phase for all electrons can be installed \( \varphi_m = \pi/2 \). It corresponds to electrons arriving kicker undulator in decelerating phases of theirs URWs under maximum rate of energy loss. In this case electrons will be gathered near to the synchronous electron if a moving screen opens the way only to URWs emitted by electrons with the energy higher than synchronous one. This is the case of an EOC in the longitudinal and transverse plane based on isochronous bend and screening technique. If \( \eta_{\text{pick}} = 0 \) or if amplitudes of synchrotron oscillations of electrons are small (no selection in longitudinal plane) then the cooling in the transverse direction only takes place.

2. The local slippage factor \( \eta_{l} \neq 0 \), energy gaps have the magnitudes \( \delta E < \sigma_{s,0}/M \), the RF accelerating system of the storage ring is switched on, the screen absorbs URWs emitted by electrons at a negative radial deviation of theirs position from the synchronous one, energy layers are located at positive deviations from synchronous one outside the energy spread of the beam and optical system change the delay time of the URWs to move the energy layers to the synchronous energy then the energy layers capture small part of electrons of the beam first and electrons with smaller energy are captured increasingly and loose their energy and betatron amplitudes until reaching the synchronous energy. So the cooling process takes place. The variant 2 permits to avoid any changes in the existing lattice of the ring (isochronous bend, bypass). It can work for existing ion storage rings as well.

**Example.** Below we consider an example of one dimensional EOC of an electron beam in the transverse plane in 2.5 GeV storage ring like Siberia-2 [13].

After single bunch injection in the storage ring the energy 100 MeV is established and the beam is cooled by synchrotron radiation damping for ~40 seconds (see...
Table 1). The equilibrium energy spread is $\sigma_{E,0}^{eq,SR} / E = 3.94 \cdot 10^{-3}$, the length of the bunch $\sigma_x^{eq,SR} = 2.32$ cm at the amplitude of the accelerating voltage $V_0 = 73$ V, the synchronous voltage $V_s = 1.89$ V. Following synchrotron radiation damping, the amplitudes of radial betatron oscillations $\sigma_{x,0}$ are artificially excited to be suitable for resolution of the electron beam in the experiment. The amplitudes of synchrotron oscillations must stay damped to work with short electron bunches.

In the variants of the example considered below the optical system resolution of the beam is $\Delta x_{res} = 1.9$ mm at $\lambda_{min} = 2 \cdot 10^{-4}$ cm, $M \lambda_{max} = 240$ cm, the initial energy spread $\sigma_{E,0} = \sigma_{E,0}^{eq,SR} = 3.94 \cdot 10^{-3} E$, the dispersion beam size $\sigma_{y,0} = 3.15 \cdot 10^{-2}$ mm, the length of the electron bunch $\sigma_x^{eq,SR} = 4.64$ cm, its size at pickup undulator $\sigma_x \approx 4$ mm, the laser amplification length $L_{ampl}^{pl,\sigma} = 1.5$ mm ($L_{ampl}^{pl,\sigma} < \sigma_x$), the radial betatron beam size in kicker undulator $\sigma_x \approx 1$ mm, the URW beam size $\sigma_{w,K} = 2$ mm, the number of electrons at the orbit $N_{e,\pm} = 5 \cdot 10^4$, optical parametric amplifier (OPA) is used.

In this case the number of electrons in the URW sample is $N_{e,\pm} = 129.5$, the number of non-synchronous photons in the sample is $N_{ph,\pm} = 2.5$ for the case of one noise photon at the OPA front end. In this storage ring the natural local slippage factor is $\eta_s = \eta L_{p,k} / C = \alpha L_{p,k} / C = 4.45 \cdot 10^{-3}$, the energy gap (11) is $\delta E_{gap} = 0.62$ keV. The dispersion beam size $\sigma_{x,0} << \delta x_{res}$ and that is why there is no selection of electrons in the longitudinal plane. That is why in order to prevent heating in the longitudinal plane by energy jumps determined by both synchronous and non-synchronous photons in the URWs, two kicker undulators are used which produce zero total energy jump [4], [6]. We accept the distance between pickup and first kicker undulator along synchronous orbit $L_{p,k} = 72.27$ m, ($\psi_{opt} = 9$, $k_{p,k} = 4$). It corresponds to the installation of undulators in the first and seventh straight sections which are located at a distance 72.38 m (we count off pickup undulator). For transverse cooling second kicker undulator is located on the same distance from the first one. Optical line is tuned such a way that electrons are decelerated in the first kicker undulator and accelerated in the second one.

The URWs have the number of the photons emitted in the pickup undulator (see Table 3) $N_{ph} = 1.15 \cdot 10^{-2}$ per electron in the frequency and angular ranges (1) suitable for cooling. The limiting amplitude of betatron oscillations [12] is $A_{x,lim} = 3.2$ mm. The electric field strength in the kicker undulator is $E_{k,k}^2 \equiv 2.06 \cdot 10^{-3}$ V/cm.

The power loss for the electron passing through one kicker undulator together with its amplified URW comes to $P_{loss} = 2.03 \cdot 10^6$ eV/sec if the amplification gain of OPA is $\alpha_{ampl} = 10^7$ (see Tables 2, 4). This power loss corresponds to the maximal energy jumps $\Delta E_{max} = 73$ eV and the average energy loss per turn $\Delta E_{loss} = 0.84$ eV/sec. The jump of the closed orbits is $\delta x_{\pm} = 5.8 \cdot 10^{-5}$ cm. For the parameters presented above the cooling time for transverse coordinate in the the variant 1, according to (6), comes to $\tau_{x,EQC} = 18.5$ msec and for the variant 2 $\tau_{x,EQC} = 0.57$ sec.

REFERENCES


